

A Comparison of Two Phenomenological Descriptions of Magnetization Curves Based on $T(x)$ Model

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This paper considers the effect of compaction pressure on the shape of magnetization curves of soft magnetic composite cores compacted at different compaction pressures. Two versions of the phenomenological Takács $T(x)$ model are taken into account.

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1. Introduction

This paper focuses on modeling magnetization curves of self-developed soft magnetic composite cores obtained for different compaction pressures with two possible extensions of the phenomenological $T(x)$ model [1]. The aim of the paper is to demonstrate the usefulness of the considered descriptions for this purpose and to determine which version is more useful for engineering purposes. It should be noted that both considered model extensions have the same number of parameters, which should facilitate the choice.

There is yet another important implication resulting from the study: this paper aims to resolve whether it is more important to take into account mutual interactions within the soft magnetic material or to consider reversible magnetization processes in modeling. Both concepts appear in the contemporary hysteresis models, to mention as generic examples the extensions of the classic Preisach model introduced by Della Torre [2] or the well-known formalism advanced by Jiles and Atherton [3]. This in turn might lead to some simplifications introduced to model equations, which might lead to faster numerical implementations of the developed codes.

The $T(x)$ description is chosen deliberately because of its simple mathematical foundations. The model developer provided a general modeling framework [1]; however, no experimental verification of his conjecture can be found either in the aforementioned reference book or in his subsequent papers. The present paper aims to fill the gap also in this context.

2. $T(x)$ model description

The $T(x)$ description relies on algebraic operations based on hyperbolic tangent mapping. For brevity the model equations are recalled for symmetric loops only.

For a more comprehensive discussion of other cases of practical importance the readers are referred to the textbook [1].

The branches of symmetric hysteresis loops are given with the relationships

$$f_+ = \tanh(x - a_0) + b, \text{ for } dx/dt > 0, \quad (1)$$

$$f_- = \tanh(x + a_0) - b, \text{ for } dx/dt < 0, \quad (2)$$

where the pseudo-parameter b is introduced in order to match the ends of the branches at loop tips

$$b = 0.5[\tanh(x + a_0) - \tanh(x - a_0)]. \quad (3)$$

Since the description is purely phenomenological, the model user may interpret the quantities f_+ , f_- , and x in accordance to his/her needs. The quantity a_0 is interpreted as reduced coercive field strength.

2.1. The reversible magnetization processes

The original $T(x)$ model describes the irreversible magnetization processes only. A possible extension to include the reversible effects is mentioned in the textbook [1], Sect. 3.2. The model developer claims that it is possible to add a linear term to capture the effects. If we interpret the x variable as the applied field strength and f_+ , f_- as magnetization for the ascending/descending branches, then the following formulae in physically meaningful quantities are valid:

$$M = M_s \tanh\left(\frac{H \mp H_c}{a}\right) \pm b + cH, \quad (4)$$

$$b = 0.5 M_s \left[\tanh\left(\frac{H_{\text{TIP}} + H_c}{a}\right) - \tanh\left(\frac{H_{\text{TIP}} - H_c}{a}\right) \right]. \quad (5)$$

In the above-given relationships a plays the role of a normalization constant, M_s is saturation magnetization, H_c is quasi-static coercive field strength, c is the new parameter responsible for reversible processes, whereas the index "TIP" denotes loop tip.

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This extension resembles to some extent those existing in the literature. Mészáros [4] considered a description based on weighted response of two $T(x)$ models. Similarly, Włodarski [5] summed the contributions of two Langevin functions in his description. Varga et al. [6] considered the overlapping irreversible and reversible magnetization processes for Finemet-type nanocrystalline alloys in a somewhat similar way like in Ref. [4]. Nová and Zemánek [7] proposed an analytical model for modeling dynamic hysteresis loops in steel with four adjustable terms, where the third term was responsible for the reversible effects and assumed as linear, like in the present paper.

In the low-dimensional GRUCAD model [8–11], considered by us for the description of hysteresis curves of soft magnetic composites (SMCs) [12, 13], the irreversible and reversible magnetization effects are decoupled and described with separated sets of equations, which also reminds the extension of $T(x)$ model described in this section.

The recent paper by Birčáková et al. [14] illustrated the importance of consideration of reversible magnetization processes in the descriptions related to SMCs. A comparison of three possible extensions of the $T(x)$ model to take into account reversible magnetization processes is provided in Ref. [15].

