Papers submitted to the Conference "Applications of Physics in Mechanical and Material Engineering"

Influence of Heat Exchange with Environment on the Results of Heat Transfer Simulations in Periodic Surface Structures

R. Dyja* and E. Gawronska

Czestochowa University of Technology, 42-201 Czestochowa, Poland

Doi: 10.12693/APhysPolA.142.44 *e-mail: robert.dyja@icis.pcz.pl

The presented paper focuses on a problem of heat transfer in periodic surface structures. The periodic structure surfaces are the compound structures described by mathematical formulas that represent areas similar to porous structures. The interest in them surged recently due to increased possibilities of their production using additive manufacturing technology. Elements designed with the use of periodic surface structures seem to have such advantages as decreased weight or good general mechanical properties. The most famous example of the periodic surface structure is gyroid. The main goal of this paper is to evaluate the impact of convective heat transfer on the temperature distribution in periodic surface structures. There are some papers with numerical simulations on this topic but they usually exclude heat exchange with the environment. The simulations of heat transfer in the periodic structure surfaces are important because of the high energy flux required for additive manufacturing to work properly. Importantly, high energy flux results in large temperature gradients that can affect the quality of the obtained part in properties like porosity or residual stresses. The heat transfer simulations in this paper are conducted by solving the heat transfer equation accompanied by the proper boundary conditions using our authorial software, which implements solution obtained by the finite element method together with the task of generation of the periodic surface structure domain.

topics: FEM, heat transfer, periodic surface structure, numerical simulation

1. Introduction

Periodic surface structures are geometrical entities that can fill a 3D space and typically split a finite volume into tightly interwoven surfaces so that they do not leave closed voids (open structure). Typically, periodic surface structures present better qualities in their structural and mechanical performance compared to structures mostly filled with a solid mass [1, 2]. Periodic surface structures are attracting more and more attention in the design community in applications such as high stiffness structures or medical bone implants [3]. Due to the complex shape of the usual periodic surface structures, the only possible way of manufacturing is to use additive manufacturing method, which is still an evolving technology. The process of creating an element in additive manufacturing technology is quite different, so the element designs that were previously considered difficult to make now become easy to manufacture [4, 5].

Examples of the periodic structure surfaces are the structure of gyroid [6, 7] or the Neovius surface [8]. While they have distinctly different shapes, some common properties are true for both of them, like smooth rib connections and no reflection symmetry or straight lines. The use of such structures in the design of elements offers such advantages as reducing the cost of the material used, direct acceleration of the production time of the element, reduction of the weight of the manufactured element, and thus greater flexibility in the design of elements [9].

In this article, numerical simulations are made with properties for a sample material Al–2%Cu. The paper deals with the thermal analysis of periodic surface structures, in which the structures were subjected to one-side heat flow. The advantage of the present study is also the assessment of the impact of heat exchange with the environment for such structures. The simulations were performed by solving the heat transfer equation using the finite element method (FEM), implemented in our own code developed in C++ programming language with the use of the PETSc numerical library. Our software module generated periodic surface structures using the CGAL library [10] and simulated the temperature distribution in them.

2. Periodic Surface Structures

In order to represent the periodic surface structures, a proper representation is needed. For surface representation, it is possible to use explicit, parametric, or implicit formulas. However, periodic surfaces are most naturally represented by an implicit formula.

Implicit surfaces are defined as a set of points satisfying the equation

$$f(x,y,z)=0, \tag{1}$$

where x, y, z are the spatial coordinates.

Using the above form, in this paper two periodic surface structures are considered. These are the gyroid (G) structure

$$\sin(x)\cos(y)+\sin(y)\cos(z)+\cos(x)\sin(z)=\delta, \tag{2}$$

and the Neovius (N) structure

$$3\big(\cos(x)+\cos(y)+\cos(z)\big) +4\cos(x)\cos(y)\cos(z)=\delta, \tag{3}$$

where δ specifies the thickness of the structure.

Both (2) and (3) have a form convenient for the preparation of finite element meshes necessary for numerical calculations. Finite element meshes were generated directly for a given surface (G or N), filling shape of a cube with the use of the CGAL library [10]. The CGAL project provides easy access to efficient and reliable geometric algorithms in a form of C++ library, which is often used in places where computational geometry algorithms are necessary, such as computer-aided design, medical imaging, computer graphics, and robotics.

