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Generation of Squeezed States in a System of Nonlinear Quantum Oscillator as an Indicator of the Quantum-Chaotic Dynamics

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We consider a model of quantum nonlinear oscillator which is excited by a series of ultrashort coherent pulses. For such a system, we analyze the potential application of the normally ordered variances as a witness of quantum chaotic behavior. We concentrate on the relations between the generation of the squeezing effects and the appearance of the quantum chaotic behavior of the system.

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1. Introduction

Although the chaotic phenomena have been known since the nineteenth century, nowadays they are still intensively studied [1–4]. It is known that the methods of the analyzing of the chaotic evolution of the classical systems are well developed and established [5, 6], but still, those for identifying quantum chaos are a subject of intense discussions. For instance, Peres [7] applied the fidelity of quantum states as the indicator of quantum-chaotic behavior. Then, such procedure was developed by Weinstein et al. [8–10]. Other proposals are allowing to check whether the system exhibits quantum-chaotic evolution or not. For instance, the analysis of the nonclassicality parameter [11], the Kullback–Leibler quantum divergence [12], the Wehrl entropy [13], and others have been proposed in that context.

In this paper, we propose another procedure which can be used in the analysis of quantum systems. We consider the application of the normally ordered variances as indicators of quantum chaotic evolution appearance. In particular, we discuss the evolution of the variances in time for the various value of the external field's strength. Such variances were considered for the cases corresponding to the regular and chaotic evolution of the classical counterpart of the quantum system analyzed here.

2. The model

We consider a nonlinear Kerr-like oscillator, externally pumped by a series of ultrashort coherent pulses. The system's evolution is governed by the following Hamiltonian:

$$\hat{H} = \frac{\chi}{2}\left(\hat{a}^{\dagger}\right)^{2}\hat{a}^{2} + \epsilon\left(\hat{a}^{\dagger}+\hat{a}\right)\sum_{k=1}^{\infty}\delta(t-kT), \qquad (1)$$

where the first term represents the "free" evolution of the oscillator during the period of time between the two subsequent pulses, whereas the second term describes the interaction with the coherent external field. The parameter χ represents the nonlinearity parameter describing the oscillator and ϵ is the strength of the interaction with external pulses which are modeled by the sum of the Dirac-delta functions $\delta(t-kT)$, where T is the time-interval between two subsequent pulses labeled by k. The operators $\hat{a}^{\dagger}$ ($\hat{a}$) are the photon creation (annihilation) operators. We assume here that the system initially is in the vacuum state $|\psi(t=0)\rangle = |0\rangle$, and all damping processes are absent. In consequence, we can use the wave-function approach to describe the system's dynamics.

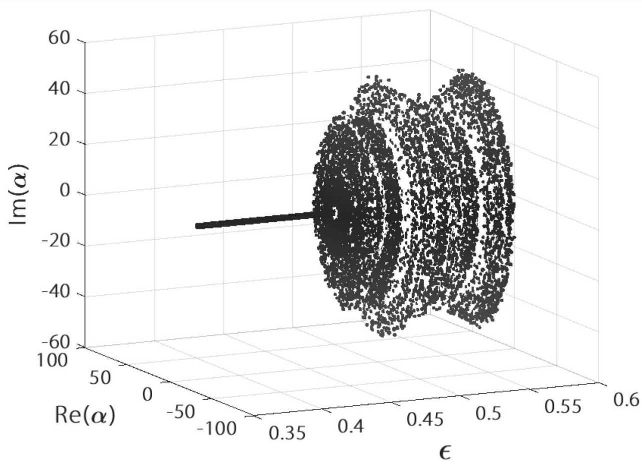

Fig. 1. Bifurcation diagram for the real and imaginary parts of α. We assume that $T=\pi$.

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In this paper, we compare the dynamics of the quantum system with the evolution of its classical counterpart. Such classical counterpart can exhibit regular or chaotic behavior. To determine whether the dynamics of the classical system is regular or chaotic, we plot the bifurcation diagram for various values of ϵ (see Fig. 1). To draw such diagram we use the procedure described in [14].

From Fig. 1, we see that for the weak external excitation ($\epsilon \lesssim 0.46$) behavior of the classical system is regular, whereas for stronger excitation strengths ($\epsilon \gtrsim 0.46$) it evolves chaotically.