2.2. The effective field

An alternative interpretation of the variable x as the reduced effective field was proposed in Ref. [16]. The effective field is a concept from theoretical physics, which incorporates the effects from different considered phenomena that affect the shape of magnetization curve using some averaged equivalent contributions to the true magnetic field strength present in the magnetic material. In the first approximation the effective field accounts only for the cooperative interactions between magnetic moments, which can be written as $H_{\text{eff}} = H + \alpha M$, where α is the so-called Weiss' constant. Such definition of the effective field plays a crucial role in the Jiles–Atherton hysteresis model [3]. Exactly the same rendering of the effective field was used when hysteresis curves in SMCs were described with $T(x)$ model in Refs. [17, 18]. An interesting geometrical interpretation of the effective field may be deduced from Della Torre's publications [2, 19]. When discussing the experimentally observed noncongruency feature of minor hysteresis loops (their average susceptibilities differ depending on their locations in the M – H plane, even when they are traced between the same values of field strengths) the author introduced a skewed coordinate system, where one of the axes was set at a certain angle from the original one. The new axis corresponds to the effective field strength and the tangent of the aforementioned angle is simply α . Thus the introduction of effective field means an affinity transformation to the M – H coordinate system.

From the implementation point of view, this extension of the $T(x)$ model is a bit more awkward, as it is

necessary to apply a numerical inversion based, e.g., on the Newton–Raphson scheme (magnetization appears implicitly on both sides of nonlinear equation). This problem is pronounced during the estimation of model parameters, because it relies on the minimization of deviations between the measured and the modeled magnetization values at preset field strength values for an assumed a priori set of model parameters. In recent versions of Matlab this problem may be overcome by using anonymous function handles and built-in *fzero* and *lsqnonlin* functions. It may be remarked at this point that there was an attempt to apply both considered concepts simultaneously to the $T(x)$ model [20]. The identification of parameters for hysteresis curves of non-oriented steels required an exhaustive search based on evolution strategies.

3. Modelling

Several cylinder-shaped SMC cores have been prepared from iron powder (99.5% pure Fe, different granulations) and suspense polyvinyl chloride PVC-S (granulation 15–100 μm) using a hydraulic press. During processing different compaction pressures were applied. In Figs. 1–4 the results for coarse granulation, i.e., grain sizes in the range 100–150 μm are shown. The notation C50, C70 denotes pressure applied to the samples, i.e., C50 — 50 tons, C70 — 70 tons. More details on the compaction process are outlined in the paper [13]. For estimation of model parameters the measured major hysteresis curves were used. The same values of model parameters were used next while modeling exemplary minor loops.

From the modelling results presented in Figs. 1–4 it follows that both approaches yield comparable results in terms of accuracy. Slightly better representation of the major loop was achieved with the use of the second extension, i.e., $T(H_{\text{eff}})$ (cf. Figs. 1–3 and Tables I, III). On the other hand, the errors are lower for minor loops for the first model (cf. Tables II, IV). From Figs. 1–4 in some case one could get the impression that the measured coercive field values are different for the ascending

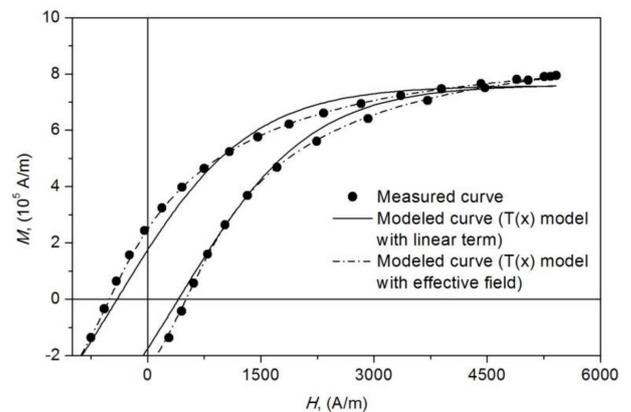


Fig. 1. Sample C50 — modelling of the major loop.

and the descending loop branches. This is merely an artefact, the differences are not significant. In the computations for Tables II–IV we have used the averaged values from both branches.

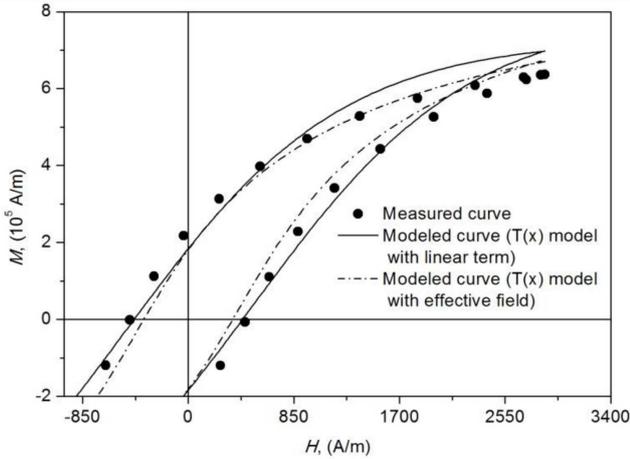


Fig. 2. Sample C50 — modelling of a minor loop.