3. Physical process description

The model of the physical process is based on the non-stationary heat transfer equation

$$\rho c\frac{\partial T}{\partial t}-k\nabla^2 T=0, \tag{4}$$

where ρ is the density, c is the specific heat capacity, k is the thermal conductivity, t is time, and T is the temperature.

Next, (4) is accompanied by an appropriate set of boundary conditions, where the Neumann boundary condition (boundary condition of II type) describes the heat flux flowing through the selected surface accordingly

$$-k\nabla T=q, \tag{5}$$

where q is the value of the heat flux density.

The Newton boundary condition (boundary condition of III type) represents the heat exchange with the environment on a selected surface

$$-k\nabla T=\alpha\left(T-T_{ot}\right), \tag{6}$$

where α is the heat exchange coefficient, T is the temperature of a body on the selected surface, and T_{ot} is the temperature of the environment.

TABLE I

Parameters used to generate different variants of the structures (G — gyroid, N — Neovius) together with the value of the Neumann boundary condition for a given case.

Name	Relative thickness	Volume ($\times 10^{-5}$) [m^3]	Side area ($\times 10^{-4}$) [m^2]	Heat flux $\left[\frac{W}{m^2}\right]$
cube	–	6.4	16	6250
N, 2 cells	0.15	1.678	3.578	27950
N, 2 cells	0.2	2.232	5.591	27950
N, 4 cells	0.15	1.686	3.419	29251
N, 4 cells	0.2	2.244	5.532	12170
G, 2 cells	0.15	2.490	5.155	19400
G, 2 cells	0.2	3.343	6.603	13731
G, 4 cells	0.15	2.492	5.012	19952
G, 4 cells	0.2	3.347	5.876	13844

Note that together with the boundary conditions (5) and (6), (4) is accompanied by the initial condition

$$T=T_0, \tag{7}$$

where T_0 is the initial temperature.

We solve (4) numerically with the use of FEM, where the necessary integration time is performed using the backward Euler scheme.

4. Simulations setup and results

The conducted simulations present results of the heat conduction problem in various domains, in which there is also heat exchange with the environment (in form of convection). The simulations were carried out for a cube, gyroid, and Neovius structures — all with a side length of 0.04 m. The following physical properties for the Al–2%Cu alloy were used in the simulations: the heat transfer coefficient $k = 260$ W/(m K), the density $\rho = 2800$ kg/m^3, and the specific heat $c = 1000$ J/(kg K).

Boundary conditions were imposed in the following way. One side of the region had forced heat flux with the Neumann boundary condition from (5). The exact values of the heat flux are summarized in Table I. The heat flux values for the Neumann boundary condition are chosen to obtain the same amount of energy that flows into the region, regardless of exact side area of given structure. For this reason, the amount of energy that flows into the region is always constant, i.e., $q = 10$ W.

For the gyroid and Neovius structures, the Newton boundary condition (6) was imposed on the internal surfaces (all surfaces that were not formed by the intersection of the given structure with a cube of a side length of 0.04 m). The following parameters were used in Newton's boundary condition: the environment temperature

TABLE II

Minimum and maximum temperatures obtained for different variants of heat transfer simulations.

Name	Relative thickness	Heat exchange coeff. $\left[\frac{\mathrm{W}}{\mathrm{m^2\,K}}\right]$	$\min T$ [K]	$\max T$ [K]
cube	–	0	313.8	314.3
	–	10	312.9	313.4
	–	100	307.4	308.1
N, 2 cells	0.15	0	360.3	363.5
	0.15	10	319.9	323.1
	0.15	100	301.5	304.2
N, 2 cells	0.2	0	343.0	345.3
	0.2	10	330.8	333.1
	0.2	100	305.6	307.8
N, 4 cells	0.15	0	367.3	370.9
	0.15	10	302.5	305.6
	0.15	100	300.0	301.7
N, 4 cells	0.2	0	348.0	350.5
	0.2	10	305.5	307.9
	0.2	100	300.2	301.9
G, 2 cells	0.15	0	338.7	341.3
	0.15	10	328.0	330.6
	0.15	100	305.2	307.7
G, 2 cells	0.2	0	327.8	329.5
	0.2	10	322.1	323.8
	0.2	100	305.6	307.2
G, 4 cells	0.15	0	342.3	344.7
	0.15	10	322.6	325.0
	0.15	100	302.5	304.7
G, 4 cells	0.2	0	329.3	331.0
	0.2	10	318.9	320.5
	0.2	100	302.7	304.3

