Analytical Approach to Temperature Distribution in Current Leads to Superconducting Electromagnets

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Received: 23.04.2021 & Accepted: 06.08.2021

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In this paper, an analytical approach to the temperature profile in the normal current leads to the superconducting electromagnets has been proposed. The heat diffusion equation was solved for a one-dimensional case. The static and dynamic temperature distribution in linear leads through which the current flows has been determined. The influence of the thermal intercepts on the current lead temperature distribution during current flow has been investigated, especially the temperature and position of the intercept, and the current lead cross-section.

topics: superconducting electromagnets, current leads, temperature profile, energy losses

1. Introduction

Nowadays, large nuclear physics scientific laboratories are constructed in which the most important physical discoveries are made with the use of advanced, huge specialized apparatus constructions. The famous examples are large colliders, such as LHC in CERN, which allowed to find the Higgs boson. The discoveries of the excited physical phenomena in these modern accelerators used in nuclear physics are possible due to large rings of superconducting electromagnets, controlling the ionized bunch movement, which are equipped in current leads, delivering the current to them. These devices containing superconducting electromagnets have huge circumferences, e.g., 27 km in the case of LHC in CERN. The transport current there reaches even 40 kA, which shows the importance of the proper construction of the current leads to these high power devices [1–9].

In the present paper, we have investigated low current density normal conducting leads to small electromagnets with the stored energy of several kJ, which are usually constructed in the form of monolithic wire, additionally cooled by returning helium vapor to the recovery system. High current density current leads have more advanced structure and are built in the hybrid form of the normal metal-HTc superconductor. A thermal intercept, considered in the present paper, is also used, decreasing the heat load at the cold end of the lead. Such a type of the current lead is used in [1] while the temperature of the intercept is dependent then on the efficiency of the helium recovery system. Physical phenomena occurring in the materials from which current leads are constructed are of the same nature as in other materials of the condensed matter physics, such as semiconductors, magnets and insulators, while their analysis belongs, therefore, to the applied physics. In the case of the current leads for nuclear accelerators, they particularly concern the heat conductivity issues, which determine energy losses generated in cryogenic temperatures during the work of the superconducting coils. The analysis of the heat conductivity process in cryogenic current leads allows to determine their proper work conditions as an important part of the superconducting electromagnets. On the other hand, the analysis presented here has a purely scientific meaning, because heat conductivity is one of the most significant processes in classical physics. It was the base for formulating the famous transform by Joseph Fourier, the essential tool for scientists, from mathematicians and physicists to engineers.

Therefore, the issues considered in this paper are both of technical and applied physics significance. The presented research is directed at finding an analytical solution to the temperature profiles in the resistive, copper current leads, while composite normal metal-HTc superconductors current leads will be considered in the next paper. While these kinds of issues are usually solved in a numerical way using computer codes, especially for the 3D case, in the present paper, the analytic approach is adopted.

This approach should allow to follow directly the influence of geometrical and thermal parameters on the temperature distribution in the linear current lead, determining the energy losses generated then.

2. Basic equations

The temperature distribution inside the current lead is described by the general heat diffusion equation

$$\rho c_m \frac{\partial T}{\partial t} = \lambda \nabla^2 T, \tag{1}$$

where T is the temperature, ρ is the density, c_m is the specific heat per unit mass, λ is the heat conductivity constant, and t is the time. This equation is then reduced by inserting the heat diffusion constant

$$D = \frac{\lambda}{\rho c_m} \tag{2}$$

to the following form

$$\frac{\partial T}{\partial t} = D \nabla^2 T. \tag{3}$$

For the copper-made current leads, the numerical values of the material parameters used in calculations of the temperature profile have been taken as equal to $\rho = 8.96$ kg/m^3, $c_m = 386$ J/(kg K) in room temperature, $\lambda = 400$ W/(m K). With them, the diffusion constant $D = 1.2 \times 10^{-4}$ m^2/s at room temperature was deduced. The specific heat of copper is very sensitive to temperature, for instance, at 4 K it is equal to 0.09 J/(kg K), while at 300 K it significantly increases to the value $c_p(300\text{ K}) = 386$ J/(kg K) [7]. However, such an increase of D value does not influence the results of the present considerations. Note that in the static approach the term containing the coefficient D will not occur, while in a dynamic case discussed later, when time approaches zero, it leads to the value of the exponent equal to unity, for arbitrary D. On the other hand, strong temperature variation of the resistivity has been explicitly taken into account.

