

# Experimental Determination Features of Acousto–Optical Figure of Merit for Crystalline Materials

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The paper specifies the Dixon–Cohen method for the experimental determination of the acousto–optical figure of merit  $M_2$ . In particular, the conditions are formulated when the Dixon–Cohen method application for  $M_2$  determination is possible without significant errors. Cases are considered when the condition of the “weak field” in the studied material is not satisfied, as well as when the transmission of the studied and reference materials is less than 100%. The error in determining the acousto–optical figure of merit  $M_2$  in the case of unenlightened test material, using  $\text{CaWO}_4$  and GaP crystals as an example, is estimated, and the limits of application of the “weak field” approximation in these two crystals are established.

topics: figure of merit, Dixon–Cohen method, Bragg diffraction

## 1. Introduction

Acousto–optical devices are widely used in various fields of science and technology to control the parameters of optical radiation [1]. One of the main parameters of such devices is the efficiency of the acousto–optical interaction. For the diffraction efficiency of the acousto–optical interaction, in the case of the interaction of plane–acoustic and plane–optical waves, the following equation was obtained [1] (the case of Bragg diffraction)

$$\eta = \sin^2 \left( \frac{\pi}{\lambda} \sqrt{\frac{l M_2 P_a}{2h}} \right), \quad (1)$$

where  $\lambda$  is the wavelength of incident light,  $l$  is the path of acoustic and optical beams interaction,  $h$  is the optical beam width,  $P_a$  is the power of the acoustic wave propagating in the light–sound conductor material, and  $M_2$  is a figure of merit of the studied material.

Note that (1) is valid when  $\cos(\theta_B) \approx 1$ , where  $\theta_B$  is the Bragg angle. The condition  $\cos(\theta_B) \approx 1$  is satisfied in the case when the frequencies of the acoustic waves do not exceed 400 MHz. It follows from (1) that the larger the value of  $M_2$ , the smaller the value of the acoustic power consumption  $P_a$  is required to obtain a certain value of the diffraction efficiency  $\eta$  at other constant parameters. Therefore, the calculation of  $M_2$  on the basis of filled matrices of piezooptic coefficients in the studied crystal (see as an example [2–5]) and its experimental determination are important tasks for acousto–optics.

The common method of experimental determination of  $M_2$  is the method of comparing the acousto–optical properties of the test and reference materials — the Dixon–Cohen method [6, 7]. However, the formula for calculating  $M_2$  of the studied material, based on experimental data, is given in [6, 7] without derivation, and the calculation in [8] does not give a complete understanding of the conditions of using the formula.

The present study is devoted to the refinement of the Dixon–Cohen method for the experimental determination of the acousto–optical figure of merit for crystalline materials.

## 2. Experimental measurements of $M_2$ by the Dixon–Cohen method

In the Dixon–Cohen method, the test sample is glued to the reference sample with a piezoelectric transducer. The test piece is glued to the end face of the reference sample, opposite to the face on which the piezoelectric transducer is glued. Fused quartz is usually used as a reference sample. When determining the coefficient  $M_2$  of the test material, the intensity of diffracted light in the reference and test samples is registered when the acoustic pulse passes through them in the forward ( $I_{Q1}, I_{S3}$ ) and back ( $I_{S4}, I_{Q5}$ ) directions after reflection from the free face of the test material. Here  $I_{Q1}, I_{Q5}$  and  $I_{S3}, I_{S4}$  are the intensities of diffracted light passed through the reference material (with generally used indices 1 and 5) and through the test sample (with generally used indices 3 and 4).

If the measurements of the intensities of diffracted light for the forward and reverse acoustic pulses are carried out at the same points of the samples, the following expression is proposed to find  $M_2$  [5, 6]

$$M_{2S} = M_{2Q} \sqrt{\frac{I_{S3} I_{S4}}{I_{Q1} I_{Q5}}}. \quad (2)$$

The Dixon-Cohen method in the “weak field” approximation is grounded in [9]. When Bragg diffraction is satisfied, we have the expression for the diffraction efficiency  $\eta$  in the case of “weak field” [1]

$$\eta = \frac{\pi^2}{2\lambda^2} \frac{P_0 l}{h} M_2. \quad (3)$$

Note that (3) is satisfied when the condition  $\eta \leq 40\%$  [1] is valid. Let us formulate the conditions for calculation of  $M_2$  by the Dixon-Cohen method in the case of Bragg diffraction. We can use (2) without significant errors [9] when:

- the transmission of optical radiation by the reference and test samples is equal to 100%;
- the value of the diffraction efficiency  $\eta$  both in the reference sample and in the test material satisfies the condition  $\eta \leq 40\%$ ;
- the intensity of incident radiation in the measurement process is constant  $I_0 = \text{const}$  during the diffraction efficiency measurement;
- the wear angle  $\xi$  of the acoustic wave in the test material must be zero ( $\xi = 0^\circ$ );
- the faces of the reference and test samples must be strictly parallel. Non-parallelism of the faces must be less than  $10''$ ;
- the faces must be polished and there are no scattering centers;
- the polarization of the incident optical radiation must be recorded and must not be changed during the measurement process.

