

Nonlinear Wave Solutions of Cylindrical KdV–Burgers Equation in Nonextensive Plasmas for Astrophysical Objects

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In this paper, the time-dependent cylindrical Korteweg-de Vries–Burgers equation has been derived using hydrodynamic equations with the Poisson equation for nonextensive ultracold neutral plasmas containing ions and nonextensive electrons, various kinds of analytical solutions have been obtained for cylindrical Korteweg-de Vries–Burgers equation using extended homogeneous balance method. Numerical analysis for the nonlinear shock wave solution revealed that its profile is significantly affected by nonextensive and the ion temperature. This theoretical study could provide a better frame-idea about the laboratory plasma systems as observed in the space for the astrophysical compact objects. This study also shows that further deep investigations are needed in future for better understanding of the nonlinear wave propagation for astrophysical compact objects in space.

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1. Introduction

Plasma, a quasi-neutral gas of charged and neutral particles, exhibits collective behavior being ionized [1]. Thus the substances in plasmas become highly electrically conductive to the point that long-range electric field dominates the behavior of the matter [2]. Basically, in turn, this governs collective behavior with many degrees of variation [1, 2]. However, plasma is rare on the Earth surface under normal conditions and is mostly artificially generated from neutral gases [3], but in this universe 99% are in plasma state [1–4]. Thus it is interesting and important to know the plasma conditions for the laboratory to understand the environmental conditions in space.

In the previous work [5], a fluid model for ions (both positively and negatively charged), electrons (with electron degenerate pressure), and stationary dust was considered and further modified Korteweg-de Vries (KdV) equation (fmKdV) was derived following reductive perturbation technique [6–9]. The investigation was done to study the small but finite amplitude for dust-ion-acoustic shock waves using “ G'/G ” method to obtain a new class of solutions [6–11]. Another study [12] derived extended homogeneous balance method [13, 14] obtaining the exact traveling wave solutions for KdV equation [15, 16]. This model discussed both small amplitude and the Sagdeev potentials for large amplitude

in nonlinear wave structures for the plasma systems containing both superthermal electrons and ions with kappa distribution and heavy dust particles which are negatively charged. Very recently, a dusty plasma model containing negatively charged dust particles, isothermal electrons, and two-temperature isothermal ions has been considered. The extended tanh method (ETM) is used to solve the reduced nonlinear ordinary differential equation from the fmKdV equation deriving KdV equation with a nonlinear wave solution providing shock wave characteristics [6–11, 15, 16].

Recently, non-extensive distribution has got the attention in plasma systems for its unique and different characteristics. Basically, the non-extensive statistics or Tsallis statistics is based on the derivation of the Boltzmann–Gibbs–Shannon (BGS) entropic measurement and is only studied in such plasma conditions/cases where the Maxwell distribution is considered inappropriate [17]. This entropic index is symbolized with q which characterizes the degree of non-extensivity of the considered system. The parameter q has been classified into three categories where (i) $q < 1$ means superextensivity, (ii) $q > 1$ means subextensivity, and (iii) $q < -1$ means that the q distribution is unnormalizable. It is important to note here that in the extensive limiting case ($q \rightarrow 1$), the q distribution reduces to the well-known Maxwell–Boltzmann velocity distribution [18–24]. The non-extensive distribution for any plasma species is given by

$$n_e = [1 + (q - 1)\psi]^{\frac{1+q}{2(q-1)}}, \quad (1)$$

where q is the nonextensive parameter characterizing the degree of nonextensivity.

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In weakly non-ideal plasmas, like the solar interior, both nonextensivity and quantum uncertainty are found into account to derive equilibrium ion distribution functions and to estimate nuclear reaction rates and solar neutrino uses [25]. Later, particles with statistical behavior as non-extensive distribution were studied in the early stage of heavy-ion collisions [26] and high-energy collisions [27]. Thus, we are interested to study the time-dependent (nonplanar [28]) nonlinear propagation for an ultracold neutral (UCN) plasma system considering q -nonextensive distribution in nonplanar cylindrical geometry because the ion-acoustic wave has been identified in the UCN plasma but in the absence of q -nonextensive distribution of electrons [29]. Thus this study could influence the future plasma experiment to observe such nonlinear dynamics.