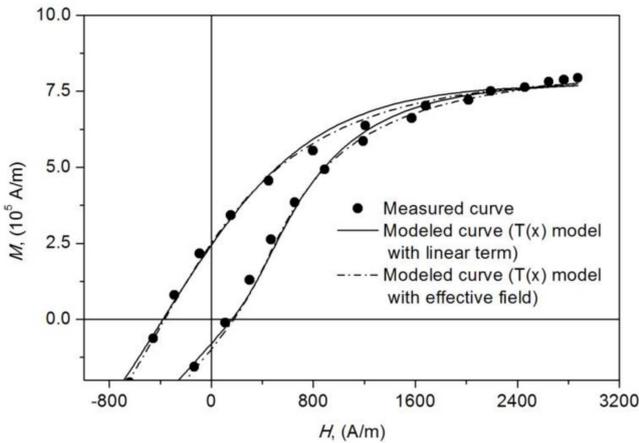


Fig. 3. Sample C70 — modelling of the major loop.

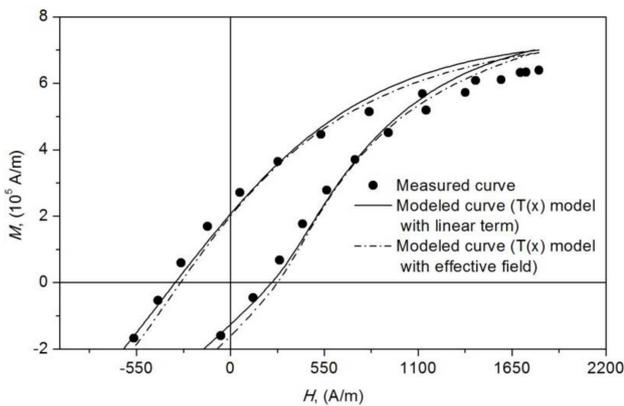


Fig. 4. Sample C70 — modelling of a minor loop.

TABLE I

Modelling errors of both approaches for sample C50 — major loop

	[%]	H_c	M_r	M_{\max}	ΔP
$T(x)$ with linear term	20.7	26.0	4.7	15.4	
$T(x)$ with effective field	0.2	4.6	0.8	2.8	

TABLE II

Modelling errors of both approaches for sample C50 — minor loop

	[%]	H_c	M_r	M_{\max}	ΔP
$T(x)$ with linear term	5.6	26.4	9.6	8.5	
$T(x)$ with effective field	23.3	26.0	5.4	24.6	

TABLE III

Modelling errors of both approaches for sample C70 — major loop

	[%]	H_c	M_r	M_{\max}	ΔP
$T(x)$ with linear term	0.3	8.2	3.2	11.9	
$T(x)$ with effective field	4.0	5.6	2.3	12.7	

TABLE IV

Modelling errors of both approaches for sample C70 — minor loop

	[%]	H_c	M_r	M_{\max}	ΔP
$T(x)$ with linear term	10.8	15.3	9.6	5.8	
$T(x)$ with effective field	24.2	17.8	8.3	6.7	

However there is yet another important feature favouring the $T(H_{\text{eff}})$ version, namely the possibility to take into account other physical phenomena easily, for example the residual stress using Sablik's modification of the effective field, originally used as a component of the Jiles–Atherton model [21, 22], but later also applied to the $T(x)$ description [23]. In the forthcoming research we plan to carry out additional studies on the possibility to tailor up the $T(H_{\text{eff}})$ model with the aforementioned Sablik term.

4. Conclusions

In this paper two possible extensions of hyperbolic model were considered. These modifications were applied to describe hysteresis curves of self-developed SMC cores obtained at different compaction pressures. A reasonable agreement between the measurement and modelling results was obtained. It should be noticed that slightly better results were obtained by using $T(x)$ with effective field especially for the sample C50. Differences between both extensions were lower in the second case i.e. for the sample C70. Thus significant differences between extensions in the first case could be effect of an imperfect parameter set obtained for the case of $T(x)$ with linear term

(the estimation procedure might have got stuck in a local minimum). A general conclusion from the study that both approaches yield reasonable accuracy sufficient for simple engineering computations. We suspect both approaches are comparable in terms of accuracy and from this perspective they might be considered as equivalent. However we point out that the description based on the effective field concept might be more advantageous, since it allows one for a natural and straightforward introduction of the effects of processing technology (e.g. stress) by using a proper extension of the effective field. In the forthcoming work we would like to explore this possibility in modelling.

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