3. The results

One of the problems in quantum chaos theory is to identify the chaotic dynamics and borders between the regular and chaotic evolution's regions of the quantum system. For the classical models, there are already developed various methods of detection the chaotic behavior of the system, such as those based on the Lyapunov exponents, power spectra or entropy [6]. For the quantum systems, the situation is not so clear as for the classical models. Here, we propose the normally ordered variances of the quadrature operators as indicators of the quantum-chaotic evolution. Such operators can be defined by the following equations:

$$\hat{X}_1 = \frac{1}{2}\left(\hat{a} + \hat{a}^\dagger\right),$$
$$\hat{X}_2 = \frac{1}{2\mathrm{i}}\left(\hat{a} - \hat{a}^\dagger\right), \tag{2}$$

where $\left[\hat{X}_1, \hat{X}_2\right] = \frac{\mathrm{i}}{2}$. To determine whether the state is a squeezed state or not, we apply the normally ordered variances of the quadratures operators

$$\left\langle : \left(\Delta \hat{X}_i\right)^2 : \right\rangle = \left\langle : \left(\hat{X}_i\right)^2 : \right\rangle - \left\langle : \left(\hat{X}_i\right)^2 : \right\rangle^2. \tag{3}$$

The colons appearing here denote the normal ordering. Then, the variances corresponding to the two quadrature operators can be written in the following form:

$$\left\langle : \left(\Delta \hat{X}_1\right)^2 : \right\rangle = \frac{1}{4}\left[\langle \hat{a}^2\rangle + 2\langle \hat{a}^\dagger \hat{a}\rangle + \langle \hat{a}^{\dagger^2}\rangle - \left(\langle \hat{a}\rangle^2 + 2\langle \hat{a}^\dagger\rangle\langle \hat{a}\rangle + \langle \hat{a}\rangle^{\dagger^2}\right)\right]$$
$$\left\langle : \left(\Delta \hat{X}_2\right)^2 : \right\rangle = -\frac{1}{4}\left[\langle \hat{a}^2\rangle - 2\langle \hat{a}^\dagger \hat{a}\rangle + \langle \hat{a}^{\dagger^2}\rangle - \left(\langle \hat{a}\rangle^2 - 2\langle \hat{a}^\dagger\rangle\langle \hat{a}\rangle + \langle \hat{a}\rangle^{\dagger^2}\right)\right]. \tag{4}$$

When one of those variances is negative, the squeezed state is generated.

Here, we study how the time-evolution of variances changes with the varying strength of external pulse ϵ. We chose three values of strength 0.1, 0.45, and 0.5. The first of them corresponds to the case when the classical system evolves regularly, the second value is close to the border of the chaotic region, whereas the third one is related to the chaotic behavior of classical counterpart of our quantum system. In Fig. 2 we show the time-dependence of variances for the three values of ϵ. We see that the character of the evolution of variances strongly depends on the value of the external excitation. When $\epsilon = 0.1$ (Fig. 2a) we observe regular oscillation of

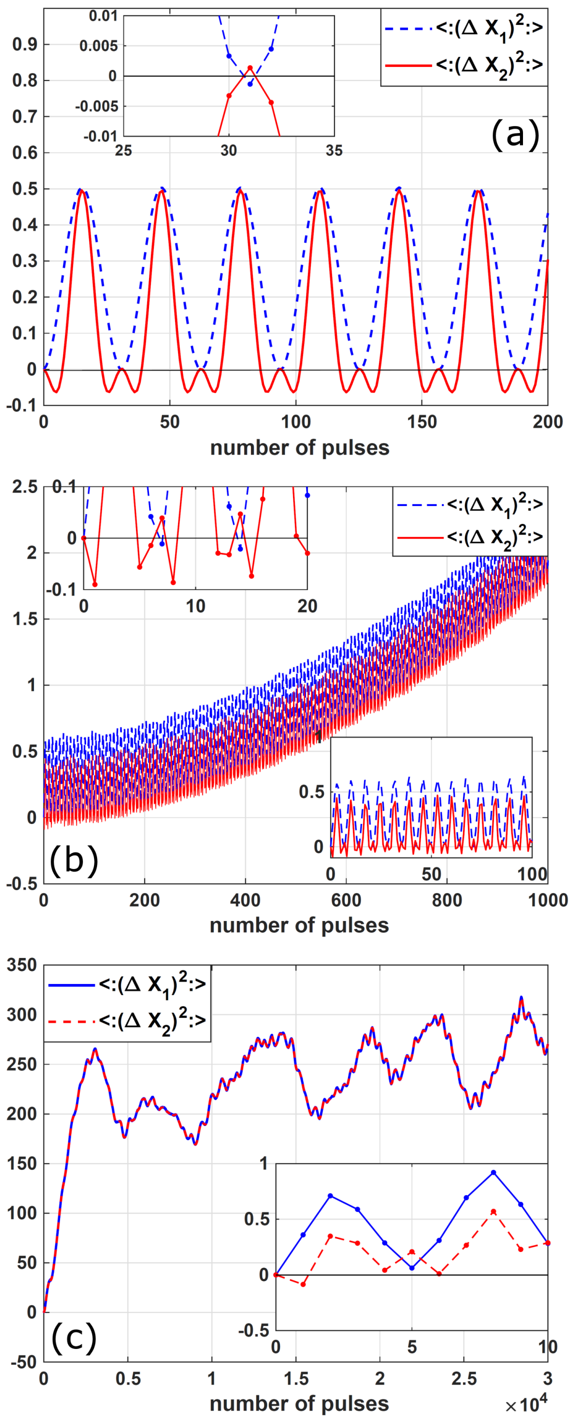

Fig. 2. Normally ordered variances of the quadratures operators versus number of pulses (time) for various values of the excitation strength ϵ: (a) 0.1, (b) 0.45, (c) 0.5.