We introduce the considered model, model equations, and methods to solve the problem in Sect. 2. The steps for the solutions of cylindrical Korteweg-de Vries–Burgers (cKdV–Burgers) equation are discussed in Sect. 3. Finally, a broad numerical analysis and ending discussion are presented in Sects. 4 and 5.

2. Model and method

Let us consider collisionless, unmagnetized, coupled plasma consisting of ion-fluids and electrons with q -nonextensive distribution. The normalized basic fluid equations of such plasma are governed by the equations of continuity and the generalized viscoelastic ion momentum equation, which are, respectively, given by following the time-dependent cKdV–Burgers equation [28]:

$$-6mm \frac{\partial n_i}{\partial t} + \frac{1}{r^\alpha} \frac{\partial}{\partial r} (r^\alpha n_i u_i) = 0, \quad (2)$$

$$D_\tau \left(m_i n_i D_t u_i + Z_i e n_i \frac{\partial \phi}{\partial r} + k_B T_f \frac{\partial n_i}{\partial r} \right) = \frac{\eta}{r^\alpha} \frac{\partial}{\partial r} \left(r^\alpha \frac{\partial u_i}{\partial r} \right) + \left(\zeta + \frac{\eta}{3} \right) \frac{\partial}{\partial r} \left(\frac{1}{r^\alpha} \frac{\partial}{\partial r} (r^\alpha u_i) \right), \quad (3)$$

and Poisson's equation

$$-6mm \frac{1}{r^\alpha} \frac{\partial}{\partial r} \left(r^\alpha \frac{\partial \phi}{\partial r} \right) = 4\pi e (n_e - n_i), \quad (4)$$

where $\alpha = 0$ for one-dimensional geometry and $\alpha = 1$ and $\alpha = 2$ for cylindrical and spherical geometry, respectively [29]. In Eqs. (2)–(4), n_i , u_i , n_e , and ϕ are the ion number density, the ion fluid radial velocity, the electron number density and the wave potential, respectively, and t (r) is time (space) variable. Furthermore, we have denoted $D_\tau = 1 + \tau_m \partial_t$, $D_t = \partial_t + u_i \partial_r$, τ_m is the viscoelastic relaxation time, Z_i — the ion charge state, e — the magnitude of the electron charge, m_i — the ion mass, T_f — the effective ion temperature arising from the electrostatic interaction among strongly correlated positive ions, k_B — the Boltzmann constant, η and ν are the bulk and shear viscosity coefficients, respectively [29].

We use a q -nonextensive distribution for electrons which were given in Eq. (1). To study cylindrical

solitary waves in a strongly coupled UCN plasma [described by Eqs. (1)–(4)] by the reductive perturbation technique [10], we first re-scale the stretched coordinates [30]:

$$\begin{cases} X = -\epsilon^{1/2}(r + \lambda_p t), \\ T = \epsilon^{3/2}t, \end{cases}, \quad (5)$$

where ϵ is a smallness parameter measuring the weakness of the dispersion and nonlinearity, and λ_p is the phase speed of the ion-acoustic shock waves, which expands the variables n_i , u_i , and ϕ about their equilibrium values in the power series of ϵ , viz.

$$\begin{cases} n_i = n_{i0} + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \dots, \\ u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \dots, \\ \phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots, \end{cases} \quad (6)$$

and develop equations in various powers of ϵ . To the lowest order in ϵ , one obtains the first-order ion continuity equation, ion momentum equation, and Poisson's equation, which give

$$n_i^{(1)} = -\frac{n_{i0}}{\lambda_p} u_i^{(1)} = -\frac{Z_i e n_{i0}}{k_B T_f - m_i \lambda_p^2} \phi^{(1)}, \quad (7)$$

$$\lambda_p = \left[\frac{1}{m_i} \left(T_f + \frac{2Z_i k_B T_e}{1+q} \right) \right]^{1/2}. \quad (8)$$

Equation (8) describes the phase speed for the ion-acoustic shock wave propagating in UCN plasmas with a q -nonextensive electron distribution function.

