The Effect of Umklapp Processes on Magnetoplasma Waves on the Surface of a Semiconductor Nanotube with a Superlattice

G.I. Rashba

Kharkiv National University named after V.N. Karazin, 4 Svobody Sq., Kharkiv, 61022, Ukraine

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The spectra of plasma waves in the electron gas on the surface of a semiconductor nanotube with a superlattice in a parallel magnetic field have been studied using the self-consistent field method. The analytical results for the dispersion relation of the plasmon branch are derived in a tight-binding approximation which takes into account the umklapp effects in the superlattice direction. The long-wave intra- and interband magnetoplasmon frequencies in a degenerate electron gas are calculated both in quasiclassical and in quantum limits. In case of a large number of the filled electron levels, associated with the orbital motion of electrons, the magnetoplasmon frequencies exhibit the oscillations, which are similar to the de Haas–van Alphen oscillations upon variation of nanotube parameters and the Aharonov–Bohm oscillations upon variation of the magnetic flux through the nanotube cross-section.

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1. Introduction

The idea of Keldysh [1] to control the band spectrum of a semiconductor by creation of an additional periodicity, so-called superlattice, has been intensively developed as a result of the recent technological advances [2, 3]. In a one-dimensional (1D) superlattice the electron energy along the axis is a structure of periodical potential barriers and wells with a periodicity larger than the lattice constant. As a result, the structure of minibands separated by energy gaps appears in the energy spectrum of the electrons travelling along the axis of the superlattice. This behavior of the energy spectrum manifests itself in various other phenomena.

The physical properties of superlattices with flat geometry have been considered in many research papers, reviews and books. The de Haas–van Alphen oscillations in superlattices and layered systems have been studied in Ref. [4]. Reference [5] provides a comprehensive review of superlattice physics and contains an extensive discussion of the research papers on plasma oscillations in superlattices in zero magnetic field. The metal–dielectric transitions in superlattices in magnetic field have been considered and the photoconductivity has been calculated in Ref. [6]. References [7] and [8] address in detail the high-frequency properties of semiconductor superlattices. Reference [9] describes classification of superlattices, their fabrication, and multiple applications. One can grow sequences of different semiconductor layers, which are smooth on an atomic scale and have the layer thickness in the range from below 10 Å and up to a few hundred Å, by using molecular beam epitaxy techniques or metal organic chemical vapor deposition. A compositional superlattice, which consists of two semiconductor materials with different band gaps, can be considered as a sequence of electron quantum wells separated by energy barriers [9]. One can produce the superlattices with thick barrier layers, causing the electrons to be fully confined in individual quantum wells, or superlattices with thin barrier layers, causing the electrons to tunnel through them and to form extended states in the superlattice direction, can be created [9]. The behavior and the properties of the 2D electron gas in a square superlattice are considered in Ref. [10].

The great interest to collective excitations in semiconductor superlattices exists since the beginning of semiconductor superlattice research. Magnetoplasma waves in semiconductor superlattices using the random phase approximation approach were considered in Refs. [11–13]. It was assumed that the magnetic field is oriented parallel to the axis of the 1D-superlattice and the broad and high barriers alternate with quantum wells. This causes the electrons to be localized inside the wells and the tunneling between the wells is not possible. The system can be approximated by a set of independent parallel planes containing 2D electron gas. We note that the Das Sarma–Quinn and Bloss theories assumed delta-shaped wave functions localized in separate quantum wells in order to approximate electronic states in the direction of the superlattice axis [12, 13]. The retardation effects during the wave propagation through the structure were not considered. These effects were taken into account in Ref. [14].

For semiconductor superlattices with thin barrier layers, electron tunneling between quantum wells can play an important role, because the system can no longer be

e-mail: georgiy.i.rashba@gmail.com

described as a combination of quasi-2D systems, but instead it has to be considered as a single 3D structure. The electron tunneling between the wells was studied in Ref. [15], where a semiconductor with 1D superlattice in the magnetic field, which is oriented along the axis of the lattice, was considered. The spectrum of magnetoplasmons was calculated based on the self-consistent field method [16] in the framework of the model of the electron effective mass and strong electron binding within the wells. It was assumed that the electron tunneling is weak. The authors of Ref. [15] have considered both the long-wave limit where the magnetoplasmon wave number is small compared to the width of the Brillouin zone along the axis of the superlattice and also the umklapp scattering [17, 18]. The authors of Ref. [15] demonstrated that the electron drift along the superlattice axis causes the appearance of new branches in the wave spectrum. The corresponding superlattices and magnetoplasmons in Ref. [15] were named "tunneling". The electromagnetic and magnetoimpurity waves in superlattices and layered conductors in the conditions of the quantum Hall effect were studied in Ref. [19].

In all references mentioned above, the superlattices with the planar geometry were considered. However, the modern technology allows the fabrication of planar, and also of radial and longitudinal cylindrical superlattices [20, 21]. The radial cylindrical superlattice contains a structure with coaxial cylinders with alternating properties while a longitudinal superlattice can be modelled as a structure with coaxial rings.

In Ref. [22], the spectrum of magnetoplasma waves in an electron gas on the surface of a semiconductor nanotube with a longitudinal superlattice in a magnetic field parallel to the axis of the tube and superlattice is calculated on the basis of the effective mass model and the strong-binding method of electrons in the random phase approximation. In the quantum and quasiclassical cases, the frequencies of long-wavelength intraband and interband magnetoplasmons in a degenerate electron gas were calculated.

In Ref. [23], the Kubo formula was obtained for the conductivity tensor of an electron gas on the surface of a nanotube with a superlattice in a magnetic field. In the same work, the components of the high-frequency conductivity tensor were calculated in the quantum and quasiclassical cases. The results of Ref. [23] were used in [24] to obtain spectra of plasma waves in the hydrodynamic approximation on the surface of a semiconductor nanotube with a longitudinal superlattice.

