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Local Loss of Rectilinear Form of Static Equilibrium of Geometrically Nonlinear System with Non-Prismatic Element under Force Directed towards Pole

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The paper considers a geometrically non-linear column constructed as an overbraced flat rod system loaded with a force directed towards the positive pole. The analyzed system is characterized by two forms of static equilibrium: rectilinear and curvilinear. It was proved in scientific studies that the loss of rectilinear form can be local and is directly related to the distribution of bending stiffness. In this paper, it is proposed to use elements with a variable cross-section as a way to reduce the phenomenon of local loss of rectilinear static equilibrium. The obtained results allow for the statement that proper shaping of the non-prismatic rod which is a component of the geometrically non-linear system may cause an increase in bifurcation load, and thus limiting the scope of local loss of rectilinear form of static equilibrium.

topics: stability, bifurcation load, non-linear system

1. Introduction

In the construction of modern machinery and equipment, complex elements are used. The chances of using them will be the greater, the better the models describing their behavior at work are. Continuous realization of undertaken scientific problems, e.g. by improving physical and mathematical models, allows observation of significant phenomena, and consequently the design of lighter, stronger and safer structures. One of the most important aspects in analyzing the behavior of slender rod systems is the issue of their stability. This problem was taken up in many scientific works. Particularly important are works in which non-linear models were analyzed, which in the most accurate way reflect the behavior of real systems. The work [1] presents the problem of stability and transverse vibrations of a geometrically non-linear system loaded by Euler's force. The issue of geometrically non-linear vibration of a beam made of viscoelastic material defined on the basis of the classical Zener rheological model is presented in the article [2]. Buckling analysis of geometrically non-linear curved beams is presented in [3]. The topic of local and global loss of rectilinear form of static equilibrium has been addressed, among others in works [4–6].

2. Boundary problem formulation

A model of a geometrically non-linear KNP column (Fig. 1) constructed of two prismatic rods and one rod of variable cross-section connected by a concentrated mass is considered in the paper. It is assumed that the flexural stiffness of the entire system is constant and known, and its distribution is given by the flexural stiffness asymmetry coefficient μ:

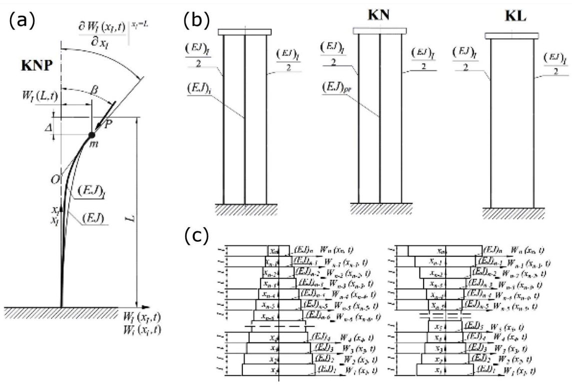

Fig. 1. Physical models of the considered systems: (a) geometrically non-linear column with non-prismatic element KNP, (b) comparative systems KN and KL, (c) modeling of the non-prismatic rod.

$$\mu = \frac{(EJ)_{pr}}{(EJ)_{pr} + (EJ)_I}. \quad (1)$$

The variable cross-section of the central bar is determined by dividing into n prismatic segments whose widths describe the approximating functions: linear and quadratic, observing the condition of total constants: volume and length.

The column was subjected to a load directed towards the positive pole.

In addition, comparative systems are also considered in the work: a geometrically non-linear column KN built of three prismatic bars and a KL linear column built of two external bars with diameters resulting from the value of the coefficient μ.