To the next higher order in ϵ , we obtain a set of coupled equations for $n_i^{(2)}$, $u_i^{(2)}$, and $\phi^{(2)}$, which could be reduced to the cKdV–Burger equation

$$\frac{\partial \phi^{(1)}}{\partial T} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial X} + \frac{1}{2T} \phi^{(1)} + B \frac{\partial^3 \phi^{(1)}}{\partial X^3} = E \frac{\partial^2 \phi^{(1)}}{\partial X^2}, \quad (9)$$

where

$$B = \frac{(T_f - \lambda^2 m_i)^2}{2\lambda_p m_i Z_i e n_{i0}}, \quad (10)$$

$$E = \frac{\eta_0}{2m_i n_{i0}}, \quad (11)$$

where we have assumed that the longitudinal viscosity coefficient $\eta = \epsilon^{1/2} \eta_0$ and the nonlinear coefficient A is

$$-0.2cmA = \frac{Z_i e}{2m_i \lambda_p} \left[1 - \frac{3 + 2q - q^2}{(1+q)^2} \right]. \quad (12)$$

3. cKdV–Burger equation solutions

Using extended homogeneous balance method (see [12]) we find various solutions for the cylindrical Korteweg-de Vries–Burgers equation. Let us first use the transformation

$$\phi^{(1)} = \frac{u}{T} + \frac{X}{2T} \quad (13)$$

and

$$\xi = XT^{-1/2} \tau = -2T^{-1/2}. \quad (14)$$

Finally, we get the KdV–Burger equation

$$u_\tau + Au u_\xi + Bu_{\xi\xi\xi} - Eu_{\xi\xi} = 0. \tag{15}$$

Then we apply the transformation as follows: $u(\xi\tau) = U(\zeta)$, $\zeta = \xi - \lambda\tau$ to Eq. (15).

Then it is reduced to the following ordinary differential equation:

$$-\lambda U' + AUU' + BU''' - EU'' = 0. \tag{16}$$

Integrating Eq. (16) with respect to ζ once, we get

$$-\lambda U + \frac{A}{2}U^2 - EU' + BU'' = 0. \tag{17}$$

Balancing U'' with U^2 yields $m = 2$. Therefore, we are looking for the solution in the form

$$U = a_0 + b_0 + a_1\omega + b_1(1 + \omega)^{-1} + a_2\omega^2 + b_2(1 + \omega)^{-2} \tag{18}$$

and

$$\omega' = k + M\omega + P\omega^2, \tag{19}$$

where a_i and b_i are constants, while k , M and P are parameters to be determined later, $\omega = \omega(\zeta)$, and $\omega' = d\omega/d\zeta$.

Substituting Eqs. (18) and (19) in Eq. (17), we get a polynomial equation ω . Hence, equating the coefficient of ω^j (i.e., $j = 0, 1, 2, \dots$) to zero and solving the obtained system of overdetermined algebraic equation using symbolic manipulation package MATHEMATICA, results:

The first set

$$\begin{aligned} k &= \frac{-E^2}{4PB^2}a_1 = 0, \quad a_2 = 0, \quad a_0 = \frac{3E^2}{5AB}, \\ b_1 &= -\frac{12}{5A}(Ek - EM + 5BkM - 5BM^2 + EP \\ &\quad - 10BkP + 15BMP - 10BP^2), \\ b_2 &= -\frac{12}{A}(Bk^2 - 2BkM + BM^2 + 2BkP \\ &\quad - 2BMP + BP^2), \\ \lambda &= \frac{1}{5}(6EM - 12EP + 60BkP - 60BMP \\ &\quad + 60BP^2 + 5Aa_0). \end{aligned} \tag{20}$$

The second set

$$\begin{aligned} k &= \frac{E^2 + 25B^2M^2}{4B^2P}, \quad a_1 = \frac{12(EP - 5BMP)}{5A}, \\ b_1 &= 0, \quad a_2 = -\frac{12BP^2}{A}, \quad b_2 = 0, \quad a_0 = \frac{3E^2}{5AB}, \\ \lambda &= \frac{1}{5}(-6EM + 60BkP + 5Aa_0). \end{aligned} \tag{21}$$

For the first set, Eq. (20), if $M = 0$, $P = 1$, we get the solutions satisfying case I.

