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Quantum Correlations in System of Kerr Nonlinear Coupler

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We consider a system composed of two linearly mutually interacting, identical Kerr-like nonlinear oscillators. Assuming that the system initially is in a coherent state, we discuss the system's dynamics and concentrate on the field's mutual correlations. As a measure of such correlations, we apply the normalized first- and second-order coherence functions. We derive the analytical formulas for such functions showing that their time-evolution strongly depends on the linear coupling parameter.

topics: Kerr coupler, quantum correlations, first- and second-order correlation functions

1. Introduction

The quantum aspects of the properties of physical systems can be analyzed in various ways. Apart from the commonly discussed last time quantum entanglement or quantum steering, it can be done, for example, with an application of the studies of other quantum correlations appearing in the system. For instance, the first- and second-order coherence functions can be applied for such purposes. The concept of the degree of coherence was proposed in 1938 by Zernike [1]. Next, Hanbury-Brown and Twiss in 1956 applied the intensity correlation function in the statistical description of measurements of photons emitted by spatially coherent light [2], including that originating from the distant stars [3]. Those experiments have initiated the studies devoted to the fluctuations of the electromagnetic field. In the next years, various measurements and devoted to them considerations were performed in the research related to various physical systems. Examples of such investigations concerned sodium atoms [4], optomechanical systems [5], polariton condensates [6], ultraintense twin beams [7], and many others.

In this paper, we will study correlations between two subsystems of the Kerr-like quantum nonlinear coupler. We will derive the analytical formulas describing the first- and second-order cross-correlation functions with an application of the operators' time-dependent form, analogously as in [8]. Then, we will analyze the time-evolution of such correlation functions and focus on the influence of the strength of interaction between these subsystems on the generation of the first- and second-order correlations.

2. The model

We consider a system composed of two identical nonlinear Kerr-type oscillators (subsystems) labeled as 1 and 2. The system can be described by the following Hamiltonian:

$$\hat{H} = \omega\big(\hat{a}_1^\dagger\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2\big) + \chi\big(\hat{a}_1^{\dagger 2}\hat{a}_1^2 + \hat{a}_2^{\dagger 2}\hat{a}_2^2\big)$$
$$+\epsilon\big(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_2\hat{a}_1^\dagger\big) + \kappa\big(\hat{a}_1^\dagger\hat{a}_1\hat{a}_2^\dagger\hat{a}_2\big), \qquad (1)$$

where the operators $\hat{a}_j^\dagger$ and $\hat{a}_j$ are the bosonic creation and annihilation operators for the mode $j = 1, 2$, respectively. The frequency of oscillatory modes of the field is equal to ω, whereas ϵ represents the strength of the linear interaction between the subsystems. The parameters χ and κ are proportional to the third-order susceptibility and characterize the nonlinear oscillators. They are related to the self-action and cross-action processes, respectively.

It should be noted that the same Hamiltonian as ours was considered by Korolkova and Peřina to describe two parallel waveguides with optical Kerr-like media [8], and by Kuang et al. to discuss two Bose–Einstein condensates involving weak nonlinear interatomic interactions [9].

In our considerations, we assume that all parameters appearing in the Hamiltonian (1) are real. Additionally, the Kerr-like nonlinearity indexes are the same for both modes and equal to χ. We assume that the time-evolution of the system starts from the two-mode coherent state $|\alpha\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle$, where α_1 and α_2 are the complex amplitudes of the coherent states corresponding to the first and the second mode of the field, respectively.

3. Results and discussion

We concentrate here on the quantum properties of the Kerr nonlinear coupler expressed by the first- and second-order correlation functions $g^{(1)}$ and $g^{(2)}$. These functions are cross-correlation functions of different order, and specify the coherences between fields' amplitudes and intensities, respectively. Each of them can be written in a specific form [10, 11].

Let us start with the definition of

$$g_{12}^{(1)}(t) = \frac{\left|\left\langle \hat{a}_1^\dagger(t)\hat{a}_2(t)\right\rangle\right|}{\sqrt{N_1(t)N_2(t)}}, \tag{2}$$

where $N_1(t) = \langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle$ and $N_2(t) = \langle \hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle$ are the mean numbers of photons in the first and second mode of the field, respectively. It should be emphasized that the first-order correlation function takes values from zero to unity. Moreover, if $g_{12}^{(1)}$ reaches zero, the coherence between the two modes is not observed, whereas if it is equal to unity, the full coherence is present.

