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Modeling of the Parties' Vote Share Distributions

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Competition between varying ideas, people and institutions fuels the dynamics of socio-economic systems. Numerous analyses of the empirical data extracted from different financial markets have established a consistent set of stylized facts describing statistical signatures of the competition in the financial markets. Having an established and consistent set of stylized facts helps to set clear goals for theoretical models to achieve. Despite similar abundance of empirical analyses in sociophysics, there is no consistent set of stylized facts describing the opinion dynamics. In this contribution we consider the parties' vote share distributions observed during the Lithuanian parliamentary elections. We show that most of the time empirical vote share distributions could be well fitted by numerous different distributions. While discussing this peculiarity we provide arguments, including a simple agent-based model, on why the beta distribution could be the best choice to fit the parties' vote share distributions.

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1. Introduction

There are numerous ways every individual can be unique. Some of the personal degrees of freedom are wholly predetermined at birth and are not influenced by the socio-economic or cultural context, e.g., biological features like skin color, while some of them could be referred to as the economic, social, or cultural variables as they are not strictly predetermined and may change to different extent depending on individual's behavior and personal experiences, e.g., wealth, influence, religion or political affiliation. Most of these variables are extremely interesting, from the perspective of social science and sociophysics, as their dynamics reflect the ongoing competition between varying ideas, people, and institutions [1–4].

Development of simple agent-based models of socio-economic and cultural interactions attracted a lot of interest from physicists in the recent couple of decades [5, 6]. While these models can be well used as a controlled testing ground for qualitative theories from the social sciences [2, 7–9]. They are also of utmost interest to physicists as these models exhibit different complex dynamical and statistical phenomena, reminiscent of the complex phenomena, such as phase transitions [10, 11], dissipative structures [12, 13] or non-extensive thermodynamics [14, 15], common in statistical physics. This interest is well grounded in the empirical data, too, as complex statistical and dynamical patterns are observed in the empirical data from socio-economic systems as well [8, 16, 17]. While there is a well established set of stylized facts for the financial markets, e.g., [18], numerous empirical studies in opinion dynamics have not yet helped to establish even a basic set of stylized facts, which could be seen as a goal for the theoretical models.

In this contribution we will focus on the parties' vote share distribution in a country as a proxy of the political attitude dynamics of the country's population. Here we will focus on empirical data from the Lithuanian parliamentary elections. Empirical data from the Lithuania parliamentary elections was previously considered in many works by Lithuanian political and social scientists, e.g., [19–21], but in most of the approaches highly aggregated data was analyzed and the analysis itself was just a means to an end. Numerous other previous approaches have already considered empirical data gathered during the various types of elections in the well-established democracies, e.g., [22–31]. Different statistical features were considered in these and numerous other papers, e.g., turnout distributions, spatial distributions, open list ranking statistics, while some of the papers were dedicated to the analysis of the vote share distributions. In these different approaches different theoretical fits for the marginal vote share distributions were proposed. In some of these works the empirical vote share data was fitted using the log-normal distribution [22], the normal distribution [25, 27], distributions based on the Weibull distribution [28, 30], and the beta distribution [29, 31]. In this paper we would like to argue that it may be hard to distinguish between these distributions, but the beta distribution is likely to be the best choice. A similar observation that numerous distribution may fit the empirical statistics of religious populations was made in [3]. It is worth to note that in order for these statistical marginal distribution models to reasonably fit the data, the respective multivariate distributions would be needed to be defined on simplex as was done in some sophisticated approaches in political science literature [32, 33]. The Dirichlet distribution, marginal distributions are the beta distributions, and the logit-normal distribution, marginal distributions are similar to the normal distribution, seem to achieve this goal, which could serve as an argument for their wider usage. But here in our analysis we will limit ourselves to the comparison of the statistical models of the marginal distributions. We will address the empirical analysis from this point of view in Sect. 3.

Most likely inspired by the theory of coarsening and the first-passage phenomena numerous modelling approaches in sociophysics have considered consensus formation in differently formulated models of the opinion dynamics [34, 39]. These approaches are primarily interested in whether the agents will reach the uniform consensus state or if the agent population will remain heterogeneous in opinion. These models are usually based on varying interpretations of the Ising model, such as the voter model [34, 35], the Sznajd model [36, 37] or the Galam model [38]. Most of time these models converge to the fixed states which are either consensus or coexistence states. Yet it is well-known that opinion heterogeneity is rather ubiquitous trait as well as that it is of rather dynamic nature. The aforementioned models can be easily extended to include dynamism by introducing exogeneous shocks, certain degree of contrarian behavior or certain type of inflexibility into the model [40–42]. Also most of these models are two-state models while in some cases there are more than two viable options to choose from, e.g., usually more than 15 parties participate in Lithuanian parliamentary election (with more than 4 of them winning seats in the parliament by the popular vote). In the political science and mathematical literature one would find more varied approaches [43, 44], but usually their primary goal is to provide election procedures, which would represent the opinion of the electorate the best. In Sect. 2 of this paper we will discuss an alternative possibility, based on Kirman's model [45], to formulate an agent-based model for the voting behavior.

