Proceedings of the Online Conference "Applications of Physics in Mechanical and Material Engineering"

Natural Frequency of Elastically Mounted Column Axially Loaded with Mass Element

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In this work, the influence of the loading method of an elastically mounted column on its natural frequency was investigated. The classic way to load these types of systems is to apply an axial force. In the presented approach, the load of the system in the form of a mass element was adopted, which much better reflects the real slender support system, whose task is to support a structure with a specific own weight. During the formulation of the boundary problem, Hamilton's principle and the perturbation small parameter method were used. A series of numerical simulations were carried out, taking into account the influence of the system parameters and the method of loading on the non-linear natural frequency. The main task was to determine the impact of the change in stiffness at mounting points of the system ends on the dynamic behaviour of the structure. It was shown that this stiffness has a significant impact on the natural frequency. It was also indicated that in the problem formulated in this way, the amplitude level of the induced system vibrations is of significant importance — which is not taken into account in the case of a force load. The knowledge of potential resonance frequencies in the case of slender support systems is one of the basic data taken into account in the design process of this type of structures due to their susceptibility to vibrations.

topics: free non-linear vibrations, column, mass element

1. Introduction

Vibrations are one of many physical problems that determine the periodic change of certain physical quantities. Mechanical vibrations refer to the periodic change of position (which can be defined as the vibration amplitude) of the system occurring around a certain balance point with a specific frequency. If the frequency of the system's vibrations (resulting from the action of the external exciting force) coincides with the frequency of natural vibrations, a resonance phenomenon occurs, resulting in a rapid increase in the vibration amplitude. In the case of mechanical systems (e.g., supporting pillars, industrial machines), the occurrence of resonance is a negative and very dangerous phenomenon.

For this reason, the issues of vibrations of physical and mechanical systems are the subject of many scientific and research works. In the case of the analysis of free vibrations, the influence of various system parameters or loads on the change of natural frequency is determined. Such issues are raised, among others, in [1–10]. Stability and free vibrations of a compound column with a piezoelectric rod are considered in [1]. A geometrically nonlinear two-member column under an eccentric, partially follower load is analysed. Characteristic curves are shown for different cases of loading and actuation. A free vibration analysis of sandwich columns with homogeneous core materials is described in [2]. The problem is formulated analytically and also the finite elements models are developed. In [3], the studies cover free vibration and buckling analysis of tapered columns made of axially functionally graded materials. The governing differential equations of the problem are derived and solved using the direct integral method combined with the determinant search technique. The obtained results are compared with those in the literature and calculated in FE software ADINA. Vibration of nonprismatic linear beam-columns, with semi-rigid connections on elastic foundation, is considered in [4]. The effect of a variable two-parameter elastic foundation is discussed. Further, the non-linear vibrations of a slender system subjected to an external force applied between the elements of a structure are discussed in [5]. The relation between the amplitude and the natural vibration frequency is obtained. Stability and free vibrations problems of stepped columns with cracks are presented in [6]. The cracks in the column are represented by massless rotational springs. The frequency equation is obtained by using properties of the Green functions.

Moreover, studies [7] cover the free vibration problem of a non-uniform column using the differential quadrature method. The results are compared with the exact solution and the FEM method. In [8], the optimum design process of thermally loaded beam-columns for maximum vibration frequency or buckling temperature is shown. In turn, the vibration problem of beam with mass at free end is presented in [9]. The system is additionally supported at various distances from the rigid mounting. Further, the problem of non-linear vibrations of a simply supported column loaded by the mass element is studied in [10]. The results show the impact of the amplitude and slenderness of the system on the free vibrations frequency.

In this work, an elastically mounted column under mass load is considered. The problem is nonlinear due to the mass loading. In such a case, a significant effect on the natural frequency has the vibration amplitude.

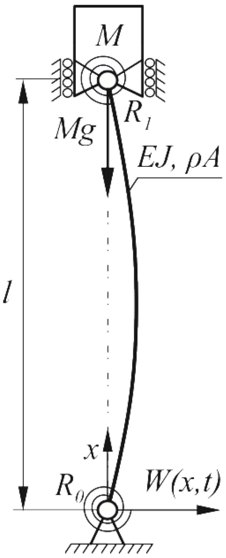

Fig. 1. Scheme of the considered system.