Potential energy of the KNP system is the sum of the bending elasticity energy of individual bars and potential energy resulted from external load:

$$V_1=\sum_{i=1}^{n}\frac{(EJ)_i}{2}\int_0^l\left(\frac{\partial^2 W_i\left(x_i,t\right)}{\partial x_i^2}\right)^2\mathrm{d}x_i+\frac{(EJ)_I}{2}\int_0^L\left(\frac{\partial^2 W_I\left(x_I,t\right)}{\partial x_I^2}\right)^2\mathrm{d}x_I$$
$$+\sum_{i=1}^{n}\frac{(EA)_i}{2}\int_0^l\left[\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2+\frac{\partial U_i(x_i,t)}{\partial x_i}\right]^2\mathrm{d}x_i+\frac{(EA)_I}{2}\int_0^L\left[\frac{1}{2}\left(\frac{\partial W_I(x_I,t)}{\partial x_I}\right)^2+\frac{\partial U_I(x_I,t)}{\partial x_I}\right]^2\mathrm{d}x_I$$
$$+PU_I(L,t)+\frac{1}{2}PW_I(L,t)\frac{W_I(L,t)}{l_b}. \tag{2}$$

Kinetic energy is given by the formula

$$T=\frac{1}{2}\sum_{i=1}^{n}(\rho A)_i\int_0^l\left(\frac{\partial W_i\left(x_i,t\right)}{\partial t}\right)^2\mathrm{d}x_i+\frac{(\rho A)_I}{2}\int_0^L\left(\frac{\partial W_I\left(x_I,t\right)}{\partial t}\right)^2\mathrm{d}x_I+\frac{m}{2}\left(\frac{\partial W_I\left(L,t\right)}{\partial t}\right)^2. \tag{3}$$

The issue was formulated on the basis of the Hamilton principle [1, 4–6], obtaining differential equations of motion

$$(EJ)_i\frac{\partial^4 W_i\left(x_i,t\right)}{\partial x_i^4}+S_{II}\frac{\partial^2 W_i\left(x_i,t\right)}{\partial x_i^2}+(\rho A)_i\frac{\partial^2 W_i\left(x_i,t\right)}{\partial t^2}=0, \tag{4}$$

where $i=1,2,\ldots,n$, and

$$(EJ)_I\frac{\partial^4 W_I\left(x_I,t\right)}{\partial x_I^4}+S_I\frac{\partial^2 W_I\left(x_I,t\right)}{\partial x_I^2}+(\rho A)_I\frac{\partial^2 W_I\left(x_I,t\right)}{\partial t^2}=0, \tag{5}$$

and a system of geometric and natural boundary conditions, including continuity conditions

$$W_1\left(0,t\right)=W_I\left(0,t\right)=0, \tag{6}$$

$$W_I(L,t)=W_n(l,t), \tag{7}$$

$$\left.\frac{\partial W_1\left(x_1,t\right)}{\partial x_1}\right|_{x_1=0}=\left.\frac{\partial W_I\left(x_I,t\right)}{\partial x_I}\right|_{x_I=0}=0, \tag{8}$$

$$\left.\frac{\partial W_I(x_I,t)}{\partial x_I}\right|_{x_I=L}=\left.\frac{\partial W_n(x_n,t)}{\partial x_n}\right|_{x_n=l}, \tag{9}$$

$$(EJ)_I\left.\frac{\partial^3 W_I(x_I,t)}{\partial {x_I}^3}\right|_{x_I=L}+(EJ)_n\left.\frac{\partial^3 W_n(x_n,t)}{\partial {x_n}^3}\right|_{x_n=l}+P\left(\left.\frac{\partial W_I(x_I,t)}{\partial x_I}\right|_{x_I=L}-\frac{W_I(x_I,t)}{l_b}\right)-m\frac{\partial^2 W_I(x_I,t)}{\partial t^2}=0, \tag{10}$$

$$(EJ)_I\left.\frac{\partial^2 W_I(x_I,t)}{\partial {x_I}^2}\right|_{x_I=L}=-(EJ)_n\left.\frac{\partial^2 W_n(x_n,t)}{\partial {x_n}^2}\right|_{x_n=l} \tag{11}$$

$$W_j\left(l,t\right)=W_{j+1}\left(0,t\right),\quad \left.\frac{\partial W_j\left(x_j,t\right)}{\partial x_j}\right|_{x_j=l}=\left.\frac{\partial W_{j+1}\left(x_{j+1},t\right)}{\partial x_{j+1}}\right|_{x_{j+1}=0}, \tag{12}$$