Using (2) and the forms of operators $\hat{a}_1(t)$ and $\hat{a}_2(t)$ derived in [8], we obtain the formula for the $g^{(1)}$ function. It can be written as

$$g_{12}^{(1)}(t) = \frac{\left||\beta_1|^2 - |\beta_2|^2 + (V - V^*)\right|}{\sqrt{\left(|\beta_1|^2 + |\beta_2|^2\right)^2 - \left(V + V^*\right)^2}}, \tag{3}$$

where

$$V = \beta_1\beta_2^* \exp\Big(2\mathrm{i}\epsilon t + f(-\mathrm{i}\theta t)\,|\beta_1|^2 + f(\mathrm{i}\theta t)\,|\beta_2|^2\Big), \tag{4}$$

and $\theta = \chi - \frac{\kappa}{2}$, $\beta_1 = \frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2)$ and $\beta_2 = \frac{1}{\sqrt{2}}(\alpha_1 - \alpha_2)$. Moreover, one applies the function $f(\gamma) = -1 + \exp(\gamma)$ which is valid for the arbitrary value of the variable γ.

To calculate the second-order correlation function, we applied the definition

$$g_{12}^{(2)}(t) = \frac{\left\langle \hat{a}_1^\dagger(t)\hat{a}_2^\dagger(t)\hat{a}_1(t)\hat{a}_2(t)\right\rangle}{N_1(t)N_2(t)}, \tag{5}$$

followed by [10, 11]. For correlated subsystems, $g^{(2)} > 1$, whereas, for the anticorrelated ones, it takes values smaller than unity. When the second-order correlation function is equal to unity, we have uncorrelated modes.

Using (5) and the explicit form of operators $\hat{a}_1(t)$ and $\hat{a}_2(t)$ derived in [8], we get

$$g_{12}^{(2)}(t) = \frac{|\beta_1|^4 + |\beta_2|^4 - (J + J^*)}{\left(|\beta_1|^2 + |\beta_2|^2\right)^2 - (V + V^*)^2}, \tag{6}$$

where

$$J = (\beta_2^*)^2\beta_1^2 \exp\Big(4\mathrm{i}\epsilon t + f(-2\mathrm{i}\theta t)\,|\beta_1|^2 + f(2\mathrm{i}\theta t)\,|\beta_2|^2\Big). \tag{7}$$

The time-evolution of both types of quantum correlations discussed here is presented in Fig. 1.

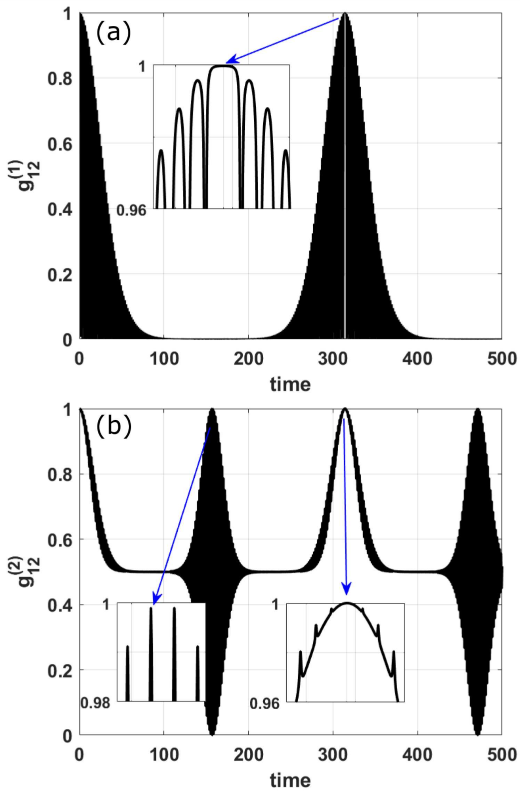

Fig. 1. The time-evolution of (a) the first-order and (b) second-order correlation functions for $\kappa = 0.16$, $\chi = 0.1$, $\epsilon = 1$. Time is scaled in the units of $1/\epsilon$.

In Fig. 1a, we see that the first-order correlation function changes periodically. Such changes in the value of $g_{12}^{(1)}$ exhibit oscillations characterized by high and low frequencies. Additionally, the first-order correlation function reaches values from zero to unity, which is related to the disappearance of coherence and the full coherence's appearance, respectively.

Analogously as for $g_{12}^{(1)}$, the second-order correlation function exhibits periodic oscillations (see Fig. 1b). The function $g_{12}^{(2)}$ takes a value equal to or smaller than unity. It means that for our system, the uncorrelated and anticorrelated modes can be observed, respectively. The second-order correlation function becomes equal to unity when the $g_{12}^{(1)}$ reaches its maximal value, and thus when the full first-order coherence between modes appears.