In Refs. [22–24], the magnetoplasmons are investigated in nanotubes with a superlattice, but without the influence of umklapp processes. In these papers, a plane wave is used as the basic wave function of longitudinal motion on the tube. Meanwhile, in the case of an artificially created periodicity in superlattices, it is more natural to use the Bloch-type wave function.

In Sect. 2, the self-consistent field method is applied to a tube with a superlattice, and in Sect. 3 the derivation of the dispersion equation for plasmons on a tube with a superlattice with allowance for umklapp processes is given. Section 4 presents the results of calculating the spectra of tunnel intraband and interband magnetoplasmons with allowance for the influence of umklapp processes. Section 6 summarizes the main results of the present article.

2. Self-consistent field method for an electron gas of a semiconductor nanotube with a tunnel superlattice

Let us consider magnetoplasma waves based on the effective mass model and the strong electron binding in the framework of Ehrenreich and Cohen self-consistent field approach (SCF-formalism) [15, 16]. The presence of a one-dimensional superlattice on the surface of a semiconductor nanotube with a period d in the presence of an external magnetic field with induction $\boldsymbol{B}$ is taken into account. The magnetic field is assumed to be directed along the axis of the tube z, which coincides with the axis of the superlattice. A longitudinal superlattice causes alternation of potential barriers and wells for electrons moving along the tube. They can be described by expression

$$\varepsilon_k = \varDelta\left(1-\cos kd\right), \tag{1}$$

which is usually used in the study of the properties of semiconductor superlattices [6–8]. In Eq. (1), $\varDelta$ is the amplitude of the modulating potential, d is period, k is the quasi-wave number. The energy of the circular motion of the electron is [25] $\varepsilon_l = \varepsilon_0(l+\eta)^2$, where $l = 0, \pm1, \dots$ are the azimuthal quantum number, $\varepsilon_0 = (2m_* a^2)^{-1}$ is rotational quantum, m_* is effective mass of an electron, a is tube radius, $\eta = \varPhi/\varPhi_0$ is magnetic flux ratio with $\varPhi = \pi a^2 B$ through the cross-section of the tube and the flux quantum $\varPhi_0 = 2\pi c/e$ (e is the magnitude of the electron charge, c is the speed of light) [25, 26]. It is assumed that the longitudinal effective mass of an electron is equal to the transverse mass. Thus, the energy of an electron on a tube with a longitudinal tunnel superlattice is [22–24]:

$$\varepsilon_{lk} = \varepsilon_0\left(l+\eta\right)^2 + \varDelta\left(1-\cos kd\right). \tag{2}$$

The spin splitting of the levels in Eq. (2) is not taken into account. The quantum constant is taken equal to unity. Thus, the energy spectrum of an electron is a set of minizones with a width $2\varDelta$ with root singularities of the density of states at their boundaries $\varepsilon_l, \varepsilon_l + 2\varDelta$ [27].

In this section, the quantum states of electrons on the tube surface in the absence of umklapp processes can be described using a wave function of the form

$$\psi_{lk}\left(\varphi, z\right) = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{\mathrm{i}l\varphi}\frac{1}{\sqrt{L}}\,\mathrm{e}^{\mathrm{i}kz}, \tag{3}$$

where φ, z are cylindrical coordinates, L is length of the tube.

More generally, in a periodic tunneling superlattice, the z-component of the electron wave function should be of the Bloch form [15] $|k_z\rangle = \mathrm{e}^{\mathrm{i}k_z z}\sum_n A_n \mathrm{e}^{\mathrm{i}nGz}$, where $G = 2\pi/d$. Thus, here we take into account only the term $n = 0$ in this expansion. The $n \neq 0$ terms, which are referred to as "umklapp terms", will be considered in Sect. 3 of this article. It will be shown in Sect. 3 that the "effective-mass"-theory results (3) are valid for long-wavelength plasmons with $q_z \ll G$.

In the framework of SCF-formalism, we start from the integral form of the Poisson equation with the potential

$$V(\boldsymbol{r},t)=\int \mathrm{d}\boldsymbol{r}'\frac{e^2 n_1(\boldsymbol{r}',t)}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{4}$$

where $n_1(\boldsymbol{r},t)=n-n_0$ is deviation of the surface concentration of electrons from its equilibrium value n_0. This approach is based on the theory of linear response, in which the collective states appear as resonances in the linear response of the gas to an external perturbation [16, 28].

We start by considering an electron gas on the surface of a semiconductor nanotube in a longitudinal (along the axis of the tube) uniform magnetic field, initially in thermal equilibrium, which is subsequently perturbed by a weak external field, whose dependence on coordinates and time in a cylindrical coordinate system is represented in the form [22–24]:

$$U(\boldsymbol{r},t)\sim \exp\left(-\mathrm{i}\omega t+\mathrm{i}m\varphi+\mathrm{i}qz\right). \tag{5}$$

Under the influence of perturbation (5), the surface density of the electron gas is redistributed. It is determined by the linear non-equilibrium correction to the density matrix $\hat{\rho}_1$ due to this perturbation, averaged over the statistical ensemble: $n_1(\varphi,z,t)=\langle\varphi z|\hat{\rho}_1(t)|\varphi z\rangle$. The values of the frequency and the wavelength, at which the perturbing potential (5) leads to a resonant response, are determined by the dispersion equation for density waves, which can propagate in an electron gas [16, 28]. In this method, we interpret the Coulomb interaction between electrons on the nanotube surface as a self-consistent field of the form

$$V(\varphi,z,t)=\int \mathrm{d}^2r'\frac{e^2\left[n\left(\varphi',z',t\right)-n_0\right]}{\sqrt{4a^2\sin^2\frac{\varphi-\varphi'}{2}+\left(z-z'\right)^2}}, \tag{6}$$

where $\mathrm{d}^2r=a\,\mathrm{d}\varphi\,\mathrm{d}z$. Equation (6) defines the integral form of the Poisson equation in the tube.

