

Technique of Specific Conductivity Measurement of Anisotropic Semiconductor Plates and Films

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Received: 23.05.2018 & Accepted: 17.07.2020

Doi: [10.12693/APhysPolA.138.759](https://doi.org/10.12693/APhysPolA.138.759)

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The paper presents a fast technique to identify the components of specific conductivity tensor of thin semiconductor planes. The proposed technique is based on a well-known method with a linear location of a four-probe tester. It is scientifically backed up by solving the relevant boundary problems of electrodynamics and characterized by the simplicity of measurements and calculations. Moreover, the suggested method does not require the use of complicated equipment and also the boundary conditions have been taken into account. Experimental testing has been carried out on monocrystals of diarsenide of cadmium and diarsenide of zinc. Practical recommendations are given on how to make the necessary measurements more precise.

topics: anisotropic semiconductor, electroconductivity tensor, potential distribution, diarsenide of cadmium, fourprobe method

1. Introduction

Researchers now have a thorough study of methods available to characterize semiconducting materials which do not show anisotropy of physical properties [1, 2]. At the same time, in modern electronics promising semiconducting compounds of the type A_2B_5 and A_2B_6 are more and more widely used [3–6].

In crystals, we observe anisotropy of electric, thermoelectric and galvanomagnetic properties due to the complexity of crystal lattice. In some cases, stimulated anisotropy of electric properties of atomic semiconductors under pressure or under the influence of external field can also be observed [5, 6]. As a consequence, the development of anisotropic semiconductor characterization methods becomes ever more important.

Probe methods [1, 2] confirmed their efficiency in measuring specific electroconductivity. Difficulty of such measurements consists in the fact that a current spreading effect appears in these materials, depending on a correlation of the value of conductivity tensor. That is why the methods which are well-developed and sufficient for isotropic materials are not applicable to anisotropic ones.

Scientific literature describes methods intended for specific conductivity measurement like the Montgomery method [7]. Its essence lies in a numerical identification of electric potential distribution on the lateral lanes of the sample. The Montgomery method puts rigid requirements

on the sample geometry and quality of the lateral planes. The method has proved its efficiency for rectangular samples and only in such a case it is possible to create high quality low-resistance contacts on the flat lateral planes, although it is not always possible to achieve it in practice. A certain problem arises when one needs to analyze conductivity but the lateral planes are not available or there are difficulties with an application contact place. In fact, there are some methods [8] which are further derived from the Montgomery method. Such methods increase the accuracy of a specific conductivity measurement, however they still set high requirements for the quality of surfaces of the lateral planes and necessitate the geometrical location of the contacts at the sample angles.

Quantitative methods identifying specific conductivity tensor of anisotropic samples were already described in scientific literature [9–11]. However, they do not allow to identify the required values with high accuracy and they are characterized by an error of 10–15%. Such methods require lengthy resistance contacts, the creation of which often results in the destruction of a semiconductor crystal which renders it unfit for future work.

This paper suggests a technique to identify the components of specific conductivity tensor of anisotropic planes and films which is based on a method with a linear location of a four-probe tester. This technique is also assessed in terms of its application and measurement accuracy.

2. Theoretical background of the technique

Let us identify the potential distribution with probe measurement when the current is permanent. In the fixed mode with absent sources and drain charge, the vector of current density \mathbf{j} , the intensity of field \mathbf{E} and the potential φ are related to each other in the following way [12]:

$$\mathbf{j} = \hat{\sigma} \mathbf{E}, \quad \mathbf{E} = -\nabla \varphi, \quad \nabla \cdot \mathbf{j} = 0, \quad (1)$$

where $\hat{\sigma}$ is the symmetrical tensor of specific conductivity. For the sample oriented, in particular, in a way that the tensor has a diagonal shape

$$\tilde{\sigma} = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}, \quad (2)$$

we can get the differential equation for the potential:

$$\sigma_x \frac{\partial^2 \varphi}{\partial x^2} + \sigma_y \frac{\partial^2 \varphi}{\partial y^2} + \sigma_z \frac{\partial^2 \varphi}{\partial z^2} = 0. \quad (3)$$