In the present analysis, (1) has been extended next onto the case of the heat Q generated in a time unit in the current lead to the form, which in the Cartesian coordinate system is expressed by (4) for volume dV

$$\rho c_m \frac{\partial T}{\partial t} \mathrm{d}V = \left[\frac{\partial T}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial T}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial T}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) \right] \mathrm{d}V + q \mathrm{d}V. \tag{4}$$

Here, parameter q is introduced — describing the local heat density related to the total heat generated in the sample Q at the time unit as

$$Q = \int \mathrm{d}V \, q. \tag{5}$$

For a one-dimensional case and current lead volume V we therefore obtain

$$\rho c_m V \frac{\partial T}{\partial t} = \lambda V \frac{\partial^2 T}{\partial x^2} + Q. \tag{6}$$

3. Analysis of thermal model in stationary case

In stationary conditions, for which the temperature profile in the current leads has already been established, in the one-dimensional, homogeneous case (4) — after integration — is reduced to the form

$$-Q = \lambda V \frac{\partial^2 T}{\partial x^2}. \tag{7}$$

The solution of (7) is received then in the general parabolic form

$$T(x) = -\frac{Q}{2\lambda_v} x^2 + bx + c, \tag{8}$$

where the new parameter $\lambda_v = \lambda V$ is introduced. After determining the coefficients b and c, (8) is transformed into the following expression

$$T(x) = -\frac{Q}{2\lambda_v} x^2 + \left(T_1 - T_0 + \frac{Q}{2\lambda_v} x_1^2 \right) \frac{x}{x_1} + T_0. \tag{9}$$

In (9), T_0 is the temperature of the warm end of the current lead, while T_1 is the temperature at the distance x_1, usually the point of thermal intercept or temperature of the cold end of the lead. Thus, for the simple current lead without thermal intercept, x_1 is equal to its length, while S is the cross-section. The heat Q generated in the current lead, through which the current I flows, is given by the coupled relation

$$Q = \frac{I^2}{S} \frac{\partial \rho}{\partial T} \int_0^{x_1} \mathrm{d}x \, T(x) = \left[-\frac{Q x_1^3}{6\lambda_v} + \frac{1}{2} \left(T_1 - T_0 + \frac{Q}{2\lambda_v} x_1^2 \right) x_1 + T_0 x_1 \right] \propto. \tag{10}$$

For the constant derivative $\frac{\partial \rho}{\partial T}$ resistivity ρ versus temperature T, the specific property of normal metals, the solution of (10) is

$$Q = \frac{(T_1 + T_0) x_1}{2} \left(\frac{1}{\propto} - \frac{x_1^3}{12\lambda_v} \right)^{-1}, \tag{11}$$

where parameter α is given as

$$\propto = \frac{I^2}{S} \frac{\partial \rho}{\partial T}. \tag{12}$$

The results of calculations of temperature distribution in the normal current lead in this stationary model are shown in Figs. 1–6.

Figure 1 shows the influence of the current on the temperature profile of the copper current lead. Quite high values of the temperature seen in this figure are connected with the current amplitude and result from the fact that the case of a vacuum cryostat is considered in the model. Then, cooling of the wire takes place through both ends only, while not through the surface.

Figure 2 demonstrates the comparison of the temperature profiles for the simple current lead and the lead with the thermal intercept at 80 K. A reduction of the temperature of the current lead is then well seen in the second case, and this effect is the reason for using such a construction of leads [1]

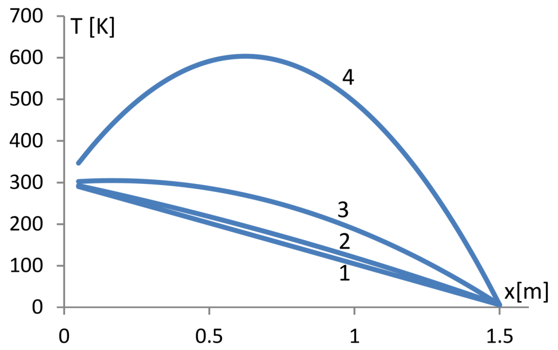

Fig. 1. Influence of the transport current on the temperature profile at the current lead, without intermediate thermal intercept: 1 — 1 A, 2 — 5 A, 3 — 10 A, 4 — 15 A.