For $k > 0$, the solutions of KdV–Burgers equation, Eq. (15), will be

$$u_1(xt) = a_0 + \frac{12E\sqrt{k} \tan(\sqrt{k}\zeta)}{5A} - \frac{12Bk \tan(\sqrt{k}\zeta)^2}{5A}, \tag{22}$$

$$u_2(xt) = a_0 + \frac{12E\sqrt{k} \cot(\sqrt{k}\zeta)}{5A} - \frac{12Bk \cot(\sqrt{k}\zeta)^2}{5A}. \tag{23}$$

For $k < 0$,

$$u_3(xt) = a_0 - \frac{12E\sqrt{-k} \tanh(\sqrt{-k}\zeta)}{5A} + \frac{12Bk \tanh(\sqrt{-k}\zeta)^2}{5A}, \tag{24}$$

$$u_4(xt) = a_0 - \frac{12E\sqrt{k} \coth(\sqrt{k}\zeta)}{5A} + \frac{60Bk \coth(\sqrt{k}\zeta)^2}{5A}. \tag{25}$$

Now for the solutions satisfying cases II and III and IV, we have the compatibility condition

$$Pk = \frac{M^2 - p_1^2}{4}. \tag{26}$$

Therefore, substitute for P and k , from Eq. (20) into Eq. (25) and solve for p_1 . It is found that

$$p_1 = \frac{E}{5B}. \tag{27}$$

Hence, for case II, we get the following solutions:

$$u_5(xt) = a_0 + \frac{12p_1 \tanh(\zeta p_1)(E + 5Bp_1 \tanh(\zeta p_1))}{5A} \tag{28}$$

and

$$u_6(xt) = a_0 + \frac{12 \coth(\zeta p_1) p_1 (E + 5B \coth(\zeta p_1) p_1)}{5A}. \tag{29}$$

In the same manner case III, results in the solution

$$u_7(xt) = a_0 - \frac{6E(\sqrt{-1+r^2} + \sinh(\zeta))}{5A(r + \cosh(\zeta))} - \frac{3B(\sqrt{-1+r^2} + \sinh(\zeta))^2}{A(r + \cosh(\zeta))^2} \tag{30}$$

with the condition that $p_1 = 1$.

For case IV, the solution form is

$$u_8(xt) = a_0 - \frac{48B}{A(2 + \coth(\zeta) + \operatorname{csch}(\zeta))^2} - \frac{24(-10B + E)}{5A(2 + \coth(\zeta) + \operatorname{csch}(\zeta))}, \tag{31}$$

with the condition that $p_1 = 1$.

$$u_9(xt) = a_0 + \frac{3 \coth\left(\frac{\zeta}{2}\right) \left(2E - 5B \coth\left(\frac{\zeta}{2}\right)\right)}{5A} \tag{32}$$

with the condition that $p_1 = 2$.

For the second set, if $M = 0$, $P = 1$, we get the solutions satisfying case I.

For $k > 0$, the solutions of KdV–Burgers equation for Eq. (15) will be

$$u_{10}(xt) = a_0 - \frac{12B(1+k)^2}{A \left(1 + \sqrt{k} \tan(\sqrt{k}\zeta)\right)^2} - \frac{12(-10B+E)(1+k)}{5 \left(A + A\sqrt{k} \tan(\sqrt{k}\zeta)\right)}, \quad (33)$$

$$u_{11}(xt) = a_0 - \frac{12B(1+k)^2}{A \left(1 + \sqrt{k} \cot(\sqrt{k}\zeta)\right)^2} - \frac{12(-10B+E)(1+k)}{5 \left(A + A\sqrt{k} \cot(\sqrt{k}\zeta)\right)}. \quad (34)$$