One should take into account that the normal constituent of the current density on the sample surface is equal to zero everywhere except for the points under the current electrodes [12]. For the probes located according to Fig. 1, the following boundary conditions apply:

$$\begin{aligned} \left. \frac{\partial \varphi}{\partial x} \right|_{x=0,a} &= 0, & \left. \frac{\partial \varphi}{\partial y} \right|_{y=0,b} &= 0, & \left. \frac{\partial \varphi}{\partial z} \right|_{z=0} &= 0, \\ \left. \frac{\partial \varphi}{\partial z} \right|_{z=d} &= \frac{I_{14}}{\sigma_z} \delta(x - x_1) \delta(y - y_1) \\ &\quad - \frac{I_{14}}{\sigma_z} \delta(x - x_4) \delta(y - y_4), \end{aligned} \quad (4)$$

where $\delta(x)$ is the Dirac delta-function. Its usage is justified for the current probes with small-size entry area. The coordinates of the current probes are (x_1, y_1) and (x_4, y_4) .

The boundary problem of (3) and (4) is solved with the method of separation of variables. As a result, the final expression for potential φ on the sample surface can be represented in the form of the double Fourier series

$$\begin{aligned} \varphi = & -\frac{4I_{14}}{ab\sigma_z} \sum_{k,n=0}^{\infty} A_{nk} \frac{\text{ch}(\eta_{kn}(d-z))}{\eta_{kn} \text{sh}(\eta_{kn}d)} \\ & \times \cos(\alpha_k x) \cos(\beta_n y), \end{aligned} \quad (5)$$

where

$$\begin{aligned} A_{nk} = & \Theta_n \Theta_k \cos(\alpha_k x_1) \cos(\beta_n y_1) \\ & - \Theta_n \Theta_k \cos(\alpha_k x_4) \cos(\beta_n y_4), \end{aligned} \quad (6)$$

$$\begin{aligned} \alpha_k = & \frac{\pi k}{a}, \quad \beta_n = \frac{\pi n}{b}, \quad \eta_{kn} = \sqrt{\frac{\sigma_x}{\sigma_z} \alpha_k^2 + \frac{\sigma_y}{\sigma_z} \beta_n^2}, \\ \Theta_{i\{i=k,n\}} = & \begin{cases} 1, & i \neq 0, \\ \frac{1}{2}, & i = 0. \end{cases} \end{aligned} \quad (7)$$

To identify the components of electroconductivity tensor σ_x , σ_y , it is necessary to carry out two

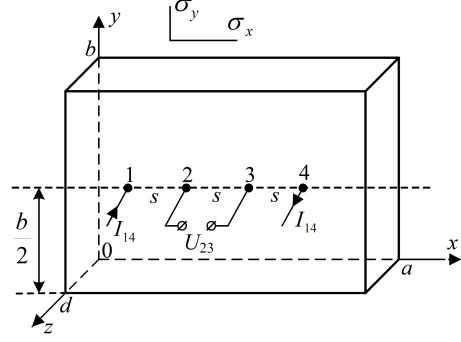


Fig. 1. The scheme of probe location on a rectangular sample.

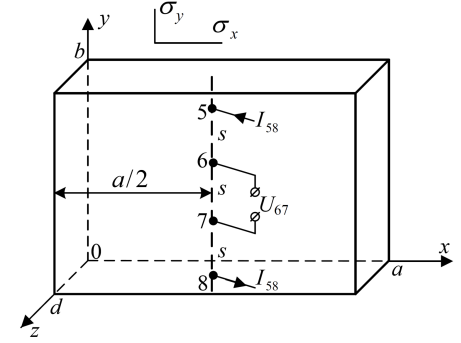


Fig. 2. The scheme of probe location on a rectangular sample.

independent measurements of current and voltage with different locations of probes. Firstly, we place a tester with a linear location of probes on the sample surface along the line which is parallel to axis x , then we measure current I_{14} and potential difference U_{23} (in Fig. 1: $x_1 = (a-s)/2$, $x_4 = (a+3s)/2$, $y_1 = y_4 = b/2$). The obtained potential distribution enables us to find the theoretical potential difference between probes 2 and 3:

$$U_{23} = \frac{I_{14}a}{\sigma_x bd} L_1, \quad (8)$$

$$\begin{aligned} L_1 = & \frac{16\sigma_x}{a^2\sigma_z} \sum_{\substack{k=1,3,5,\dots, \\ n=0,2,4,\dots}} \Theta_n \frac{\text{cth}(\eta_{kn}d)}{\eta_{kn}} \\ & \times \sin\left(\frac{1}{2}\alpha_k s\right) \sin\left(\frac{3}{2}\alpha_k s\right). \end{aligned} \quad (9)$$