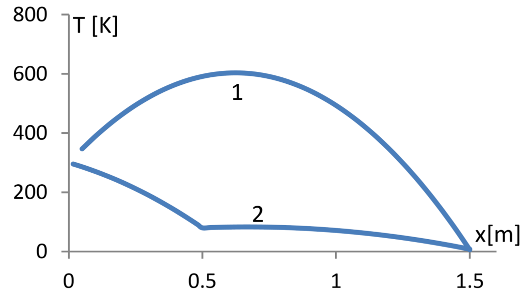

Fig. 2. The comparison of the temperature profile in the current lead for $I = 15$ A: 1 — current lead without a thermal intercept, 2 — current lead with a thermal intercept.

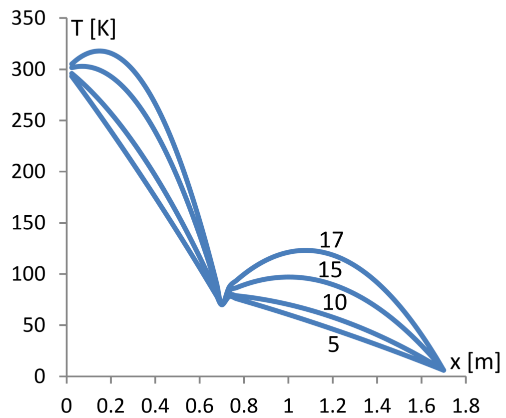

Fig. 3. Temperature distribution in the current lead with the thermal intercept at 70 K for various currents, i.e., 5, 10, 15 and 17 A.

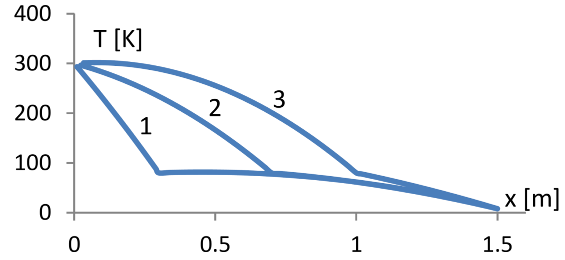

Fig. 4. The dependence of the calculated temperature profile of the current lead as the function of the position of the thermal intercept at 80 K: 1 — $x_1 = 0.3$ m, 2 — 0.7 m, 3 — 1 m.

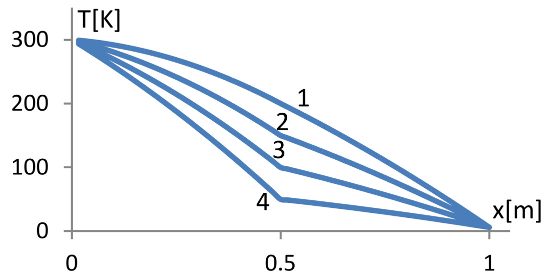

Fig. 5. The dependence of the calculated temperature profile at the copper current lead as the function of the temperature of the thermal intercept: 1 — 200 K, 2 — 150 K, 3 — 100 K, 4 — 50 K.

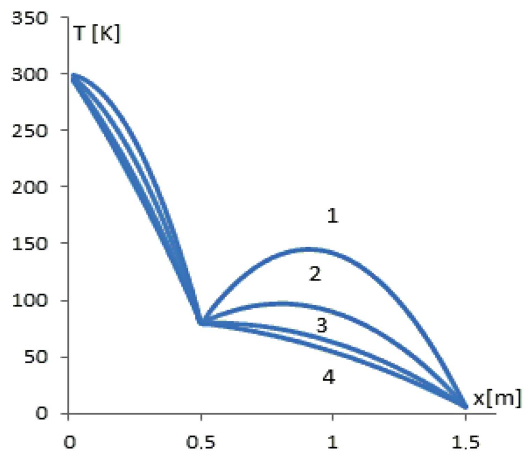

Fig. 6. Temperature distribution in the current lead with the thermal intercept at 80 K for the different cross-sections: 1 — 2.5×10^{-6} m^2, 2 — 3×10^{-6} m^2, 3 — 4×10^{-6} m^2, 4 — 5×10^{-6} m^2 and $I = 18$ A.