For $k < 0$,

$$u_{12}(xt) = a_0 + \frac{12B(1+k)^2}{A \left(-1 + \sqrt{-k} \tanh(\sqrt{-k}\zeta)\right)^2} + \frac{12(-10B+E)(1+k)}{5A \left(-1 + \sqrt{-k} \tanh(\sqrt{-k}\zeta)\right)}, \quad (35)$$

$$u_{13}(xt) = a_0 - \frac{12B(1+k)^2}{A \left(-1 + \sqrt{k} \coth(\sqrt{k}\zeta)\right)^2} + \frac{12(-10B+E)(1+k)}{5A \left(-1 + \sqrt{k} \coth(\sqrt{k}\zeta)\right)}. \quad (36)$$

Now, we left with solutions satisfying cases II and III and IV, since the main criteria for these cases to be applicable is the compatibility condition

$$Pk = \frac{M^2 - p_1^2}{4}. \quad (37)$$

From Eq. (50), it is found that

$$p_1 = \frac{\beta}{5\alpha} \quad (38)$$

Therefore, solutions to equation for the type of Eq. (15), will be

$$u_{14}(xt) = a_0 - \frac{12(5B - E + (-10B + E)p_1 \tanh(\zeta p_1))}{5A(-1 + p_1 \tanh(\zeta p_1))^2} \quad (39)$$

and

$$u_{15}(xt) = a_0 - \frac{12(5B - E + (-10B + E)\coth(\zeta p_1) p_1)}{5A(-1 + \coth(\zeta p_1) p_1)^2}. \quad (40)$$

In the same manner, case III results in the solution

$$u_{16}(xt) = a_0 - \frac{24(-10B + E)(r + \cosh(\zeta))}{5A(-2r + \sqrt{-1 + r^2} - 2\cosh(\zeta) + \sinh(\zeta))} - \frac{12B}{A \left(1 - \frac{\sqrt{-1+r^2+\sinh(\zeta)}}{2(r+\cosh(\zeta))}\right)^2}, \quad (41)$$

where $p_1 = 1$.

For case IV, the solution form is

$$u_{17}(xt) = a_0 - \frac{48B}{A(2 + \coth(\zeta) + \operatorname{csch}(\zeta))^2} - \frac{24(-10B + E)}{5A(2 + \coth(\zeta) + \operatorname{csch}(\zeta))} \quad (42)$$

with $p_1 = 1$.

$$u_{18}(xt) = a_0 + \frac{12(5B - E + (-10B + E)\coth(\zeta))}{5A(-1 + \coth(\zeta))^2} \quad (43)$$

with the condition that $p_1 = 2$.

The extended HB method is applied to give the traveling wave solutions for the cKdV–Burger equation. The obtained solutions cover many types like periodical, rational, solitary and shock wave solutions like Eqs. (22), (24), and (30), some of them cannot be recovered using methods like tanh-method, the extended tanh method, the G'/G method method, etc. [6–12, 15, 16, 18]. To investigate the nonlinear properties of solitary waves we can study the soliton solution, Eq. (30) and the shock wave solution, Eq. (24). From the solution of Eq. (30), we can express the solution in the following form:

$$\phi_1 = \frac{\zeta}{2AT} + \frac{3u_7}{AT}, \quad (44)$$

which is a soliton solution as plotted in Fig. 1. The shock wave solution can be expressed from Eq. (24) as

$$\phi_1 = \frac{\zeta}{2AT} + \frac{3a_0}{AT}(\operatorname{sech}^2(\sqrt{-k}\zeta) \pm \tanh(\sqrt{-k}\zeta)). \quad (45)$$

4. Analysis

From Fig. 1 we can see that in the initial period the nonlinear wave structure shows a soliton wave characteristics with a nonlinear increasing and decreasing shape with respect to position. But with the increase of the time, the nonlinear wave structure is going to flat after a sudden increase in the profile where the decreasing rate is really slower than the increasing rate, not shock structure but tends to be nonlinear shock wave structure. This result makes the significance of this work that time plays an important role in this model where electrons are taken with q -nonextensive distribution.