With the ratio value $d/s < 0.7$, the sample can be considered to be thin with the calculation error of less than 2% [1, 2]. In this case, we can do summation over “ k ” [13]. As a result, in the approximation of the thin sample we get the expression for multiplier L_1 , included into (8), namely:

$$\begin{aligned} L_1 = & \frac{s}{a} + \sum_{n=2,4,6,\dots} \frac{2}{\gamma\beta_n \text{ash}(\gamma\beta_n a)} \\ & \times \left[\text{ch}(\gamma\beta_n(a-s)) - \text{ch}(\gamma\beta_n s) \right. \\ & \left. + \text{ch}(2\gamma\beta_n s) - \text{ch}(\gamma\beta_n(a-2s)) \right]. \end{aligned} \quad (10)$$

Here, $\gamma = \sqrt{\sigma_y/\sigma_x}$ reads as the parameter of electroconductivity anisotropy.

For the second measurement, we place a probe on the same plane of the sample along the line parallel to axis y (in Fig. 2: $x_5 = x_8 = a/2$, $y_5 = (b + 3s)/2$, $y_8 = (b - 3s)/2$) and measure current I_{58} and distribution potential U_{67} . Analogically, we get the expression for the correlation of current I_{58} and voltage U_{67} :

$$U_{67} = \frac{I_{58}}{\sigma_y} \frac{b}{ad} L_2, \quad (11)$$

where L_2 is the geometrical factor corresponding particularly to the considered case of the probe location. In the case of thin samples we get:

$$L_2 = \frac{s}{b} + \sum_{k=2,4,6,\dots} \frac{2}{\alpha_k b / \gamma \operatorname{sh}(\alpha_k b / \gamma)} \times \left[\operatorname{ch}(\alpha_k (b - s) / \gamma) - \operatorname{ch}(\alpha_k s / \gamma) + \operatorname{ch}(2\alpha_k s / \gamma) - \operatorname{ch}(\alpha_k (b - 2s) / \gamma) \right] \quad (12)$$

Using the measurement data, we identify the value of the dimensionless parameter Q :

$$Q = \frac{R_1}{R_2} = \frac{U_{23}/I_{14}}{U_{67}/I_{58}}, \quad (13)$$

which we need to calculate the value γ , necessary for getting the value of the components of specific electroconductivity tensor.

According to (8) and (11), the “theoretical” relation $Q(\gamma)$ can be expressed in the following way:

$$Q(\gamma) = \left(\frac{\gamma a}{b} \right)^2 \frac{L_1}{L_2}. \quad (14)$$

3. Technique of measurement procedure

The suggested technique aimed at the measurement of components of special electroconductivity tensor of a crystal or film can be summarized as follows:

1. With a certain value of current I_{14} measure voltage U_{23} , then using the same linear probe with current value I_{58} measure voltage U_{67} (see Fig. 1 and Fig. 2).
2. Calculate from the experiment the value of parameter $Q = R_1/R_2$ according to (13).
3. Using the relation plot $Q(\gamma)$ identify the value of parameter γ by putting in correspondence the “theoretical” and experimental values of parameter Q (obtained with (14) and (13)). For some values of relations a/s , b/s , we showed Q related to anisotropy parameter γ in Fig. 3.
4. Identify the value of the geometrical factors L_1 and L_2 using (10) and (12).
5. Calculate the value of a component of electroconductivity tensor σ_x and σ_y using:

$$\sigma_x = \frac{I_{14}}{U_{23}} \frac{a}{db} L_1, \quad \sigma_y = \frac{I_{58}}{U_{67}} \frac{b}{da} L_2. \quad (15)$$