and PolFEL. For the sake of comparison of these current leads, the heat flowing to the cryogenic liquid through the cold end has been determined. Losses connected with the heat diffusion through the cold end of a 5 mm diameter lead are equal to 4.6 W for the simple lead and 0.72 W for the lead with a thermal intercept, which clearly points to the advantage of the more complicated solution.

Figure 3 demonstrates the temperature characteristics of the copper current leads with the thermal intercept as the function of the current, while Fig. 4 — as the position of the thermal intercept at 80 K.

In Fig. 5, the dependence of the calculated temperature profile on the copper current lead is shown as the function of the temperature of the thermal

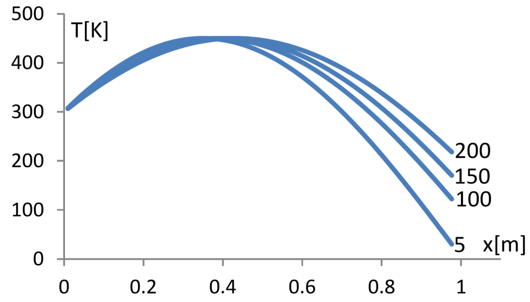

Fig. 7. Temperature distribution inside the current lead determined from the dynamic approach for different cold end temperatures, i.e., 5, 100, 150 and 200 K.

intercept. It is an important result because in the construction of the current leads, the thermal intercept is frequently realized by the thermal contact with flowing helium gas returning to the recovery system. Consequently, the choice of the temperature of the intercept influences the heat balance of the cryogenic cooling system.

Figure 6 presents the temperature profile of the current lead as the function of its cross-section, which is important for the current lead design. The boundary conditions used in these calculations were such that $T = 300$ K at the warm end of the current lead, $T = 80$ K for the temperature of the intercept and for the data shown in Fig. 3, it was 70 K and the temperature of the cold end was 6 K. The analysis of the current leads was also made previously in [5], where, however, a more engineering approach was adopted, dealing with other issues as compared to the present, more phenomenological study.

4. Dynamic approach to analysis of current lead temperature distribution

In this part of the paper, a different analytical method of approximation of the temperature profile distribution in the linear current lead to the superconducting electromagnet, based on the dynamic approach, is discussed. For this purpose, a one-dimensional solution of basic (4) has been found for the dynamic case, which reduces then to (6). We consider the situation in which during a steady current flow the static, stable temperature distribution in the current lead is reached. Then, the current is switched off and the temperature starts to decrease dynamically from this initial distribution. The shape of the static temperature distribution is obtained in the limit of $t = 0$. Mathematically, such a situation is described by the following dynamic part of the heat diffusion equation

$$\rho c_m \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}. \tag{13}$$

The solution of (13) for the time equal to zero will approach a steady, initial temperature profile, determined by the equilibrium between heating during the current flow and heat released in the process of thermal conductivity. The goal of this attempt is such that we investigate in this way a new physical approach to the heat transfer in the current lead, in which the total knowledge of the generated heat is not necessary but only the maximum temperature is estimated. We search for the solution of (13) for that case in the form of the separation of the variables

$$T(x,t) = Z(t)\, F(x) \tag{14}$$

which leads to the dependence

$$\frac{1}{DZ}\frac{\partial Z}{\partial t} = F\frac{\partial^2 F(x)}{\partial x^2} = -\gamma. \tag{15}$$