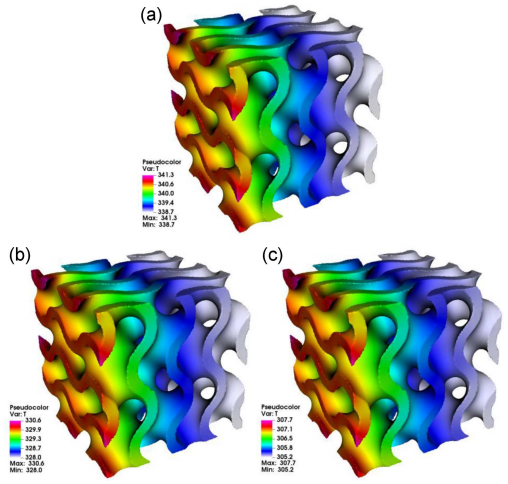

Fig. 1. Example of temperature field in gyroid after 250 s with 2 cells and the relative thickness $\delta = 0.15$. (a) No heat exchange through convection, (b) the heat exchange coefficient $\alpha = 10$ W/(m^2 K), and (c) the heat exchange coefficient $\alpha = 100$ W/(m^2 K).

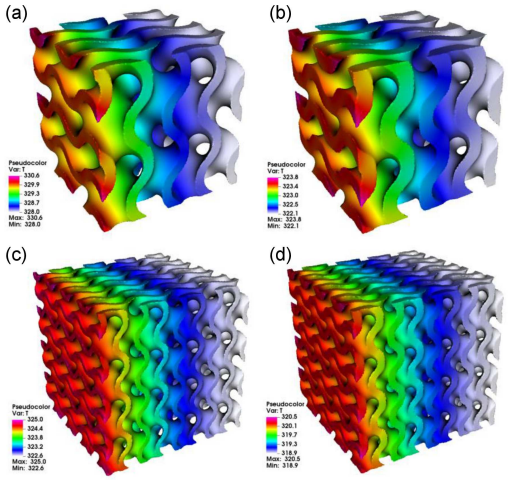

Fig. 2. Example of temperature field in gyroid after 250 s for a structure (a) with 2 cells and relative thickeness $\delta = 0.15$, (b) with 2 cells and $\delta = 0.2$, (c) with 4 cells and $\delta = 0.15$, and (d) with 4 cells and $\delta = 0.2$. The temperature profiles were obtained with the heat exchange coefficient value of $\alpha = 10$ W/(m^2 K).

$T_{ot} = 300$ K and the heat exchange coefficient equal to $\alpha = 0$ W/(m^2 K) (no convection), $\alpha = 10$ W/(m^2 K) or $\alpha = 100$ W/(m^2 K). The α values were chosen to reflect free and forced convection.

For the cube, the Newton boundary condition was imposed only on the remaining five external surfaces. Additionally, the simulations results with the Newton boundary condition turned on were confronted with the case where no heat exchange with the environment occurred.

The initial temperature for all cases was $T_0 = 300$ K. The presented results show the temperature distribution at time $t = 250$ s.

In the current setup, finite elements were generated only for the structures. The empty space was left with no finite elements because the heat exchange with the environment was simulated only in a simplified way by the Newton boundary condition, without using the Navier–Stokes equations.

All simulations were conducted with heat equation solved numerically with the finite element method. All FEM meshes used tetrahedral linear finite elements. The number of finite elements was equal to 261824 for a cube, ranged from 1496875 to 2445280 for Neovius, and from 1720478 to 2727576 for gyroid. The increased number of finite elements for Neovius and gyroid was caused by the complex shapes of these structures and the difficulties in correctly representing them using a smaller number of finite elements.