2. Boundary problem of free non-linear vibrations

The column (with the length l) elastically mounted on both sides and loaded by the mass element M is under consideration. The load fulfils the Euler load conditions with the longitudinal inertia of the loading element additionally taken into account. The elastic mounting of the system is modelled by means of two rotational springs R_0 and R_1 (see Fig. 1).

The boundary problem was formulated using Hamilton's principle and taking into account the following dimensionless parameters:

$$w(\xi,\tau)=\frac{W(x,t)}{l},\quad \xi=\frac{x}{l},$$

$$u(\xi,\tau)=\frac{U(x,t)}{l},\quad \tau=\omega t, \tag{1}$$

$$k^2(\tau)=\frac{S(t)\,l^2}{EJ}\quad \theta=\frac{Al^2}{J},\quad \Omega^2=\frac{\omega^2(\rho A)\,l^4}{EJ},$$

where ω is the natural frequency, $S(t)$ — the internal force, E — the Young modulus, J — the geometric moment of inertia, ρ — the density, A — the cross-section of the column.

Based on Hamilton's principle, and also taking into account the geometric boundary conditions

$$w(0,\tau)=w(1,\tau)=u(0,\tau)=0, \tag{2}$$

and the natural boundary conditions of the form

$$\frac{EJ}{l}\left.\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}\right|_{\xi=0}-R_0\left.\frac{\partial w(\xi,\tau)}{\partial\xi}\right|_{\xi=0}=0 \tag{3}$$

$$\frac{EJ}{l}\left.\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}\right|^{\xi=1}+R_1\left.\frac{\partial w(\xi,\tau)}{\partial\xi}\right|^{\xi=1}=0 \tag{4}$$

$$\frac{k^2(\tau)\,EJ}{l^2}-M\omega^2 l\frac{\partial^2 u(\xi,\tau)}{\partial\tau^2}=0, \tag{5}$$

then the equations of motion of the system are determined in the respective, transverse and longitudinal directions to its axis, i.e.,

$$\frac{\partial^4 w(\xi,\tau)}{\partial\xi^4}+k^2(\tau)\frac{\partial^2 w(\xi,\tau)}{\partial\xi^2}$$
$$+\Omega^2\frac{\partial^2 w(\xi,\tau)}{\partial\tau^2}=0 \tag{6}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial u(\xi,\tau)}{\partial\xi}+\frac{1}{2}\left(\frac{\partial w(\xi,\tau)}{\partial\xi}\right)^2\right)=0. \tag{7}$$

The non-linear term appearing in (7) is developed into a series of small vibration amplitude parameters ε. Then, (6) and (7) are grouped with respect to the same powers of the small parameter. The obtained equations are solved sequentially and, based on them, the following parameters are determined:

- linear component of internal force in the column,
- linear component of natural frequency,
- nonlinear component of internal force in the column,
- non-linear component of natural frequency.

3. Results of numerical simulations

The main problem in this study was to determine the effect of the mounting stiffness of the considered column on its nonlinear natural frequency. The results of the numerical calculations were presented with the use of dimensionless parameters:

$$\lambda=\frac{M}{M_{\mathrm{E}}},\quad \Omega^*=\sqrt{\Omega_0^2+\varepsilon^2\Omega_2^2},$$

$$\zeta_A=\frac{\mathrm{Amp}}{r},\quad r_0=\frac{R_0 l}{EJ},\quad r_1=\frac{R_1 l}{EJ}, \tag{8}$$

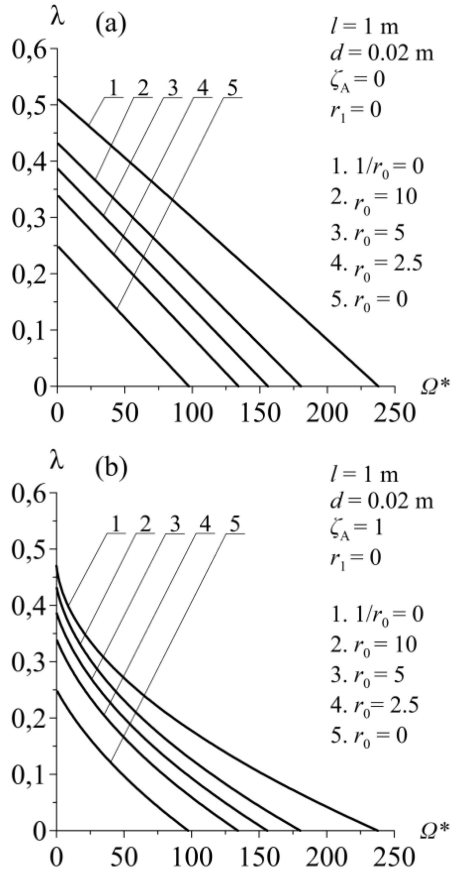

Fig. 2. The influence of one-side mounting rigidity on natural frequency: (a) $\zeta_A = 0$, (b) $\zeta_A = 1$.