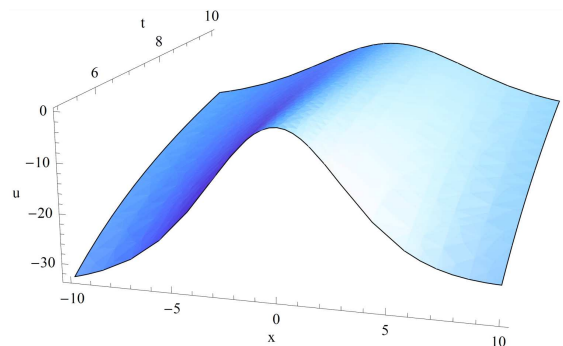


Fig. 1. 3D plot of the soliton solution, Eq. (30).

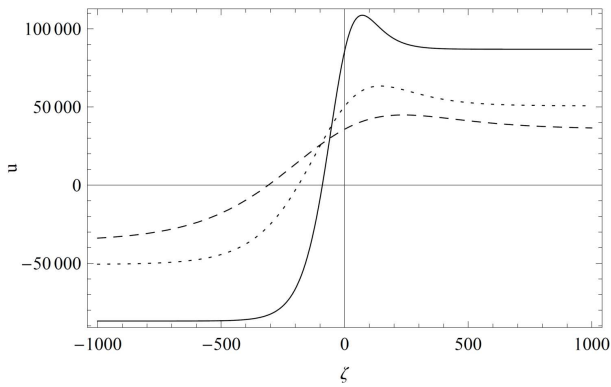


Fig. 2. Shock wave profile, Eq. (24), for different values of $T_f = 300$ (solid curve), $T_f = 450$ (dotted curve), and $T_f = 600$ (dashed curve).

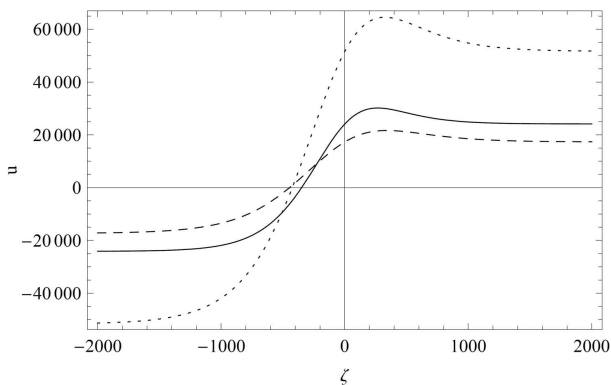


Fig. 3. Shock wave profile from Eq. (24) for different values of $q = 0.01$ (solid curve), $q = 0.3$ (dotted curve), and $q = 0.9$ (dashed curve).

In the next figure, Fig. 2, we show how the viscosity coefficient (η) in the fluid changes when the temperature is changing. At low temperature (solid line in Fig. 2), the viscosity coefficient does not change at the beginning but makes a sudden big jump with a high increasing rate and a comparatively low peak, and then moves with a constant rate as no change in η . Note here that changing position also refers that change in time; i.e., without changing the time it is not possible to refer to two continuous positions. For the next high value of T_f , η starts to increase slowly and then moves as a constant with position providing a perfect nonlinear shock wave profile (dotted curve in Fig. 2). But at high temperature (dashed curve), the change in the nonlinear wave profile also provides a shock structure but the change in the viscosity coefficient (η) is not so significant like other cases as shown in Fig. 2. This phenomenon indicates that temperature also has a great effect on the q -nonextensive of the UCN plasma containing q -nonextensive electrons. Now, we are interested to see what happens if the value of q changes where T_f is fixed. We observe a different and interesting phenomenon at the same time.