Expressing the field V through the non-equilibrium contribution $n_1(\boldsymbol{r},t)$, we obtain the nonlocal expression

$$V(\varphi,z,t)=\int \mathrm{d}^2r'\,\upsilon\left(\varphi-\varphi',z-z'\right)n_1\left(\varphi',z',t\right), \tag{7}$$

where

$$\upsilon(\varphi,z)=\frac{e^2}{\sqrt{4a^2\sin^2\frac{\varphi}{2}+z^2}},$$

which is the potential of the Coulomb field on the nanotube surface. Taking the Fourier components in (7) and using the integral convolution theorem, we obtain the local relation

$$V(m,q,\omega)=\upsilon(m,q)\,n_1(m,q,\omega), \tag{8}$$

where

$$\upsilon(m,q)=4\pi e^2 a I_m(|q|a)\,K_m(|q|a), \tag{9}$$

which is the cylindrical harmonics of the Coulomb potential of electrons on the tube [22], I_m and K_m are modified Bessel functions.

Small longitudinal oscillations of the electron gas on the tube can be described with the help of the linearized Liouville equation for the density matrix [28] with a self-consistent field (6):

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\hat{\rho}_1(t)=\left[\hat{H}_0,\hat{\rho}_1\right]+\left[\hat{V},\hat{\rho}_0\right]+\left[\hat{U},\hat{\rho}_0\right], \tag{10}$$

where $\hat{H}_0$ is the Hamiltonian of one electron, $\hat{U}$ is contribution of the external field to the interaction energy. We consider the diagonal matrix elements $\hat{\rho}_0$ in the representation of the eigenstates of the Hamiltonian $\hat{H}_0$ (3). They are determined by the Fermi distribution function

$$\langle l'k'|\,\hat{\rho}_0\,|lk\rangle=f\left(\varepsilon_{lk}\right)\delta_{l'l}\delta_{k'k},$$

$$f_{lk}=f\left(\varepsilon_{lk}\right)=\frac{1}{\mathrm{e}^{\beta(\varepsilon_{lk}-\mu)}+1},$$

where β is reverse temperature, μ is chemical potential.

From Eq. (10) it is easy to see that the matrix elements $\hat{\rho}_1$ in the basis $\hat{H}_0$ satisfy the equations

$$\left(\mathrm{i}\hbar\frac{\partial}{\partial t}-\varepsilon_{lk}+\varepsilon_{l'k'}\right)\langle lk|\,\hat{\rho}_1(t)\,|l'k'\rangle= \left(f_{l'k'}-f_{lk}\right)\langle lk|\,\hat{V}+\hat{U}\,|l'k'\rangle. \tag{11}$$

The Fourier transform on the time of expression (11) has the form

$$\langle lk|\,\hat{\rho}_1(\varphi,z,\omega)\,|l'k'\rangle=\frac{f_{l'k'}-f_{lk}}{\omega-\varepsilon_{lk}+\varepsilon_{l'k'}} \times\langle lk|\,\hat{V}(\varphi,z,\omega)+\hat{U}(\varphi,z,\omega)\,|l'k'\rangle. \tag{12}$$

Let us write the Fourier component of the non-equilibrium electron density on the tube in the form

$$n_1(m,q,\omega)=\sum_{lkl'k'}\int \mathrm{d}S\,\mathrm{e}^{-\mathrm{i}m\varphi}\mathrm{e}^{-\mathrm{i}qz} \times\langle\varphi z|lk\rangle\langle lk|\,\hat{\rho}_1(\omega)\,|l'k'\rangle\langle l'k'|\varphi z\rangle, \tag{13}$$

where $\mathrm{d}S=a\,\mathrm{d}\varphi\,\mathrm{d}z$, and also take into account the equality

$$\langle lk|\,\hat{V}(\varphi,z,\omega)+\hat{U}(\varphi,z,\omega)\,|l'k'\rangle=$$

$$\langle lk|\sum_m\int\frac{\mathrm{d}^2q}{(2\pi)^2}\mathrm{e}^{\mathrm{i}m\varphi}\mathrm{e}^{\mathrm{i}qz}\Big\{\upsilon(m,q)\,n_1(m,q,\omega) +U(m,q,\omega)\Big\}\,|l'k'\rangle. \tag{14}$$

Equations (13) and (14) allow us to reduce Eq. (12) to the integral equation for $n_1(m,q,\omega)$:

$$n_1(m,q,\omega)=\sum_{lkl'k'}\frac{1}{S}\sum_{m'q'}\langle lk|\,\mathrm{e}^{-\mathrm{i}m\varphi}\mathrm{e}^{-\mathrm{i}qz}\,|l'k'\rangle \times\frac{f_{lk}-f_{l'k'}}{\omega-\varepsilon_{l'k'}+\varepsilon_{lk}+\mathrm{i}0}\langle l'k'|\,\mathrm{e}^{\mathrm{i}m'\varphi}\mathrm{e}^{\mathrm{i}q'z}\,|lk\rangle \times\left\{\upsilon\left(m',q'\right)n_1\left(m',q',\omega\right)+U\left(m',q',\omega\right)\right\}. \tag{15}$$

If the Hamiltonian $\hat{H}_0$ is a translation invariant system, the kernel of Eq. (15) contains the factor $\delta_{mm'}\delta_{kk'}$, which makes it possible to solve this equation algebraically. As a result, the solution of Eq. (15) in the basis of the functions (3) can be represented in the form

$$n_1(m,q,\omega) = \frac{P(m,q,\omega)}{1 - v(m,q)\,P(m,q,\omega)} U(m,q,\omega), \tag{16}$$

where $P(m,q,\omega)$ is polarization operator of an electron gas on the surface of a semiconductor nanotube in a magnetic field along the axis of the tube, taking into account the longitudinal superlattice. Equating the denominator in expression (16) to zero, we obtain a dispersion equation, whose solutions give a spectrum of plasmons on a tube with a superlattice without taking into account umklapp phenomena [22].