Next, we analyze how the quantum correlations depend on the strength of the linear interaction between subsystems. Thus, in Fig. 2, we show the dependence of the maximal and minimal values of first- and second-order correlation functions on the value of ϵ.

In Fig. 2a, we see that, contrary to the minimal value, the maximal value of $g_{12}^{(1)}$ depends on the parameter ϵ. For weak coupling between the subsystems, only for some values of ϵ, the maximal value

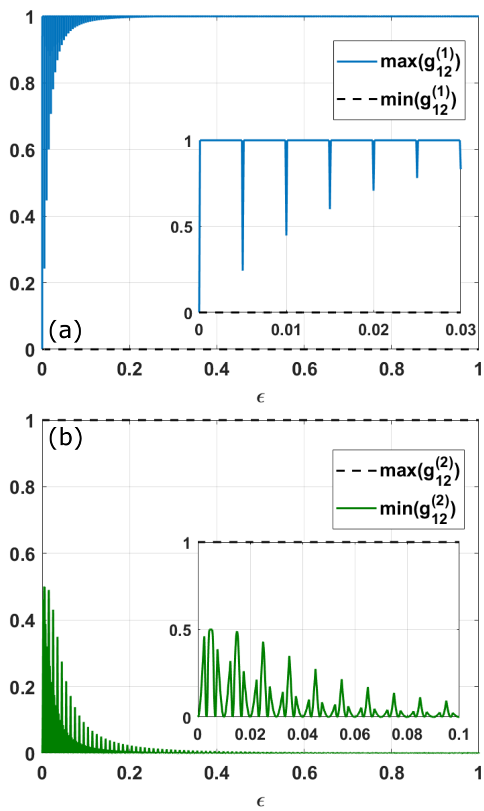

Fig. 2. The maximal and minimal values of (a) the first- and (b) second-order correlation functions vs. the value of the strength of the linear interaction between subsystems. We assume that $\kappa = 0.16$, $\chi = 0.1$.

of the first-order correlation function is not equal to unity — we do not observe full intermode coherences for such a case. However, as ϵ increases, the maximal value of $g_{12}^{(1)}$ becomes practically equal to unity, and strong first-order correlations appear in the system.

If we analyze the second-order correlation function, we see that its maximal value does not depend on the epsilon's value. On the other hand, its minimal value changes with increasing the coupling parameter. We see that the minimal value of the second-order correlation function equals zero for some values of the coupling strength. That means that the function $g_{12}^{(2)}$ changes from zero to unity, such as it is shown in Fig. 1a.

4. Conclusion

In the present paper, the model of the Kerr-type nonlinear coupler was considered. In particular, we were interested in the time-evolution of the quantum correlations present in this system. Applying the Heisenberg equation's analytical solutions derived in [8], we have found the analytical formulas determining the first- and second-order correlation functions. We studied the time-dependence of such cross-correlation functions for various values of the interaction parameters. We have shown that the maximal and minimal values of the functions $g_{12}^{(1)}$ and $g_{12}^{(2)}$, respectively, strongly depend on the coupling strength. The obtained results show that the system considered here can be a source of strong correlations, including the full coherence. Additionally, we have proved that the degree of correlations between the modes can be easily controlled by modifying the parameters describing the system.

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References

[1] F. Zernike, *Physica* **5**, 785 (1938).

[2] R. Hanbury Brown, R.Q. Twiss, *Nature* **177**, 27 (1956).

[3] R. Hanbury Brown, R.Q. Twiss, *Nature* **178**, 1046 (1956).

[4] H.J. Kimble, M. Dagenais, L. Mandel, *Phys. Rev. Lett.* **39**, 691 (1977).

[5] L.-h. Sun, G.-x. Li, Z. Ficek, *Phys. Rev. A* **85**, 022327 (2012).

[6] C. Antón, G. Tosi, M.D. Martín, Z. Hatzopoulos, G. Konstantinidis, P.S. Eldridge, P.G. Savvidis, C. Tejedor, L. Viña, *Phys. Rev. B* **90**, 081407 (2014).

[7] J. Peřina Jr., *Phys. Rev. A* **93**, 013852 (2016).

[8] N. Korolkova, J. Peřina, *Opt. Commun.* **136**, 135 (1997).

[9] L.-M. Kuang, Z.-Y. Tong, Z.-W. Ouyang, H.-S. Zeng, *Phys. Rev. A* **61**, 013608 (1999).

[10] C.C. Gerry, P.L. Knight, *Introductory Quantum Optics*, Cambridge University Press, 2004.

[11] P.R. Berman, V.S. Malinovsky, *Principles of Laser Spectroscopy and Quantum Optics*, Princeton University Press, 2011.