This paper is divided into two main parts. In Sect. 2 an agent-based model for the voting behavior is presented. In Sect. 3 we discuss the empirical data and use the model from previous section to reproduce the statistical patterns uncovered during the empirical analysis. Finally we summarize our results and provide a discussion in Sect. 4.

2. A multi-state agent-based model for the voting behavior

Originally in a seminal paper by Kirman [45] a simple two-state herding behavior model was proposed. The aim of the model was to reproduce similar behavioral patterns observed by biologists and economists. It was noted that individuals tend to imitate their peers' actions despite the lack of rational reasons to do so [46–50]. This model is in some sense very much like many other psychologically motivated models [9, 41, 42, 51–53] but, in comparison, this model is extremely simple and, as we will show later, extremely efficient.

Kirman made an assumption that agents could change their behavior on their own (acting according to the perceived attractiveness of the available choices) or due to peer influence (recruitment mechanism). In contemporary form this model is usually formulated using the one step transition probabilities [54–56]:

$$P(X \to X+1) = (N-X)\,(\sigma_1 + hX)\,\Delta t, \tag{1}$$

$$P(X \to X-1) = X\left[\sigma_2 + h\,(N-X)\right]\Delta t. \tag{2}$$

Here N is a total number of agents in the modeled two-state system, where each state represents different behavioral pattern (e.g., different trading strategies in the financial market applications [54–56]), X is a total number of agents occupying the first state (consequently there are $N-X$ agents occupying the second state), σ_i are the perceived attractiveness parameters, h is an inter-agent interaction intensity parameter, while Δt is a relatively short time step. In general, it should be as small as possible, at least so that a single agent could switch his state per time step. In the scope of this paper h parameter is not relevant, as here we will not consider the temporal trends, so we can eliminate it by introducing rescaled time $t_s = ht$ [54–56]:

$$P(X \to X+1) = (N-X)\,(\varepsilon_1 + X)\,\Delta t_s, \tag{3}$$

$$P(X \to X-1) = X\left[\varepsilon_2 + (N-X)\right]\Delta t_s. \tag{4}$$

Here $\varepsilon_i = \frac{\sigma_i}{h}$ is the rescaled attractiveness parameters. The dynamics of $x = \frac{X}{N}$, in the $N \to \infty$ limit, could be approximated by the Fokker–Planck [57, 58]:

$$\frac{\partial}{\partial t_s}p(x,t_s) = -\frac{\partial}{\partial x}\left[\Big(\varepsilon_1(1-x) - \varepsilon_2 x\Big)p(x,t_s)\right] + \frac{\partial^2}{\partial x^2}\Big[x\,(1-x)\,p(x,t_s)\Big], \tag{5}$$

or a stochastic differential equation [56]:

$$\mathrm{d}x \approx \left[\varepsilon_1(1-x) - \varepsilon_2 x\right]\mathrm{d}t_s + \sqrt{2x(1-x)}\,\mathrm{d}W_s. \tag{6}$$

From these equations it is rather straightforward to show that the stationary distribution of x is the beta distribution, $\mathcal{B}e\,(\varepsilon_1,\varepsilon_2)$, probability density function (PDF) of which is given by

$$p_{st}(x) = \frac{\Gamma(\varepsilon_1+\varepsilon_2)}{\Gamma(\varepsilon_1)\Gamma(\varepsilon_2)}x^{\varepsilon_1-1}(1-x)^{\varepsilon_2-1}. \tag{7}$$

The extension of the two-state model to describe switching between multiple states is rather straightforward, though with some noteworthy implications.

As the total number of agents, $\sum_i X_i = N$, is conserved, if an agent switches his state, one state gains an agent, while the another state loses an agent. With this in mind we can write the one step transition probabilities, to and from state i, as follows:

$$P\left(X_i \to X_i \pm 1\right) = \sum_{j\neq i} P\left(X_i \to X_i \pm 1, X_j \to X_j \mp 1\right), \tag{8}$$

where the P on the right hand side stands for the switching probability between two different states. Let us assume that this P takes the same form as in the two-state model case, if so then we obtain

$$P(X_i \to X_i+1) = \sum_{j\neq i} X_j\left(\sigma_{ji} + h_{ji}X_i\right)\Delta t, \tag{9}$$

$$P\left(X_i \to X_i-1\right) = X_i \sum_{j\neq i}\left[\sigma_{ij} + h_{ij}X_j\right]\Delta t. \tag{10}$$

Although the current form of the transition probabilities allows some flexibility, but the analytical treatment of the multi-state model seems to be impossible as the one step transition probabilities for X_i depend on other X_j $(j \neq i)$ in non-trivial manner. To eliminate this cumbersome dependence let us assume that:

- the perceived attractiveness of a state, σ_{ij}, does not depend on agent from which state is attracted to it, $\sigma_{ij} = \sigma_j$;
- the interaction intensity is symmetric and independent of the states interacting agents are in, $h_{ij} = h$.