In the framework of the formalism of the self-consistent field of Ehrenreich and Cohen [16], the total potential can be decomposed into the sum of external and induced potentials

$$V = V^{ex} + V^{in}, \tag{17}$$

which is reflected in the sum of the two terms in curly brackets in (15).

3. Dispersion equation for plasmons, accounting for the effect of umklapp processes in the case of weak tunneling

In Sect. 2 we have neglected umklapp processes [17] due to the periodicity of the superlattice in the z direction. They are important, when the z component of the plasmon wave vector is close to a reciprocal wave vector nG, where $G = 2\pi/d$. In this section we take umklapp processes into account and derive the results valid for any q_z. We will work with the Bloch states of the form

$$\psi_{lk}(\varphi,z) = \frac{e^{il\varphi}}{\sqrt{2\pi}} \frac{1}{\sqrt{N}} \sum_n e^{iknd} \Phi(z - nd), \tag{18}$$

where N is the number of cells along the axis z, $\Phi(z-nd)$ is a tight-binding (Wannier) wave function centered on the n-th quantum well.

Because of umklapp processes in the z direction, the density response on the tube $n_1(m,q,\omega)$ is no longer related to the potential $V_m(q)$ with the same wave vector q only, rather it is related to all $V_m(q')$ with $q' = q + mG$, where m is an integer. We start from Eq. (15), rewriting it in a suitable form to divide it further the longitudinal and transverse types of motion of electrons on the tube

$$n_1(m,q) = 2 \sum_{lkl'k'} \frac{f_{l'}(k') - f_l(k)}{\varepsilon_{l'}(k') - \varepsilon_l(k) - \hbar\omega - i0} \times \langle lk| V(\varphi,z) |l'k'\rangle \langle l'k'| e^{im\varphi} e^{iqz} |lk\rangle, \tag{19}$$

where V is the total potential that can be decomposed into the sum of the external and induced potentials (17). Using the Fourier transform of $V(\varphi,z)$, we can write (19) in the form

$$n_1(m,q,\omega) = \sum_{kk'q'} V_m(q') \langle k| e^{-iqz} |k'\rangle \langle k'| e^{iq'z} |k\rangle \times P_m(k,k',\omega), \tag{20}$$

where

$$P_m(k,k',\omega) = \frac{1}{S} \sum_{ll'} \frac{f_{l'}(k') - f_l(k)}{\varepsilon_{l'}(k') - \varepsilon_l(k) - \hbar\omega - i0} \left|\langle l| e^{il\varphi} |l'\rangle\right|^2. \tag{21}$$

The function $P_m(k,k',\omega)$ is periodic in k and k' with period G. Using the wave function (18), we represent the first matrix element in (20) as

$$\langle k| e^{-iqz} |k'\rangle = \delta_{k'-k-q,m_1G} A_{k'}(q), \tag{22}$$

where

$$A_k(q) = \sum_n e^{iknd} \int_{-\infty}^{+\infty} dz \Phi^*(z)\, e^{-iqz} \Phi(z - nd). \tag{23}$$

Similarly for the second matrix element in (20) we obtain

$$\langle k'| e^{iq'z} |k\rangle = \delta_{k-k'+q',m_2G} A_k(-q').$$

Here we emphasize that the tunneling between semiconductor rings reflects two aspects within the framework of the used formalism. First, an energy dispersion in the z direction is introduced, so that the function $P_m(k,k',\omega)$ depends on the bandwidth Δ. Second, the matrix element in (22) and (23) contains terms, caused by overlaps of electron wave functions $\Phi(z-nd)$, which are centered on different quantum wells. If the overlaps of wave functions are small, the dominant term in (23) is the $n=0$ term. Thus, the effect of tunneling originates mostly from the z-direction energy dispersion in the function (21). The same consideration is used in perturbation theory in quantum mechanics, namely if the energy is approximated to the first order, then the wave function can be approximated to the zeroth order only [15]. Therefore, for weak tunneling the $n \neq 0$ overlap terms in (23) can be neglected, and $A(q)$ no longer depends on k [15]:

$$A(q) = \int_{-\infty}^{+\infty} dz \Phi^*(z)\, e^{-iqz} \Phi(z). \tag{24}$$

Hence, we obtain

$$\begin{aligned} n_1(m,q,\omega) &= \\ &\sum_{kq'} V_m(q') P_m(k,k+q)\, \delta_{q'-q,(m_1-m_2)G} A(q') A^*(q) = \\ &A^*(q) P_m(q,\omega) \sum_{q'} \delta_{q'-q,nG} V_m(q') A(q') = \\ &A^*(q) P_m(q,\omega) \sum_n V_m(q+nG) A(q+nG), \end{aligned} \tag{25}$$

where

$$P_m(q,\omega) = \sum_k P_m(k,k+q) = \frac{1}{\pi a L} \sum_{lk} \frac{f\left(\varepsilon_{(l+m)(k+q)}\right) - f(\varepsilon_{lk})}{\varepsilon_{(l+m)(k+q)} - \varepsilon_{lk} - \hbar\omega - i0}$$

which is the polarization operator of an electron gas on a tube with a superlattice [22]. Here, f is the Fermi function and L is the tube length. In the last equality in (25), we used the fact that, since $\sum_{q'} = \frac{L}{2\pi}\int_{-\infty}^{+\infty} dq'$, we can sum up not by q', but by n. Further, in formula (25), we replace q by $q + pG$, we put as usual in the theory of collective excitations $V_{in} = V$ [15], and in the right-hand side of this formula we substitute the expression (25). As a result, we obtained

$$V_m(q+pG)=4\pi e^2 a I_m(|q+pG|a)K_m(|q+pG|a)$$
$$\times P_m(q+pG,\omega)A^*(q+pG)$$
$$\times\sum_{n=-\infty}^{\infty}V_m(q+nG)A(q+nG). \qquad (26)$$