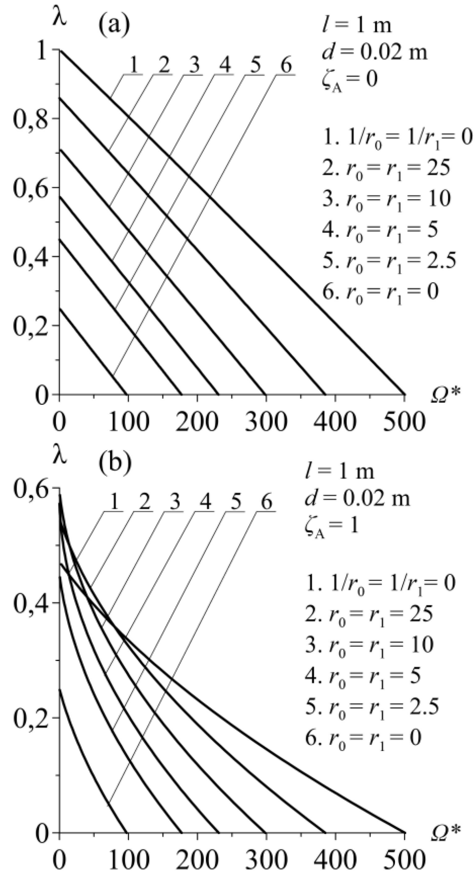

Fig. 3. The influence of two-sides mounting rigidity on natural frequency: (a) $\zeta_A = 0$, (b) $\zeta_A = 1$.

where λ is the external load parameter, M — the mass loading the column, M_{E} — the critical mass of the clamped-clamped column (calculated with the use of Euler's buckling theory), Ω^* — the parameter of natural frequency, Ω_0 — the parameter of the linear component of natural frequency, ε — the small amplitude parameter, Ω_2 — the parameter of the non-linear frequency component of natural vibrations, ζ_A — the vibration amplitude parameter, Amp — the amplitude of vibrations, r — the gyration radius, r_0 — the parameter of the stiffness in the bottom mounting, r_1 — the parameter of the stiffness in the top mounting.

The results of numerical calculations are presented in the form of characteristic curves (on the plane load — natural frequency). This form allows to analyse changes in the vibration frequency of the system in the entire range of its real load from the point of view of stability (from zero to critical load, at which the system buckles).

Different cases of stiffness change in the mounting points of the column were considered: stiffness change in only one of the mountings (see Fig. 2) and simultaneous change in the stiffness in both mounting points (see Fig. 3), taking into account the linear component of frequency (cases (a)) and taking into account the nonlinear component (cases (b)).

4. Conclusions

On the basis of the obtained results, it was found that an increase in the stiffness in the supports causes the characteristic curves to shift towards higher values. This is because the overall system stiffness has increased. The course of characteristic curves in the linear problem is linear. Increasing the stiffness in the supports may increase the critical load of the system (λ for $\Omega^* = 0$). The increase in stiffness when considering the same column load causes an increase in natural frequency. Controlling the stiffness of the support can be one way to actively counteract resonance.

Taking into account the non-linear problem (the amplitude effect) changes the course of the characteristic curves from linear to non-linear. It can be observed that with certain values of stiffness, the critical load of the system decreases. This is mainly due to the vibration amplitude and increased system stiffness. As shown in [10], an excessive increase in amplitude may result in the reduction of the critical load. Moreover, the influence of the set amplitude level on the system with higher stiffness is greater, therefore, with higher stiffness in the supports, a reduction of the critical load with regard to the linear problem can be observed. In an extreme case, the curves may intersect (see Fig. 3b). In this case, the increase in stiffness in relation to the same level of vibration amplitude resulted in a significant reduction of the critical load.

Acknowledgments

This work was supported by the Częstochowa University of Technology (BS/PB 1-100/3010/20/P).

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