$$(EJ)_j\left.\frac{\partial^2 W_j\left(x_j,t\right)}{\partial x_j^2}\right|_{x_j=l}=(EJ)_{j+1}\left.\frac{\partial^2 W_{j+1}\left(x_{j+1},t\right)}{\partial x_{j+1}^2}\right|_{x_{j+1}=0}, \tag{13}$$

$$(EJ)_j\left.\frac{\partial^3 W_j\left(x_j,t\right)}{\partial x_j^3}\right|_{x_j=l}=(EJ)_{j+1}\left.\frac{\partial^3 W_{j+1}\left(x_{j+1},t\right)}{\partial x_{j+1}^3}\right|_{x_{j+1}=0}, \tag{14}$$

where $j=1,2,\ldots,n-1$. Due to the non-linearity occurring in the system, the boundary problem was solved based on the small parameter method (perturbative method).

3. Results of numerical computation

All the results of numerical calculations obtained were presented in a dimensionless form, after introducing the bifurcation load parameters λ_b, convergence Z^* (when approximating with a linear function) and coordinates p^*, q^* of the apex of the approximating parabola

$$\lambda_b = \frac{P_b L^2}{(EJ)_{pr} + (EJ)_I}, \tag{15}$$

$$Z^* = \frac{b_1 - b_n}{L} \times 100\%, \tag{16}$$

$$p^* = \frac{p}{L}, \tag{17}$$

$$q^* = \frac{q}{L}. \tag{18}$$

The subscript "*pr*" means that the marked values refer to prismatic and comparative systems.

The comparative analysis of the bifurcation load change of the geometrically non-linear column KN and geometrically linear KL as a function of the flexural stiffness distribution parameter μ allows for the determination of local and global loss of rectilinear form of static equilibrium (Fig. 2). In terms of local instability $\mu \in (0; \mu_{gr})$, a geometrically non-linear column built of three bars carries less bifurcation load than the geometrically linear column created as a result of removal of the central bar. Above the μ_{gr} value, the linear system carries less bifurcation load.

The paper adopts the thesis that since the selection of the shape of a bar with a variable cross-section causes a change in the bifurcation load, it is possible to select approximating parameters at which an increase in the bifurcation load will result in a shift in the border between the local and global loss of rectilinear static equilibrium and a reduction of an adverse local phenomenon instability.

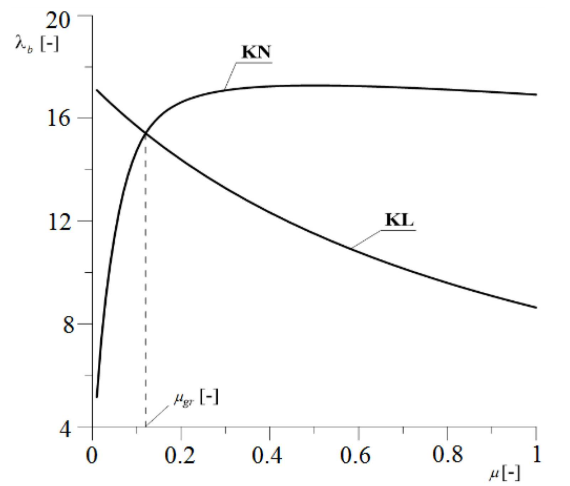

Fig. 2. Bifurcation load of KN and KL systems as a function of flexural stiffness distribution coefficient μ.

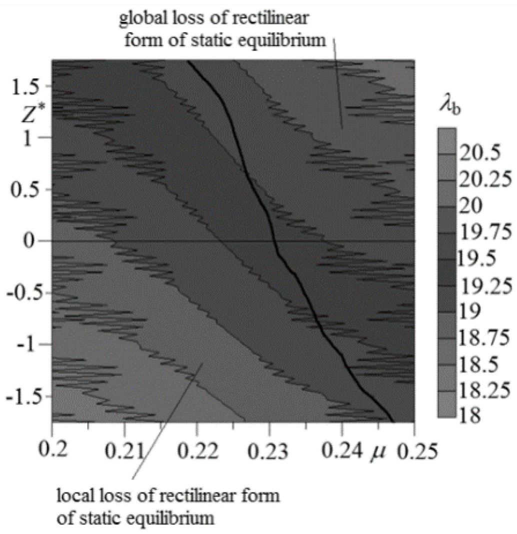

Fig. 3. Regions of local and global loss of rectilinear form of static equilibrium of column KNP $l_b^* = 0.25$ when internal rod is approximated by linear function.