Multiplying (26) by $A(q+pG)$ and summing over p, we get

$$4\pi e^2 a\sum_{p=-\infty}^{\infty}P_m(q+pG,\omega)I_m(|q+pG|a)$$
$$\times K_m(|q+pG|a)|A(q+pG)|^2=1. \qquad (27)$$

Taking into account expression (11) and periodicity of the P_m with period G, Eq. (27) can be represented in the form

$$P_m(q,\omega)\sum_{p=-\infty}^{\infty}|A(q+pG)|^2 \upsilon_m(|q+pG|a)=1. \qquad (28)$$

Equation (28) is a dispersion equation, which should be used for derivation of the plasmon spectrum in a weakly tunneling superlattice on the surface of a nanotube with allowance for umklapp phenomena. If we retain in summation $\sum_p$ only the term with $p=0$ in (28) and assume that $|A(q)|^2=1$, then we obtain the usual dispersion equation in the random phase approximation for magnetoplasma waves propagating along the tube [22, 29, 30]:

$$\upsilon(m,q)P(m,q,\omega)=1.$$

Now we consider the consequences of Eq. (28), using some simple models for the function $A(q)$.

If we consider the extremely weak-tunneling limit and assume $\Phi(z-nd)$ to be δ-function-like, $\Phi^2(z-nd)=\delta(z-nd)$, then $A=1$ [15]. If in addition, $\Delta=0$, then there is no energy dispersion in the direction of the axis z, and the tube with the superlattice behaves like a system of independent uniaxial rings. In this case, the superlattice is a layered system consisting of conducting rings separated from each other by insulators with a width d. Since electron tunneling is neglected, the electrons of different rings are correlated only by interaction with the electromagnetic field. A charge fluctuation created in one of the rings generates an electromagnetic field that polarizes adjacent rings and, in turn, creates a perturbation of the charge density in them. Repeatedly repeating, this process moves along the axis of the superlattice across the rings. This is the nature of the propagation of a plasma wave in this limiting case.

In typical semiconductor superlattices with weak or moderate tunneling the function $\Phi(z-nd)$ is not δ-function. Instead, it should behave like a packet confined mostly within a quantum well [15]. A simple, but physically plausible, wave function model has the Gaussian form [15]:

$$\Phi(z-nd)\sim\exp\left[-\frac{(z-nd)^2}{2\xi^2}\right], \qquad (29)$$

where ξ is the half-width. Assuming $\int_{-\infty}^{\infty}\mathrm{d}z\,\Phi^2(z)=1$, we get

$$\Phi(z-nd)=\frac{1}{\pi^{1/4}\sqrt{\xi}}\exp\left(-\frac{(z-nd)^2}{2\xi^2}\right).$$

We note that in Ref. [18] the localization of the electron density at the lattice nodes was also approximated by Gaussian functions. If the function $\Phi(z-nd)$ is mostly confined within a range of d, then we obtain $\xi<d/2$. For such a wave function, following [15], we obtained

$$A_k(q)=\exp\left(-\left(\frac{q\xi}{2}\right)^2\right)$$
$$\times\left[1+2\sum_{n=1}^{\infty}\exp\left(-\left(\frac{nd}{2\xi}\right)^2\right)\cos\left(\left(k-\frac{q}{2}\right)nd\right)\right]\simeq$$
$$\exp\left(-\left(\frac{q\xi}{2}\right)^2\right). \qquad (30)$$

Due to the exponential decay, the terms with $n\geq 1$ in (30) can be neglected. It is a very good approximation, if $\xi\leq d/4$. The dispersion Eq. (28), taking into account Eq. (9), then takes the form

$$4\pi e^2 aP_m(q,\omega)\sum_{p=-\infty}^{\infty}I_m(|q+pG|a)K_m(|q+pG|a)$$
$$\times\exp\left(-\frac{1}{2}(q+pG)^2\xi^2\right]=1. \qquad (31)$$

The denominator of the formula for the polarization operator $P_m(q,\omega)$ in (31) contains the frequencies of vertical electron transitions between the levels ε_l:

$$\Omega_\pm(l,m)=\begin{pmatrix}\varepsilon_{l+m}-\varepsilon_l\\ \varepsilon_l-\varepsilon_{l-m}\end{pmatrix}=\varepsilon_0\left[2m(l+\eta)\pm m^2\right]. \quad (32)$$

Every integer value m is related to a branch in the plasmon wave spectrum. At $m=0$ the solutions of Eq. (31) represent the intra-band plasmon spectrum. The values $m\neq 0$ correspond to inter-band plasmons.

In the dimensionless variables $x=(k\pm/2)d$, in the case of a completely degenerate electron gas on the tube under $T=0$ K, taking into account Eqs. (2), (18), and (25), the real part of the polarization operator can be represented in the following form:

$$\mathrm{Re}P_m(q,\omega)=-\frac{1}{2\pi^2 ad}$$
$$\times\sum_l\left(\int_{(-k_l+q/2)d}^{(k_l+q/2)d}\frac{\mathrm{d}x}{\Omega_+-\omega}\left(1+\frac{2\Delta\sin\frac{qd}{2}\sin x}{\Omega_+-\omega}\right)^{-1}\right.$$
$$\left.-\int_{(-k_l-q/2)d}^{(k_l-q/2)d}\frac{\mathrm{d}x}{\Omega_--\omega}\left(1+\frac{2\Delta\sin\frac{qd}{2}\sin x}{\Omega_--\omega}\right)^{-1}\right), \qquad (33)$$

where

$$k_l=\frac{1}{d}\arccos\left(1-\frac{\mu_0-\varepsilon_l}{\Delta}\right) \qquad (34)$$

is the maximal wave number for an electron in the l-th miniband and μ_0 is the Fermi energy. We note that this expression resembles in its form the expression for the polarization operator of a quasi-one-dimensional (chain) metal with a narrow $1D$-band [18] and the polarization operator for a degenerate two-dimensional system in the tight-binding approximation [31].