The dimensions of the reference sample and the test samples must satisfy the condition that the acoustic wave excited by the piezoelectric transducer propagates in the shadow region. This condition looks as

$$d \leq \frac{0.2 L_{\min}^2}{\Lambda}. \quad (4)$$

Here  $d$  is the total length of the standard and the test sample in the direction of propagation of the acoustic wave,  $L_{\min}$  is the minimum length of the piezoelectric transducer,  $\Lambda$  is the acoustic wavelength.

Let us consider the case when the Bragg diffraction condition for the value of diffraction efficiency ( $\eta > 40\%$ ) is not satisfied in the test material. This case is realized under the condition  $M_{2S} \gg M_{2Q}$ . When determining  $M_{2S}$ , if the approach is based on the “weak field” approximation, two buffers are used, i.e., one of fused quartz and the other of a material with a much higher figure of merit  $M_2$  [10–12]. This results in increased costs and time as well as increased in errors in determining  $M_{2S}$ .

Next, we assume that all other conditions formed by us for the Dixon-Cohen method are satisfied. Using (1) and for the condition  $\eta \leq 40\%$  in [9], we determine the values of acoustic power at the points of experimental measurements, namely

$$P_i = \left( \frac{\pi^2}{2\lambda^2} \frac{l}{h} M_{2S} \right)^{-1} \arcsin(\sqrt{\eta_i}), \quad (5)$$

for  $i = 3, 4$ , and

$$P_5 = \left( \frac{\pi^2}{2\lambda^2} \frac{l}{h} M_{2Q} \right)^{-1} \eta_5. \quad (6)$$

If the total losses on attenuation and reflection during the acoustic wave propagation through the interface between the buffer and the test material is denoted by  $\gamma$ , then the power at points  $i = 3$  and 4 can be written as

$$P_i = P_1 (1 - \gamma). \quad (7)$$

For point 5, we have

$$P_5 = P_1 (1 - \gamma)^2. \quad (8)$$

From the relations  $\frac{P_3}{P_1}$  and  $\frac{P_5}{P_1}$ , we obtain the formula for calculating the coefficient  $M_{2S}$  assuming  $M_{2S} \gg M_{2Q}$ , and thus

$$M_{2S} = M_{2Q} \sqrt{\frac{\arcsin(\sqrt{\eta_3}) \arcsin(\sqrt{\eta_4})}{\eta_1 \eta_5}}. \quad (9)$$

The proposed approach allows to determine the figure of merit of the studied material under the condition  $M_{2S} \gg M_{2Q}$  using only one buffer of fused quartz.

Let us consider another case where the transmission of optical radiation of the reference and test samples is different from 100%. In this case, the reflectance  $R$  and the absorption coefficient  $\alpha$  for optical radiation of the reference and test samples are non-zero. Then, we can use the approach proposed in [13] for the values of diffraction efficiencies  $\eta$ 's in Raman-Nath diffraction, and write them in the approximation of the “weak field” at the points of experimental measurements. Therefore,

$$\eta_i = (1 - R_Q)^2 e^{-\alpha_Q d_0} \left( \frac{\pi^2}{2\lambda^2} \frac{P_i l}{h} M_{2Q} \right), \quad (10)$$

$$\eta_j = (1 - R_S)^2 e^{-\alpha_S d_S} \left( \frac{\pi^2}{2\lambda^2} \frac{P_j l}{h} M_{2S} \right), \quad (11)$$

for  $i = 1, 5$ ,  $j = 3, 4$ . Now,  $R_{Q,S} = \left( \frac{n_{Q,S} - 1}{n_{Q,S} + 1} \right)^2$  is the reflectance of optical radiation from the reference and test samples,  $\alpha_Q, d_Q$  and  $\alpha_S, d_S$  — absorption coefficients and thicknesses of the reference and test samples, respectively. Let us denote  $(1 - R_Q)^2 e^{-\alpha_Q d_0} = T_Q$  and  $(1 - R_S)^2 e^{-\alpha_S d_S} = T_S$ . It is obvious that  $T_Q$  and  $T_S$  are the transmittance of optical radiation through the reference and test materials in the absence of an acoustic field. In the case for the value of  $M_{2S}$  when  $\eta \leq 40\%$ , we obtain

$$M_{2S} = \frac{T_Q}{T_S} M_{2Q} \sqrt{\frac{I_{S3} I_{S4}}{I_{Q1} I_{Q5}}}. \quad (12)$$

When  $\alpha_0 = 0$  and  $\alpha_S = 0$ , (12) coincides with the expression given in [11].