The diffusion constant D was defined in (2). This method of separation of the variables leads to (16), determining the time dependence of the temperature decay

$$\frac{1}{DZ}\frac{\partial Z}{\partial t} = -\gamma \tag{16}$$

which has the exponential solution

$$Z(t) = \exp(-\gamma D t). \tag{17}$$

The second differential equation describes the position-dependent temperature profile

$$\frac{1}{F}\frac{\partial^2 F(x)}{\partial x^2} = -\gamma \tag{18}$$

For $t = 0$, the solution of this equation gives, therefore, the required static temperature profile described by the function $F(x)$. The solution of (18) has been chosen in the form of

$$F(x) = A \sin\left(\sqrt{\gamma}\,(x+\beta)\right), \tag{19}$$

where the parameters γ and β have been determined from the boundary conditions

$$\sqrt{\gamma} = \frac{1}{x_1}\left[\pi - \arcsin\left(\frac{F(x_1)}{A}\right) - \arcsin\left(\frac{300}{A}\right)\right] \tag{20}$$

and

$$\beta = \frac{\arcsin\left(\frac{300}{A}\right)x_1}{\pi - \arcsin\left(\frac{F(x_1)}{A}\right) - \arcsin\left(\frac{300}{A}\right)}. \tag{21}$$

Here, x_1 is the length of the current lead. Finally the initial, static profile of temperature distribution in this model is given by

$$F(x) = A\sin\left[\left(x + \frac{x_1 \arcsin\left(\frac{300}{A}\right)}{\pi - \arcsin\left(\frac{F(x_1)}{A}\right) - \arcsin\left(\frac{300}{A}\right)}\right)\left(\frac{\pi - \arcsin\left(\frac{F(x_1)}{A}\right) - \arcsin\left(\frac{300}{A}\right)}{x_1}\right)\right], \tag{22}$$

while the coefficient A describes in this model the maximum temperature and is approximated by the relation

$$A = \frac{1}{x_1}\left(\frac{I^2R}{\lambda S}l_1l_2 + 300l_2 + T_2l_1\right), \tag{23}$$

where l_1 and l_2 are the distances at the current lead from the point of the maximum temperature x_m to the warm and cold end, respectively, while x_m is given by the relation

$$x_m = \frac{\frac{\pi}{2} - \arcsin\left(\frac{300}{A}\right)}{\pi - \arcsin\left(\frac{300}{A}\right)}. \tag{24}$$

The results of calculations of the influence of the cold end temperature T_2 on temperature profile in the current lead obtained basing on the dynamic approach are shown in Fig. 7. The shift of the parameter x_m is seen here as well as the similarity to the previous results obtained in the static approach. The elaborated model will be useful for the analysis of the hybrid current leads to be included in the next paper.

5. Conclusions

In this paper, we have discussed two analytical approaches to the heat diffusion issues appearing in the normal current leads to the superconducting electromagnets: the static and dynamic one. Analytical calculations performed basing on these approaches allowed to establish the influence of the thermal intercept on the temperature profile of the current leads and generated heat flowing through the cold end of the current lead. These results have, therefore, significance for applied physics, indicating the function of various physical parameters as the current, size of the current leads, temperature and position of the thermal intercept, cold end temperature on the temperature profile in the current lead and energy losses.

References

[1] J. Voelker, *Gun Claster Meeting: short story of a new solenoid for a Gun 1.1 bERLinPro, HZB*, 2018.

[2] Xianhao Li, Li Ren, Ying Xu, Jing Shi, Xiaogang Chen, Guilun Chen, Shuqiang Guo, Yuejin Tang, Jingdong Li, *IEEE Trans. Appl. Supercond.* **30**, 1 (2020).

[3] S.J. Lee, H.S. Yang, *IEEE Trans. Appl. Supercond.* **29**, 5402705 (2019).

[4] A. Ballarino, *Physica C* **468**, 2143 (2008).

[5] M.S. Kim, J.Y. Jang, Y.J. Hwang, Y.S. Choi, *Appl. Sci.* **10**, 6712 (2020).

[6] M.N. Wilson, *Superconducting Magnets*, Oxford University Press, Oxford 1983.

[7] Y. Iwasa, *Case Studies in Superconducting Magnets*, 2nd ed., Springer, 2009.

[8] R. Wesche, A.M. Fuchs, *Cryogenics* **34**, 145 (1994).

[9] J. Simon, E.S. Drexler, R.P. Reed, *NIST Monograph 177, Properties of Copper and Copper Alloys at Cryogenic Temperatures*, U.S. Government Printing Office, Washington 1992.