The dispersion equation (31) describes all types of plasma excitations in the system with allowance for umklapp processes along the tube axis. When m and q are not equal to zero, there are two types of related plasma excitations: inter-band plasmons and intra-band plasmons. The existence of the latter is due to electron tunneling along the axis of the tube, therefore they are called tunnel plasmons.

4. Plasmon spectra with allowance for the influence of umklapp processes

4.1. Spectrum of tunnel intraband plasmons

To obtain an analytic expression for the spectrum of a tunnel plasmon, taking into account the effect of the umklapp processes, we use the dispersion Eq. (31) under the condition $m = 0$, and we also perform the expansion in Eq. (33) with respect to the parameter $2\Delta\sin(\frac{qd}{2})/\omega$. As a result, from the formula (33) we obtain [22]:

$$\mathrm{Re}P_0\left(q,\omega\right) = \frac{4\Delta\sin^2\frac{qd}{2}}{\pi^2 a d\omega^2}\sum_l \sin k_l d, \tag{35}$$

where

$$\sin k_l d = \frac{1}{\Delta}\left[\left(\mu_0-\varepsilon_l\right)\left(\varepsilon_l+2\Delta-\mu_0\right)\right]^{1/2}.$$

The solution of the dispersion Eq. (31) with allowance for (35) gives the spectrum of axially symmetric tunnel intraband plasmons

$$\omega_0^2\left(q\right) = \frac{4}{\pi^2\hbar^2 a d}\sin^2\frac{qd}{2} \times \sum_{l=-\infty}^{\infty}\sqrt{\left(\mu_0-\varepsilon_l\right)\left(\varepsilon_l+2\Delta-\mu_0\right)} \times \sum_{p=-\infty}^{\infty} u\left(0,q+pG\right)\exp\left(-\frac{1}{2}\left(q+pG\right)^2\xi^2\right), \tag{36}$$

where $G = 2\pi/d$ is vector of the reciprocal lattice,

$$u\left(0,q+pG\right) = 4\pi e^2 a I_0\left(\left|q+pG\right|a\right)K_0\left(\left|q+pG\right|a\right). \tag{37}$$

We further assume that there are many energy levels ε_l below the Fermi level, i.e.$\mu_0 \gg \varepsilon_0$. Then, the summation in l in Eq. (35) can be done using the Poisson summation formula [22]. As a result, the plasmon spectrum contains monotone terms and terms which oscillate with the variation of the tube parameters and magnetic flux Φ. They depend on the parameter $\mu_0/2\Delta$.

If $\mu_0 < 2\Delta$, we can derive the exact expression for the monotone part of the plasmon spectrum from Eq. (36):

$$\omega_{mon}^2\left(q\right) = \frac{8\sqrt{m_*}\Delta^{3/2}}{\pi^2 d}\sin^2\left(\frac{qd}{2}\right) \times \left[\frac{2\Delta-\mu_0}{\Delta}K\left(\sqrt{\frac{\mu_0}{2\Delta}}\right) - 2\frac{\Delta-\mu_0}{\Delta}E\left(\sqrt{\frac{\mu_0}{2\Delta}}\right)\right] \times \sum_{p=-\infty}^{\infty} u\left(0,q+pG\right)\exp\left(-\frac{1}{2}\left(q+pG\right)^2\xi^2\right], \tag{38}$$

where K and E are the elliptical integrals of the 1st and 2nd kind, respectively [32]. When $a \to \infty$ is considered, the function $u_0(q)$ is equal to $2\pi e^2/(q+pG)$ and Eq. (38) does not depend on the tube radius. It is related to the case of 2D electron gas with a one-dimensional superlattice without magnetic field. It is obtained by cutting the tube along the generatrix and then unwrapping it onto the plane.

The oscillating part of the plasmon spectrum in the case of $\mu_0 < 2\Delta$ is

$$\omega_{osc}^2\left(q\right) = \frac{4\sqrt{2\Delta}\left(\mu_0\varepsilon_0\right)^{1/4}}{\pi^2 a d}\sin^2\left(\frac{qd}{2}\right) \times \sum_{r=1}^{\infty}\frac{1}{r^{3/2}}\cos\left(2\pi r\eta\right)\sin\left(2\pi r\sqrt{\frac{\mu_0}{\varepsilon_0}}-\frac{\pi}{4}\right) \times \sum_{p=-\infty}^{\infty} u\left(0,q+pG\right)\exp\left(-\frac{1}{2}\left(q+pG\right)^2\xi^2\right). \tag{39}$$

This expression describes the oscillations similar to the de Haas–van Alphen and Aharonov–Bohm oscillations. The oscillations of the de Haas–van Alphen type are conserved in the absence of the magnetic field. In the case of $\mu_0 < 2\Delta$, the oscillations are caused by the passage of the square-root singularities of the density of states at ε_l through the Fermi level, if the nanotube parameters, the radius, and electron density vary. The oscillation frequency can be extracted from the condition $\mu_0 = \varepsilon_0 l^2$. When the tube radius varies, the frequency is equal to the Fermi wave number k_F. If the dependence of Eq. (39) on the square root of the electron density is considered, the frequency is equal to $\sqrt{2\pi}a$.

