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Cross-Correlations in Transport through a Quantum Dot Cooper Pair Splitter Asymmetrically Coupled to Normal Leads

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We investigate transport properties of a quantum dot-based Cooper pair splitter with two ferromagnetic leads and one superconducting electrode. The transport quantities of the system are calculated utilizing the real-time diagrammatic technique in the sequential tunneling regime. Particularly, we calculate the Andreev current and corresponding current cross-correlations, i.e. correlations between currents flowing through two junctions with normal leads. Main goal of the paper are studies on the influence of asymmetry in couplings to the normal leads and its magnetism on the Andreev transport.

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PACS/topics: quantum dots, Andreev transport, Cooper pair splitter, current cross-correlations

1. Introduction

Recently, electron transport in hybrid systems based on quantum dots has attracted much attention mainly due to possibility of constructing a device enabling efficiently creation of nonlocal entangled electron pairs [1] and its potential application in quantum computing [2]. Moreover, splitting of Cooper pairs into two spatially separated electrodes has been demonstrated experimentally in a carbon nanotube double quantum dot system [1].

In a hybrid system consisting of QD (quantum dot) coupled to two ferromagnetic (or nonmagnetic) leads and one superconducting electrode the current flows mainly due to the Andreev reflection processes when the applied bias voltage is within the superconducting gap [3]. In such hybrid device one can distinguish two kinds of the Andreev processes: *direct Andreev reflection* (DAR) and *crossed Andreev reflection* (CAR). In the former process the hole is reflected back to the electrode from which the incoming electron arrives, whereas in the latter process the hole is reflected into the second, spatially separated electrode. Reversing the sign of the applied bias voltage results in transfer of the Cooper pair from superconductor into the same normal lead or splitting when the two electrons forming the Cooper pair end in different leads. There are also possible virtual process which do not lead to creation (or annihilation) of the Cooper pairs in the superconductor, i.e. *elastic cotunneling* (EC), when an electron is transferred between two normal-metal leads via virtual states in the superconductor. The Cooper pairs beam splitters based on QDs systems turn out to be very effective as they give possibility for easily changing device's parameters, and thus, tuning the contributions due to CAR and DAR, or EC processes or even suppress one of them.

A well-known quantity which allows to distinguish different contributions to sub-gap transport is the current cross-correlations, i.e., correlations between currents flowing through two junctions with normal leads. Generally, positive current cross-correlations can be associated with interactions supporting currents in both junctions and can be present in systems with superconducting electrodes [4, 5]. Particularly, in the Cooper pair splitters enhancement of positive current cross-correlations can be attributed to high Cooper pair splitting efficiency. However, interactions which mutually block the currents flowing through two junctions lead to suppression of positive current cross-correlations or even change their sign. Hence, negative sign of current cross-correlations corresponds to tunneling processes that occur in opposite directions.

In this paper we study dependence of current cross-correlations on asymmetry in strengths of coupling to normal electrodes. Particularly, the influence of the asymmetry in coupling strengths on current cross-correlations is examined in two distinct magnetic configurations of the external ferromagnetic leads, i.e. when magnetic moments of both leads are aligned in the same directions (parallel) or oppositely (antiparallel).

2. Model and theoretical description

We consider a system consisting of single-level quantum dot attached to two normal metal and one superconducting lead as shown in Fig. 1.

In the limit of an infinite superconducting gap, $\Delta \to \infty$, the system can be described by effective Hamiltonian

$$H = \sum_{\beta=L,R} H_\beta + + H_{QD}^{\mathrm{eff}} + H_T, \qquad (1)$$

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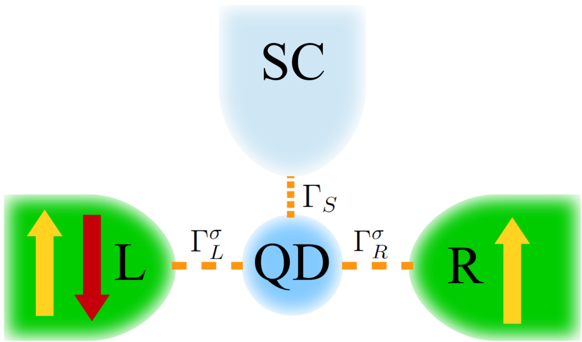

Fig. 1. Schematic picture of the QD system coupled to two normal metal (L, R) and one superconducting (SC) leads. The arrows indicate possible magnetic configurations of normal electrodes: parallel-arrows in L and R lead align in the same direction or antiparallel-arrows in L and R lead align in opposite directions.

where the first term, H_β describes the left ($\beta = L$) and right ($\beta = R$) ferromagnetic electrodes in the non-interacting quasiparticle approximation. Here, $H_\beta = \sum_{\boldsymbol{k}\sigma} \varepsilon_{\boldsymbol{k}\beta\sigma} c^\dagger_{\boldsymbol{k}\beta\sigma} c_{\boldsymbol{k}\beta\sigma}$ with $\varepsilon_{\boldsymbol{k}\beta\sigma}$ denoting the single particle energy.