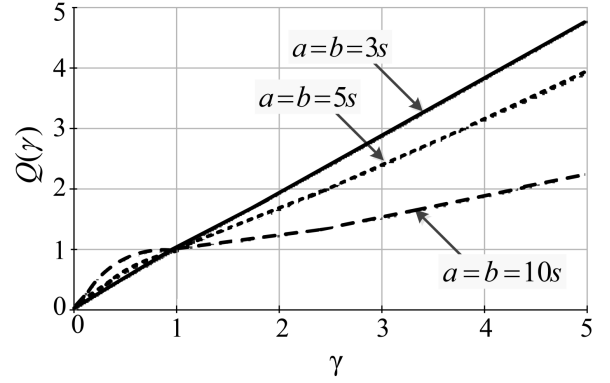


Fig. 3. The diagram showing the relation of parameter Q value to electroconductivity anisotropy coefficient γ with $a = b$ and various values of s .

For practical considerations, it is worth to identify the value of geometrical factors L_1 and L_2 for the infinite sample ($a/s \rightarrow \infty$, $b/s \rightarrow \infty$). As a result of calculations of the relevant limits, we conclude that for an indefinite plane the expressions (15) can be represented in the following way:

$$\sigma_x = \frac{0.2206}{\gamma dt} \frac{I_{14}}{U_{23}}, \quad \sigma_y = \frac{0.2206\gamma}{d} \frac{I_{58}}{U_{67}}. \quad (16)$$

For isotropic samples ($\sigma_x = \sigma_y = \sigma$, $\gamma = 1$) we get a well-known formula used to calculate specific resistance when the measurements are done with the help of a linear four-probe tester [1, 2]:

$$\rho = \frac{1}{\sigma} = 4.532 \frac{U_{23}}{I_{14}} d. \quad (17)$$

4. Experimental validation and conclusions

The suggested technique aimed at identifying the components of specific electroconductivity tensor was experimentally tested on anisotropic CdAs₂ and ZnAs₂ [14, 15] plates. We used clamping wolframium probes as current and measurement contacts.

Constant electric current passed through the samples under study. The current came from stabilized power supply B5-44, a potential difference between measuring probes was measured by voltmeter V2-34, the positions of measuring and current probes were monitored and controlled with the MBS-10 microscope. The average values of electroconductivity tensor are given in Tables I and II. The control values of these components measured by a standard two-probe method [1, 9] are also included for comparison. For the practical measurement of conductivity control values we used tinker contacts, the ohmic properties of which to diarsenide of cadmium and diarsenide of zinc were proved experimentally [3, 4].

One of restrictions that limit the use of the suggested measurement method is the size of the sample: the transverse size of semiconductors under study must be bigger than the length of a free path

Results of experimental measurements.

TABLE I

Sample	a [mm]	b [mm]	d [mm]	s [mm]	σ_x [$(\Omega \text{ m})^{-1}$]		σ_y [$(\Omega \text{ m})^{-1}$]	
					Proposed method	Two-probe method	Proposed method	Two-probe method
CdAs ₂	19.20	22.50	1.00	3.00	41.34	42.10	8.82	8.90
ZnAs ₂	18.00	20.00	0.85	3.00	2009	2021	509	520

TABLE II

Comparison of experimental measurements and results obtained with other methods, i.e., (1) — Montgomery [7], (2) — modified Montgomery [8], (3) — Schnabel [4], (4) — transverse [11].

Sample	σ_x [$(\Omega \text{ m})^{-1}$]				σ_y [$(\Omega \text{ m})^{-1}$]			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
CdAs ₂	38.25	39.72	37.33	44.14	8.05	8.14	8.03	9.32
ZnAs ₂	1908	1926	1905	2181	442	464	443	561

of an electron. The second restriction is the presence of a flat upper plane, adjacent to the measurement contacts. Also, it is necessary for general resistance of the semiconductor R (in the order of value) to meet the condition: $R_A \ll R \ll R_V$ (where R_A — the resistance of amperemeter, R_V — the resistance of voltmeter).

The advantage of the suggested method is that it is not necessary to make an ohmic connection to the sample — this procedure often creates certain difficulties leading to spoiling the crystal which becomes unsuitable for further use. The error of this technique is determined by the error of the well-known four-probe method.

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