$\langle: (\Delta\hat{X}_1)^2 :\rangle$ and $\langle: (\Delta\hat{X}_2)^2 :\rangle$ with constant amplitude. The oscillations are determined by a single frequency. On the other side, we see that both variances take negative values. It means that the squeezed states are generated. Such states appear periodically. Additionally, observed squeezing in the second quadrature is stronger than in the quadrature $\hat{X}_1$.

When $\epsilon = 0.45$ the both variances oscillate and the oscillations are modulated. The possible transition toward the chaotic evolution is manifested by the appearance of additional frequencies in their time-evolution. The same as for $\epsilon = 0.1$, the squeezing in the both quadratures appears for such a case.

For $\epsilon = 0.5$, we see that character of the time-evolution of the variances differs from that shown in the previous case. The squeezing effect appears only in the quadrature $\hat{X}_2$. The squeezed state can be observed just after the first pulse — for the longer times, the values of variances drastically increase. Apart from the initial growth, some irregular changes appears in the time-evolution of the variances. Such irregularity is characteristic for the chaotic behavior of the system.

4. Conclusion

In this paper, we have proposed a new method of detection of the quantum chaotical evolution of the system. In particular, we concentrated on the nonlinear Kerr-like system, which is externally pumped by a series of ultrashort coherent pulses. For such model, we studied the time-dependence of the normally ordered variances of the field for two cases — corresponding to the weak and strong excitations (to the regular and chaotic behaviors of the classical counterpart of the quantum system, respectively). It was proved that the time-evolution of the variances strongly depends on the strength of excitation. For the weak excitation (regular) regime, the squeezed states are periodically generated, even for a long time. When the external coupling is sufficiently strong, the squeezing effects appear only in one quadrature, and for the short period of time — only just after the first pulse. Thus, our results suggest that the normally ordered variances could be applied as a witness of the quantum-chaotic behavior. Moreover, in contrast to parameters discussed in other papers, such witnesses be useful in studies of quantum chaotic behavior also in a short-time regime.

References

[1] M.W. Jarosik, R. Szczęśniak, A.P. Durajski, J.K. Kalaga, W. Leoński, *Chaos Interdiscipl. J. Nonlin. Sci.* **28**, 013126 (2018).

[2] J.K. Kalaga, M.W. Jarosik, W. Leoński, R. Szczęśniak, *Witnesses of Quantum Chaos and Nonlinear Kerr-Like Oscillator Model*, InTech, 2018.

[3] A.V. Viatkina, M.G. Kozlov, V.V. Flambaum, *Phys. Rev. A* **95**, 022503 (2017).

[4] B.C. Yang, J. Pérez-Ríos, F. Robicheaux, *Phys. Rev. Lett.* **118**, 154101 (2017).

[5] G.L. Baker, J.P. Gollub, *Chaotic Dynamics: An Introduction*, 2nd ed., Cambridge University Press, 1996.

[6] H.G. Schuster, W. Just, *Deterministic Chaos — An Introduction*, Wiley-VCH, Weinheim 2005.

[7] A. Peres, *Phys. Rev. A* **30**, 1610 (1984).

[8] Y.S. Weinstein, S. Lloyd, C. Tsallis, *Phys. Rev. Lett.* **89**, 214101 (2002).

[9] J. Emerson, Y.S. Weinstein, S. Lloyd, D.G. Cory, *Phys. Rev. Lett.* **89**, 284102 (2002).

[10] Y.S. Weinstein, J. Emerson, S. Lloyd, D. Cory, *Quant. Inform. Process.* **1**, 439 (2002).

[11] A. Kowalewska-Kudłaszyk, J.K. Kalaga, W. Leoński, *Phys. Rev. E* **78**, 066219 (2008).

[12] A. Kowalewska-Kudłaszyk, J.K. Kalaga, W. Leoński, V. Cao Long, *Phys. Lett. A* **376**, 128012 (2012).

[13] J.K. Kalaga, W. Leoński, A. Kowalewska-Kudłaszyk, *Proc. SPIE* **9441**, 94410W (2014).

[14] W. Leoński, *Physica A* **233**, 365 (1996).