The second term in Eq. (1) is effective Hamiltonian of QD being in proximity to superconductor and acquires the form [6]:

$$H_{QD}^{\text{eff}} = \sum_\sigma \varepsilon_\sigma d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow - \frac{\Gamma_S}{2}\left(d^\dagger_\uparrow d^\dagger_\downarrow + d_\downarrow d_\uparrow\right), \quad (2)$$

where the effective pair potential Γ_S is the coupling strength between the dot and superconducting electrode and is given by $\Gamma_S = 2\pi|V^S|^2\rho_S$ with ρ_S denoting BCS density of states in the normal state. Moreover, ε_σ and U denote the spin-dependent QD's energy level and the Coulomb integral, respectively.

Finally, tunneling of electrons between the ferromagnetic leads and the QD is modelled by

$$H_T = \sum_{\boldsymbol{k}\sigma} \sum_{\beta=L,R} (V^\beta_{\boldsymbol{k}\sigma} c^\dagger_{\boldsymbol{k}\beta\sigma} d_\sigma + \text{H.c.})$$

with $V^\beta_{\boldsymbol{k}\sigma}$ denoting the relevant tunneling matrix elements.

In the wide band approximation dot's coupling to the normal metal electrodes can be assumed to be energy independent and constant, $\Gamma_L^\sigma = \Gamma_L(1 + \tilde{\sigma} p)$, and $\Gamma_R^\sigma = \Gamma_R(1 + \eta\tilde{\sigma} p)$ with $\tilde{\sigma} = 1$ for $\sigma = \uparrow$ and $\tilde{\sigma} = -1$ for $\sigma = \downarrow$. Here, p denotes the spin polarization of magnetic leads assumed to be the same for the left and right electrodes, whereas $\eta = \pm 1$ is chosen for parallel (upper sign) and antiparallel (lower sign) magnetic alignment of the leads. The Andreev bound states' energies are defined as: $E^A_{\alpha,\beta} = \alpha\frac{U}{2} + \frac{\beta}{2}\sqrt{\delta^2 + \Gamma_S^2}$, where $\alpha, \beta = \pm$ and $\delta = \varepsilon_\uparrow + \varepsilon_\downarrow + U$. These energies are the excitation energies of the dot decoupled from the normal metal leads.

In order to derive the transport properties of the system the real-time diagrammatic technique has been employed [7]. In stationary/steady state the occupation probability p^{st}_χ of a state $|\chi\rangle$ can be found from

$$\boldsymbol{W}\boldsymbol{p}^{st} = 0, \quad (3)$$

where $\boldsymbol{p}^{st}$ is the vector of probabilities p^{st}_χ and $\boldsymbol{W}$ denotes self-energy matrix with the elements $W_{\chi\chi'}$ accounting for transitions between the states $|\chi\rangle$ and $|\chi'\rangle$. The $|\chi\rangle$'s states are eigenvectors of the effective QD's Hamiltonian, i.e. two single-occupied states $|\uparrow\rangle$, $|\uparrow\rangle$ and two states $|\pm\rangle = 1/\sqrt{2}\left(\sqrt{1 \mp \delta/(2\varepsilon_A)}|0\rangle \mp \sqrt{1 \pm \delta/(2\varepsilon_A)}|2\rangle\right)$ are the superposition of empty and double occupied QD's states, where $2\varepsilon_A = \sqrt{\delta^2 + \Gamma_S^2}$.

In the sequential tunneling approximation the current cross-correlations, S_{LR}, are given by

$$S_{LR} = \frac{e^2}{\hbar}\text{Tr}\left(\left(\boldsymbol{W}^{I_L}\boldsymbol{P}\boldsymbol{W}^{I_R} + \boldsymbol{W}^{I_R}\boldsymbol{P}\boldsymbol{W}^{I_L}\right)\boldsymbol{p}^{st}\right), \quad (4)$$

where the propagator $\boldsymbol{P}$ is determined from equation, $\boldsymbol{W}\boldsymbol{P} = \boldsymbol{p}^{st}\boldsymbol{e}^{\mathrm{T}} - \boldsymbol{1}$, with $\boldsymbol{e}^{\mathrm{T}} = (1, 1, \ldots, 1)$. The self-energy matrix $\boldsymbol{W}^{I_\alpha}$ is similar to $\boldsymbol{W}$, but it takes into account the number of electrons transferred through a given junction.