At $\mu_0 \gg \varepsilon_0$, $\mu_0 - 2\Delta \gg \varepsilon_0$, $\mu_0 > 2\Delta$ the monotone and oscillating parts of the magnetoplasmon spectrum with inclusion of the umklapp processes are equal

$$\omega_{mon}^2\left(q\right) = \frac{8\sqrt{2m_*\mu_0}\Delta}{\pi^2 d}\sin^2\left(\frac{qd}{2}\right) \times \left[\frac{\mu_0-\Delta}{\Delta}E\left(\sqrt{\frac{2\Delta}{\mu_0}}\right) - \frac{\mu_0-2\Delta}{\Delta}K\left(\sqrt{\frac{2\Delta}{\mu_0}}\right)\right] \times \sum_{p=-\infty}^{\infty} u\left(0,q+pG\right)\exp\left(-\frac{1}{2}\left(q+pG\right)^2\xi^2\right), \tag{40}$$

$$\omega_{osc}^2\left(q\right) = \frac{4\sqrt{2\Delta}\left(\mu_0\varepsilon_0\right)^{1/4}}{\pi^2 a d}\sin^2\left(\frac{qd}{2}\right)\times$$

$$\times \sum_{r=1}^{\infty} \frac{1}{r^{3/2}} \cos(2\pi r\eta) \left[\sin\left(2\pi r \sqrt{\frac{\mu_0}{\varepsilon_0}} - \frac{\pi}{4}\right) - \left(1 - \frac{2\Delta}{\mu_0}\right)^{1/4} \sin\left(2\pi r \sqrt{\frac{\mu_0 - 2\Delta}{\varepsilon_0}} + \frac{\pi}{4}\right)\right]$$
$$\times \sum_{p=-\infty}^{\infty} u(0, q + pG) \exp\left(-\frac{1}{2}(q + pG)^2 \xi^2\right). \quad (41)$$

In order to calculate Eqs. (39) and (41), the asymptotic approximations of Fourier integrals have been used, when the integrand has singularities at the lower limit (in the case of Eq. (39)) or at both upper and lower limits (in the case of Eq. (41)) [22]. The second period in the oscillations of the de Haas–van Alphen type is present due to the fact that at $\mu_0 > 2\Delta$ the Fermi level crosses both the lower and the upper boundaries $\varepsilon_l + 2\Delta$ of the minibands upon the variation of the tube parameters. This case is similar to the existence of the minimal and maximal cross-sections of the corrugated cylindrical Fermi surface in the de Haas–van Alphen effect [33, 34]. The second oscillation frequency in Eq. (41) can be derived from the condition $\varepsilon_l + 2\Delta = \mu_0$. The existence of two oscillations with close frequencies at $\mu_0 \gg 2\Delta$ indicates that the plot of Eq. (41) versus the tube parameters should have a beating pattern [22].

4.2. Spectrum of the interband plasmons

At $m \neq 0$ we have the condition

$$2\Delta \sin\left(\frac{qd}{2}\right) \ll |\omega - \Omega_{\pm}|.$$

In this case the main contribution to the real part of the polarization operator (33) at $q = 0$ is equal to

$$\mathrm{Re}P_m(0, \omega) = \frac{1}{\pi^2 a} \sum_l k_l \left(\frac{1}{\Omega_- - \omega} - \frac{1}{\Omega_+ - \omega}\right).$$

Consequently, Eq. (31) has two roots in the quantum limit. The first one is located below Ω_- and the second one above Ω_+. In particular, in the case of $\eta < 1/2$, $l = 0$ we obtain from Eq. (32) that $\Omega_- < 0$ and $\Omega_+ > 0$. The spectrum of the long-wavelength interband plasmons near the frequency Ω_+ has the form

$$\omega_m(q) = 2m\varepsilon_0\eta + \left[\left(\varepsilon_0 m^2 + \frac{u_m(q) k_0}{\pi^2 a}\right)^2 - \left(\frac{u_m(q) k_0}{\pi^2 a}\right)^2\right]^{1/2}, \quad (42)$$

where it is necessary to renormalize the Fourier component of the Coulomb interaction on the tube

$$u_m(q) \to \sum_{p=-\infty}^{\infty} u(m, q + pG) \exp\left(-\frac{1}{2}(q + pG)^2 \xi^2\right). \quad (43)$$

We also emphasize that in Eq. (42) the superlattice parameters d and Δ are only contained in k_0 (Eq. (34)).

5. Discussion

Let us compare the results obtained in this article with the corresponding formulas for the plasmon spectra without taking into account the influence of the umklapp processes obtained in Ref. [22]. We will compare Eq. (15) with Eq. (36), Eq. (17) with Eq. (38), Eq. (18) with Eq. (39), Eq. (19) and Eq. (20) with Eq. (40) and Eq. (41). One can see that the effective-mass theory in Sect. 2 does give the correct result when $q \ll G$, because if $\xi \neq 0$, the summation over p in Eq. (31) is dominated by the $p = 0$ term when $q \ll G$. We see that the results for plasmon spectra without and with allowance for the umklapp processes agree within a correction factor $|A(q)| \simeq \exp(-(\frac{q\xi}{2})^2)$. This factor originates from the detailed form of the wave function in the lattice node (29) and it is close to unity when $q \ll G$ and $\xi < d/2$. The effective-mass wave function (3) used in Sect. 2 and the tight-binding wave function (18) used in Sect. 3 and Sect. 4 to take into account the umklapp processes are very different. We conclude that in the limit $q \ll G$ the detailed form of the wave function is not very important for the calculation of the spectrum of long-wavelength plasmons as long as the electron wave functions are extended in the nanotube axis direction.

6. Conclusion

This article presents the results of calculations of the spectrum of magnetoplasma waves in a degenerate electron gas on the surface of a semiconductor nanotube with a longitudinal superlattice in a longitudinal magnetic field, taking into account the umklapp processes. It was shown that the effect of the longitudinal superlattice along the tube axis mix the Fourier components of the Coulomb interaction differing by reciprocal-lattice vectors $\boldsymbol{G}$. It can be seen from Eq. (43) that the localization of the electron density at the superlattice cells in model (29) in combination with the umklapp processes along the nanotube axis has the effect of replacing the usual bare Coulomb potential on the tube Eq. (9) by the renormalized Coulomb potential, as defined in Eq. (43). The latter depends on ξ, the length of the wave function (29) in the direction of the tube axis through the "form factor" $\exp(-\frac{1}{2}(q + pG)^2\xi^2)$, which is proportional to the spatial Fourier component of the function (29).