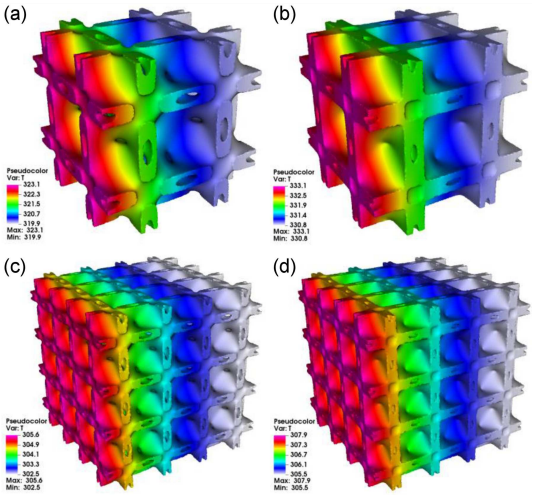

Fig. 3. Example of temperature field in Neovius after 250 s for a structure (a) with 2 cells and relative thickeness $\delta = 0.15$, (b) with 2 cells and $\delta = 0.2$, (c) with 4 cells and $\delta = 0.15$, and (d) with 4 cells and $\delta = 0.2$. The temperature profiles were obtained with the heat exchange coefficient value of $\alpha = 10$ W/(m^2 K).

Figures 1 and 2 show examples of the temperature profiles for the Neovius and gyroid structures with 2 or 4 cells and different thicknesses. The temperature profiles are presented for the case of $\alpha = 10$ W/(m^2 K). Table II summarizes maximum, minimum and difference temperature for all analyzed cases (i.e., no convection, $\alpha = 10$ W/(m^2 K) or $\alpha = 100$ W/(m^2 K)) and for a cube.

In general, the temperature profiles in Figs. 2 and 3 resemble the temperature profiles for a case with no convection. However, the heat exchange coefficient has a great impact on the values of maximum and minimum temperature. The difference between the maximum and minimum temperature values remains roughly the same, regardless of the value of heat exchange coefficient. It should be noted that a cube shows much less impact of the heat exchange coefficient on temperature than the periodic surface structures, where in some cases the minimum temperature could even be equal to the environment temperature.

5. Conclusions

The value of the heat exchange coefficient in the Newton boundary condition has a visible impact on the temperature profiles obtained for periodic surface structures. It is noticeably bigger than it is in the case of a simple cube. This observation most probably emerges from fact that periodic surface structures usually have a complicated shape of internal surfaces, which increases the surface on which heat exchange can occur. An additional observation may be that the differences between the maximum and minimum temperatures for a given shape remain generally the same, regardless of the value of the heat exchange coefficient.

Previous studies on heat transfer in periodic structures did not take into account the heat exchange with the environment in any form. This paper presents one of the first attempts to take this phenomenon into account. The authors are aware of the limitations of the chosen approach. The study shows that the heat exchange with the environment has a much bigger impact in the periodic structures than in a simple shape like a cube. For this reason, it will be very valuable to perform simulations using the Navier–Stokes equation to fully simulate the convection in order to add more details about the nature of heat exchange in the periodic structure surfaces.

References

[1] D.W. Abueidda, M. Bakir, R.K. Abu Al-Rub, J.S. Bergström, N.A. Sobh, I. Jasiuk, *Mater. Des.* **122**, 255 (2017).

[2] M. Sychov, L. Lebedev, S.V. Dyachenko, L.A. Nefedova, *Acta Astronaut.* **150**, 81 (2017).

[3] S.C. Kapfer, S.T. Hyde, K. Mecke, C.H. Arns, G.E. Schroder-Turk, *Biomaterials* **32**, 6875 (2011).

[4] C. Yan, L. Hao, A. Hussein, D. Raymont, *Int. J. Mach. Tools Manuf.* **62**, 32 (2012).

[5] K.V. Wong, A. Hernandez, *Int. Sch. Res. Notices* **2012**, 208760 (2012).

[6] E. Gawronska, R. Dyja, *Materials* **14**, 427 (2021).

[7] C. Ya, L. Hao, A. Hussein, P. Young, *J. Mech. Behav. Biomed. Mater.* **51**, 61 (2015).

[8] E.R. Neovius, *Bestimmung zweier specieller periodischer Minimalflachen, aufwelchen unendlich viele gerade Linien und unendlich viele ebene geodatische Linienliegen*, J.C. Frenckell & Sohn, Helsingsfors, 1883 (in German).

[9] X. Wang, C. Wang, X. Zhou, D. Wang, M. Zhang, Y. Gao, L. Wang, P. Zhang, *Materials* **13**, 4786 (2020).

[10] CGAL software, www.cgal.org.