3. Numerical results

We present the numerical results for the current cross-correlations assuming large superconducting-gap limit. Furthermore, we assume spin degenerate QD's level, $\varepsilon_\uparrow = \varepsilon_\downarrow$. We mainly focus on influence of asymmetry in coupling strength of QD to the normal metal leads on the aforementioned quantity. The asymmetry in couplings' strength are modeled by introducing parameter α into relevant couplings in the following way: $\Gamma_L = (1+\alpha)\Gamma/2$ and $\Gamma_R = (1-\alpha)\Gamma/2$ with $\alpha \in [0, 1]$. Thus, for $\alpha = 0$ both leads are coupled to the QD with equal strength, whereas with increasing value of the parameter α the asymmetry in couplings of the two leads grows. In the limit $\alpha = 1$, one of the normal leads (here, right one) becomes completely detached from the QD and does not play any role in transport. Meanwhile, the second normal electrode (left one) is then coupled to QD with maximum strength, $\Gamma_L = \Gamma$. Notice that defining the asymmetry in this way, the total coupling strength between QD and two normal leads becomes constant regardless of change in the asymmetry.

The Andreev current (not shown) is optimized when particle–hole symmetry holds, and thus it becomes significant only for small detuning δ. Therefore, we assume in our considerations that $\delta = 0$. Each time the electrochemical potential of normal metal leads crosses one of the Andreev levels the Andreev current reveals a step. Figure 2 shows current cross-correlations as a function of bias voltage calculated for the case with nonmagnetic leads (a) and magnetic electrodes (b, c). In the later case two magnetic configurations, parallel (b) and antiparallel (c), are considered for the same value of the spin polarization. Current cross-correlations, S_{LR}, vanish when the QD is occupied by single electron, i.e. for $E_{+-} < eV < E_{-+}$ and for $|\delta| < \sqrt{U^2 - \Gamma_S^2}$ and no Andreev current can flow as two electrons are required to form the Cooper pair. However, S_{LR} reveals, mostly positive values of bias voltage for which the Andreev current

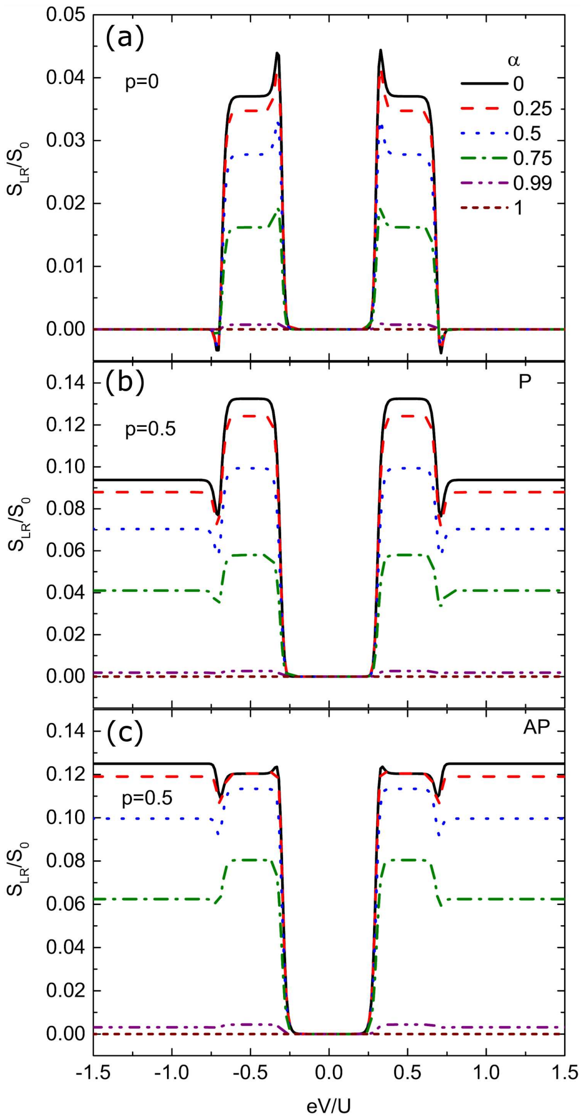

Fig. 2. Current cross-correlations as a function of bias voltage calculated for indicated values of spin polarization p (a) and for parallel (b) and antiparallel (c) magnetic configuration. The other parameters are: $\delta = 0$, $U = 1$ (used as energy unit), $\Gamma_S = 0.4$, $\Gamma = 0.01$ and $T = 0.015$, with $S_0 = e^2\Gamma/\hbar$.

flows, indicating that CAR processes make a contribution to it. Surprisingly, S_{LR} exhibits also negative values. However, this feature is not distinct for nonmagnetic case as it can be also seen for finite spin polarization p when deviating from particle–hole symmetry point, i.e. when $|\delta| > 0$ [8]. Negative values of S_{LR} indicates the presence of tunneling processes in opposite directions, i.e. single electron tunneling from the left (right) lead onto the QD and re-tunnel into the right (left) lead. Both tunneling events $L \to R$ and $R \to L$ occur with the same probability. Thus, no net charge current is observed, but these processes give contribution to the current cross-correlation, specifically to its negative values, indicating that tunneling processes by left and right junctions occur in opposite directions and compensate each other.