When q is 0.01 then the nonlinear profile is perfect shock structure with the increasing value of η shown in Fig. 3 with a solid curve. With the dotted curve in Fig. 3, we see the large shock wave profile when $q = 0.3$ in our considered model and model parameters (given in caption) and there is a big increasing change in the shock profile. When we take q close to 1 ($q = 0.9$) we see that the wave profile starts with the highest value of η comparing the other two waves. The wave structure starts increasing with a low rate (changes occur slowly) where the constant rate (after the increase) in profile shows the lowest value of η . This means that when q is equal to 1, ($q \rightarrow 1$), the q distribution reduces to the well-known Maxwell–Boltzmann velocity distribution. From this view, it looks that our model analysis does agree with well established theory and previous works.

5. Conclusion

In this study, we consider a simple UCN a system containing collisionless, unmagnetized, coupled plasma consisting of ion-fluids and electrons with q -nonextensive distribution. We have derived cKdV–Burgers equation using extended homogeneous balance method using both the standard perturbation method and stretching coordinates. From the nonlinear ion-acoustic (IA) shock wave solution we have plotted three simple figures as Figs. 1–3.

Ultracold neutral plasmas formed by photoionizing laser-cooled atoms near the ionization threshold have electron temperatures in the 1–1000 K range and ion temperatures from tens of millikelvin to a few kelvin [31]. The results of this study are expected to contribute to the understanding of the nonlinear potential excitations that may appear in the laboratory UCN plasma experiments. The main applications of the UCN plasma are industrial applications, such as in the lighting technologies, plasma processing, and the pursuit for fusion energy. Laboratory works needs lots of preparation for environmental and health-safety issues. Experimental works are always expensive for all branches of science. We are all in the point that we need an established theory and some pre-assumption before moving to the experimental/laboratory works to make it successful. So we develop a model considering some previous works from theory and experiments. It is interesting to understand the effects of q and T_f in space as found in many kinds of plasma environments like the solar wind, the Earth's magnetospheric plasma sheet, Jupiter, Saturn, supernovae shells (where the condition for soliton formation is well satisfied) [32–36].

On the other hand, in the astrophysical environments, like space, the Reynolds numbers and the initially laminar configurations can transit to turbulence, which is important and needs to be considered for any theory of magnetic reconnection [37–40]. These authors made a brief discussion about the flares predicted by turbulent reconnection and relate them to solar flares and gamma ray bursts following the process of tearing reconnection

should transfer to turbulent reconnection including solar observations, measurements in the solar wind or heliospheric current sheet, and show their correspondence with turbulent reconnection predictions. Following this, it is important to note here that the turbulent reconnection has been found to play an important role in explaining various astrophysical problems [41–43]. As we cannot directly work with real compact objects in space [39, 40, 42, 43], thus it is the time to add magnetic field in the plasma systems modeling to study more about the dynamics of the the turbulent reconnection processes in the astrophysical environments.

To conclude, it may be pointed out that the results of this studies could be useful for understanding some non-linear behaviors of dust acoustic and dust-ion acoustic waves in different regions, where the strong/weak magnetic field is present [44, 45], as well as other physical phenomena like a condensation of rogue and double layers [46], where dust grains are also reported to be found in the laboratory as well as in space and astrophysical environments [47–50]. In some cases, quantum plasmas are also considered for the polarity effect [51] and both positively and negative charged ions [52, 53]. In these works, authors directly and indirectly suggested to make a deep investigation/study about the effects of magnetic fields in space. Thus, the astrophysical objects are somehow got involved with the space condition having plasmas (with/without dust particles).

As this theoretical study is limited from lots of facts and factors, however, we have made such a simple model with some condition of dust particles and q distribution for the nonplanar geometry case. In the environment of compact cosmic sources, such as the astrophysical jets launched from the black holes or neutron stars, the effects of strong magnetic field are very important. Thus, a deep investigation with more critical conditions is needed in the future studies to know more details. The non-resonant acceleration of particles in magnetic turbulences is widely discussed in astrophysical context, in order to explain the non-thermal emission spectra of the gamma ray bursts and a quasi-blackbody spectrum depending on the acceleration mechanism “thermal or magnetic“ of the flow [54, 55].

Acknowledgments

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