In the present paper the intraband and interband plasmons have been considered in the quantum and quasiclassical cases. It has been shown that the wave frequencies exhibit oscillations similar to the de Haas–van Alphen and the Aharonov–Bohm oscillations upon the variation of the nanotube parameters and magnetic flux through its cross-section. The characteristics of the de Haas–van Alphen oscillation type are defined by the ratio of the Fermi energy to the miniband width. If the ratio is large, the plot of the plasmon frequency versus the tube radius or electron density exhibits a beating pattern. The beatings are not present if the ratio is small.

The umklapp processes considered in this article should be also taken into account in the calculation the thermodynamic functions of an electron gas on the surface of a nanotube [35], as well as the energy spectrum of electrons in a quantum dot with impurity atoms [36].

These peculiarities of the plasmon spectrum can be observed experimentally using light or electron scattering by the nanotubes with a superlattice in magnetic field.

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References

[1] L.V. Keldysh, *Fiz. Tverd. Tela* **4**, 2265 (1962).

[2] L. Esaki, R. Tsu, *IBM J. Dev.* **14**, 61 (1970).

[3] L.L. Chang, K. Ploog, *Molecular Beam Epitaxy and Heterostructures*, Martinus Nijhof, Dordrecht 1985.

[4] V.M. Gvozdikov, *Fiz. Tverd. Tela* **26**, 2574 (1984); V.M. Gvozdikov, *Fiz. Tverd. Tela* **28**, 320 (1986).

[5] A.P. Silin, *Usp. Fiz. Nauk* **147**, 485 (1985).

[6] V.N. Lutskii, M.I. Kaganov, A.Y. Shik, *Zh. Eksp. Teor. Fiz.* **92**, 721 (1987).

[7] F.G. Bass, A.P. Tetervov, *Phys. Rep.* **140**, 237 (1985).

[8] F.G. Bass, A.A. Bulgakov, A.P. Tetervov, *High-Frequency Properties of Semiconductors with Superlattices*, Nauka, Moscow 1989 (in Russian).

[9] M.A. Herman, *Semiconductor Superlattices*, Akademie-Verlag, Berlin 1986.

[10] S.Y. Glazov, S.V. Kruchkov, *Phys. Tech. Poluprovodn.* **34**, 835 (2000).

[11] M. Apostol, *Z. Phys. B* **22**, 13 (1975).

[12] D. Sarma, J.J. Quinn, *Phys. Rev.* **B25**, 7603 (1982).

[13] W.L. Bloss, *Solid State Commun.* **44**, 366 (1982).

[14] A.C. Tselis, J.J. Quinn, *Phys. Rev. B* **29**, 2021 (1984); A.C. Tselis, J.J. Quinn, *Phys. Rev. B* **29**, 3318 (1984).

[15] W.M. Que, G. Kirczenow, *Phys. Rev. B* **36**, 6596 (1987).

[16] H. Ehrenreich, M.H. Cohen, *Phys. Rev.* **115**, 786 (1959).

[17] S.L. Adler, *Phys. Rev.* **126**, 413 (1962).

[18] P.F. Williams, A.N. Bloch, *Phys. Rev.* **10**, 1097 (1974).

[19] V.M. Gvozdikov, A.M. Ermolaev, I.D. Vagner, R. Vega-Monroy, *Physica B* **284**, 1734 (2000).

[20] V.Y. Prinz, V.A. Seleznev, V.A. Samoylov, A.K. Gutakovsky, *Microelectron. Eng.* **30**, 439 (1996).

[21] V.P. Dragunov, I.G. Neizvestnyi, V.A. Gridchin, *Fundamentals of Nanoelectronics*, Logos, Moscow 2006 (in Russian).

[22] A.M. Ermolaev, G.I. Rashba, M.A. Solyanik, *Low Temp. Phys.* **38**, 511 (2012).

[23] A.M. Ermolaev, G.I. Rashba, *Physica B* **451**, 20 (2014).

[24] A.M. Ermolaev, G.I. Rashba, *Solid State Commun.* **192**, 79 (2014).

[25] I.O. Kulik, *Pisma Zh. Exp. Teor. Fiz.* **11**, 407 (1970).

[26] I.O. Kulik, *Fiz. Nizk. Temp.* **36**, 1057 (2010).

[27] A.M. Ermolaev, G.I. Rashba, M.A. Solyanik, *Fiz. Nizk. Temp.* **37**, 1033 (2011).

[28] N.D. Mermin, E. Canel, *Ann. Phys.* **26**, 247 (1964).

[29] A.I. Vedernikov, A.O. Govorov, A.V. Chaplik, *JETP* **93**, 853 (2001).

[30] R.Z. Vitlina, L.I. Magarill, A.V. Chaplik, *JETP* **106**, 793 (2008).

[31] E.A. Pashitskiy, Y.M. Malozovskiy, A.B. Semenov, *Ukr. J. Phys.* **36**, 889 (1991).

[32] I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series and Products*, Academic, New York 1980 [Nauka, Moscow, 1971 (Russian original)].

[33] I.M. Lifshitz, M.Y. Azbel', M.I. Kaganov, *Electron Theory of Metals*, Consultants Bureau, New York 1973 [Nauka, Moscow 1971 (Russian original)].

[34] I.M. Lifshits, V.G. Peschanskii, *Sov. Phys. JETP* **11**, 137 (1960).

[35] A.M. Ermolaev, G.I. Rashba, M.A. Solyanik, *Eur. Phys. J. B* **73**, 383 (2010).

[36] A.M. Ermolaev, G.I. Rashba, *Eur. Phys. J. B* **66**, 223 (2008).