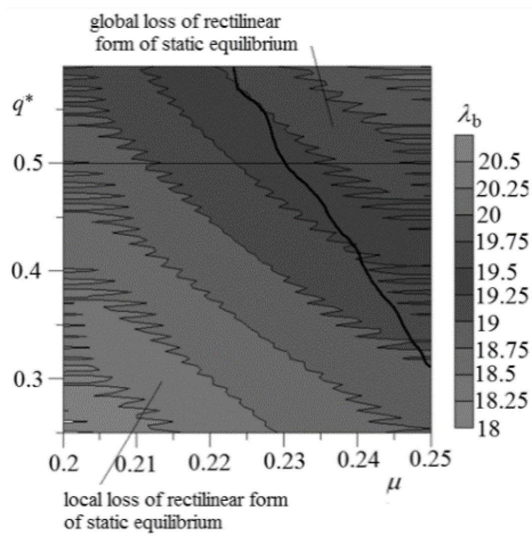

Fig. 4. Results obtained for polynomial of degree 2 and $p^* = 0.2$ — description as in Fig. 3.

A series of numerical tests carried out in the field of stability of geometrically non-linear systems with a non-prismatic element made it possible to determine the impact of the bar outline as a component of the KNP column on the change of the μ_{gr} value, and thus the areas of local and global loss of rectilinear form of static equilibrium.

Figures 3–5 show the change in the limit value of the flexural stiffness distribution coefficient μ_{gr}, which was obtained when approximating the shape by a polynomial of degree I (Fig. 3) and II (Figs. 4, 5). Depending on the adopted geometry of the load-carrying structure that determines the direction of external force, the bifurcation load distribution changes as a function of approximation

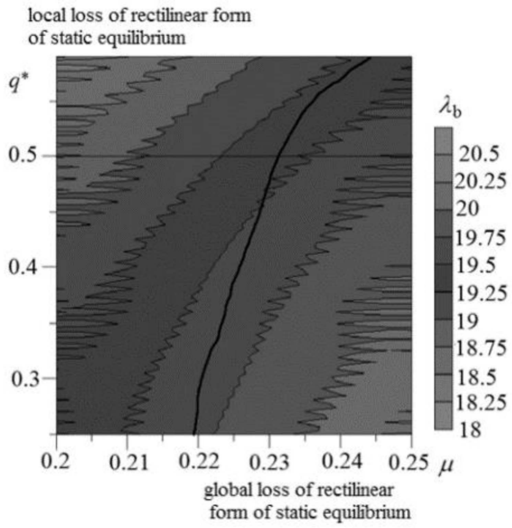

Fig. 5. Results obtained for polynomial of degree 2 and $p^* = 0.8$ — description as in Fig. 3.

function parameters. This means that for each geometry, it is possible to specify the values of the Z^* or q^* and p^* parameters that will be most favourable from the point of view of getting out of the local loss of rectilinear form of static equilibrium.

4. Conclusions

Stability of a geometrically non-linear column with non-prismatic rod subjected to a force directed to the positive pole was analyzed in this work. On the basis of the numerical calculations carried out, the following conclusions were formulated:

1. Shape approximation of the system affects the value of the bifurcation load. Depending on the parameters describing the outline of the rod, change in the shape may cause an increase or decrease in the value of bifurcation load.
2. Comparison of the bifurcation load of a geometrically non-linear column and the critical load of a linear column as a function of the coefficient of flexural stiffness asymmetry μ allows the determination of local and global loss of rectilinear form of static equilibrium.
3. After considering in the model of a geometrically non-linear column rod with a variable cross-section, a change in the value of bifurcation load of systems compared to systems made of prismatic elements was noted. It has been shown that properly selected values of parameters describing the outline of the column cause an increase in bifurcation load above the critical load of the linear system, and thus the "exit" of the system from the local instability.

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