TABLE I

Ratio of acousto-optical figure of merit determined from relations (2) and (12) for CaWO<sub>4</sub> and GaP crystals. Here,  $n_S$  indicates the refractive index of the test material and  $n_Q$  is the refractive index of the buffer SiO<sub>2</sub>. Note: \*Ref. [14] (for  $\lambda = 0.63 \mu\text{m}$ )

| Material          | $n_S$ | $n_Q$ | $(1 - R_S)^2$ | $M_{2S}^{(1)}/M_{2S}^{(2)}$ |
|-------------------|-------|-------|---------------|-----------------------------|
| CaWO <sub>4</sub> | 1.92* | 1.50* | 0.81          | 0.88                        |
| GaP               | 3.31* |       | 0.51          | 0.55                        |

Let us calculate what will be the error in determining  $M_2$  when the test samples are used without translucent coatings (transmission < 100%), using (2), not (12). To do this, find the following relationship

$$\frac{M_{2S}^{(1)}}{M_{2S}^{(2)}} = \frac{T_S}{T_Q} = \frac{(1 - R_S)^2}{(1 - R_Q)^2}. \quad (13)$$

Here  $M_{2S}^{(1)}$  and  $M_{2S}^{(2)}$  are the coefficients of the acousto-optical figure of merit for the test sample calculated by (2) and (12), respectively. Table I shows the values of the ratio  $M_{2S}^{(1)}/M_{2S}^{(2)}$  for some materials.

It follows from Table I that with the increasing refractive index of the studied unenlightened materials, the error in determining  $M_2$  by (3) increases. For GaP, this error is already  $\sim 50\%$ . In addition, in the study of  $M_2$  for unenlightened materials, the “weak field” condition for Bragg diffraction changes from  $\eta \leq 40\%$  to  $\eta \leq (1 - R_S)^2 \times 40\%$ , and for Raman-Natt diffraction according to our calculations to  $\eta \leq (1 - R_Q)^2 \times 12\%$ . Thus, for the considered materials, the “weak field” condition for Bragg diffraction has the form of  $\eta \leq 32\%$  and  $\eta \leq 20\%$  for CaWO<sub>4</sub> and GaP, respectively.

This fact must be taken into account when determining the the acousto-optical figure of merit  $M_2$  for the studied unenlightened samples.

As defined in (12),  $T_Q = I_{np.Q}/I_0$ ,  $T_S = I_{np.S}/I_0$ , where  $I_{np.Q}$  and  $I_{np.S}$  are the intensities of light that propagates through the reference and test samples, respectively, at the measurement points in the absence of an acoustic wave. Therefore, (12) can be rewritten as

$$M_{2S} = \frac{I_{np.Q}}{I_{np.S}} M_{2Q} \sqrt{\frac{I_{S3} I_{S4}}{I_{Q1} I_{Q5}}}. \quad (14)$$

Note that (14) is easier to use compared to (12), because it does not require the values of refractive indices of materials of both the reference and test samples, and also allows us to use the value of light intensities of the reference and test samples at measurement points. These measurements are performed simultaneously with the measurement of the intensities of the diffracted beams at specified points in the absence of acoustic waves. This reduces the error and requires the calculation of  $M_2$  of the test material. It should be noted that (12) and (14) are derived when considering the diffraction of a light

beam on an acoustic wave in the case of Raman-Nath diffraction. Their use in the case of Bragg diffraction, as done in [14], is not obvious since there is no strict consideration of diffraction of light on an acoustic wave, taking into account the processes of light reflection. The question of the application of (12) and (14) in the case of Bragg diffraction, and at the same time what error we make when calculating  $M_2$  of the studied material, must be solved experimentally.

### 3. Conclusions

The paper proposes a formula for calculating the figure of merit  $M_2$  when the condition of “weak field” in the studied material is not satisfied. A case is also considered and formulas are derived when the transmission of the investigated and reference samples is different from 100%. The formulae for the calculation of  $M_2$ , which relates to this case, are derived. The error in determining the acousto-optical figure of merit  $M_2$  is estimated in the case of unenlightened test material, using CaWO<sub>4</sub> and GaP crystals as an example, and the limits of application of the “weak field” approximation in these two crystals are established.

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