Regardless of spin polarization and magnetic alignment of the ferromagnetic leads the current cross-correlations decrease with increase in the asymmetry in strength of coupling to normal electrodes. Systematic reduction of dot's coupling to one of the normal electrodes leads to suppression of CAR processes which is clearly indicated by S_{LR} drop. In all cases depicted in Fig. 2 current cross-correlations vanish for $\alpha = 1$ in whole range of applied bias voltage, because then one normal metal electrode is completely decoupled from the QD, and thus, CAR processes become impossible. Comparing P and AP magnetic configurations, one can note that S_{LR} decreases slower with increase in α in the AP alignment. Moreover, in the AP configuration S_{LR} exhibits significant values for wide range of applied bias voltage, whereas for P alignment S_{LR} reveals high values only for specific regions of bias voltage. Particularly, S_{LR} achieves similar values in both magnetic configurations for $E^A_{\pm} < eV < E^A_{++}$ and for $E^A_{--} < eV < E^A_{\mp}$. However, S_{LR} for $eV > E^A_{++}$ ($eV < E^A_{--}$) decreases significantly for P-alignment, whereas for AP configuration it remains almost unchanged and even slightly amplified. Interestingly, when asymmetry in couplings increases, bias voltage dependence of S_{LR} becomes qualitatively similar in both magnetic configurations. However, S_{LR} still achieves higher values for AP alignment. It is worth noting that S_{LR} in nonmagnetic case ($p = 0$) reveals much smaller values than in magnetic one. The next feature which distinguishes magnetic case from nonmagnetic one is vanishing of S_{LR} for $eV > E^A_{++}$ and $eV < E^A_{--}$ when $p = 0$. The physical mechanism of this phenomenon has been explained in detail in Ref. [8].

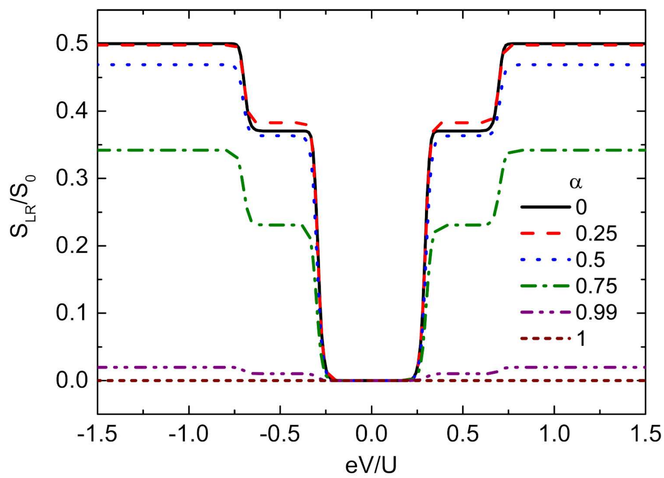

Fig. 3. Current cross-correlations as a function of bias voltage calculated for $p = 1$ in the AP configuration.

In Fig. 3 we show bias voltage dependence of current cross-correlations in the case of half-metallic leads in AP alignment. Firstly, S_{LR} acquires non-negative values in the whole range of bias voltage (and detuning parameter δ — not shown). In the case of half-metallic leads there are no available states in a given ferromagnetic lead for electrons incoming from the other ferromagnetic electrode, and thus, tunneling processes in opposite directions become totally blocked and no negative values of S_{LR} can emerge. Secondly, S_{LR} becomes maximized which is well understood as only CAR processes contribute to the Andreev current, whereas DAR processes are completely blocked since in given electrode only states with one spin orientation are available. With increase in the asymmetry in couplings the S_{LR} decreases as one of the electrode becomes successively detached from the dot leading to reduction of CAR processes and suppression of the Andreev transport.

4. Conclusions

In this paper we have studied the cross-correlations between currents flowing through two junctions with normal leads in QD-based Cooper pair splitter. We have shown that asymmetry in couplings to two normal electrodes leads to suppression of CAR processes regardless of magnetic configuration of external electrodes. Moreover, the magnetism of external electrodes leads to nontrivial behavior of current cross-correlations.

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