Note that these assumptions are the opposite of what is assumed by the well-known bounded confidence model [41]. Yet these assumptions allow us to further simplify the one-step transition probabilities

$$P\left(X_i \to X_i + 1\right) = \left(N - X_i\right)\left(\varepsilon_i + X_i\right)\Delta t_s, \tag{11}$$

$$P\left(X_i \to X_i - 1\right) = X_i\left(\varepsilon_{-i} + N - X_i\right)\Delta t_s. \tag{12}$$

Here $\varepsilon_{-i} = \sum_{j \neq i} \varepsilon_j$ is the total attractiveness of switching away from i. Because these switching probabilities have the same form as Eqs. (3) and (9), the stationary distribution of x_i is most likely to be $\mathcal{B}e\left(\varepsilon_i, \varepsilon_{-i}\right)$. In that case the stationary distribution of $\boldsymbol{x}$ is $\mathrm{Dir}\left(\boldsymbol{\varepsilon}\right)$. Though note that if the simplifying assumptions are violated the stationary distribution of $\boldsymbol{x}$ might no longer be the Dirichlet distribution, nor the marginal distribution of at least some of the x_i might no longer follow the beta distribution.

In the previous paragraphs we have defined the two-state and multi-state models ignoring the underlying interaction topology, or namely we defined the models as mean-field models. Yet these models can be easily generalized to take interaction topologies, e.g, some kind of random networks [59, 60], into account. In such case one has to define individual agent switching probabilities:

$$P_a\left(O \to D\right) = \left[\varepsilon_D + n_a(D)\right]\Delta t_s. \tag{13}$$

Here a is an index, which identifies individual agents, O and D represent origin and destination states, respectively ($O \neq D$), while $n_a(D)$ is a number of agent's a neighbors who are in state D. As long as average degree of nodes on the network is large, i.e., comparable with the total number of agents, links are uncorrelated and Δt_s is small, both the discussed mean-field models and this model should produce the same results [60]. If the average degree is significantly smaller than the total number of agents, one can still use one-step transition probabilities to describe the system at macro-level, but the probabilities will be of a slightly different form than Eqs. (11) and (12). Here we would like to note that if we set $\varepsilon_D = 0$ and restrict the model to two states, then Eq. (13) basically describes the dynamics underlying the well known Voter model [5, 34, 35].

3. Empirical analysis of the Lithuanian parliamentary elections

Let us start with a brief description of the two-tier voting system used during the Lithuanian parliamentary elections. The elections are held quadrennially (the exact date is set by the president of Lithuania). During every parliamentary election all of the seats in the parliament are being contested. Namely, some members of the parliament may serve a shorter than a four-year term, if they have replaced somebody else. 71 of the seats in the parliament are allocated to the elected representatives of the 71 electoral districts (two-round system is used to elect the representatives of the electoral districts). The remaining 70 seats are distributed according to the popular vote for an open party list. The party needs to pass the threshold of 5% of the popular vote to obtain at least 1 seat in this way.

From voters perspective the parliamentary elections take form of two ballots. One ballot is used to vote for a representative of the electoral district. Usually the district representative is not elected in the first round and the second round is held. This time the voter will be able to from a list of the top two candidates from the first round. Another ballot is used to vote for a political party or movement and optionally for the up to 5 people from the respective party's or movement's list.

In this paper we are concerned with the distribution of popular vote for the parties (from here onwards we will refer to both parties and movements simply as parties) across all of the polling stations. Each of the 71 electoral districts has multiple polling stations (there are usually ≈ 2000 polling stations in total). Every voter is assigned to one of the polling stations based on the location of residence. Due to uneven population density and consideration of the polling station proximity to the voters, the polling stations vary in the number of the assigned voters — some of the smallest polling stations could have as few as 100 assigned voters, while the largest could have 7000. In this contribution we will ignore this difference and consider each polling station as a single unit providing one data point per party per election. In our analysis we ignore voting by mail as well as voting in polling stations abroad in order to at least to some extent ensure that the polling stations are spatially separate, namely to keep chances that person lives and interacts with people from the other polling station as low as it is possible.

The data used in this analysis is freely available at the website is managed by the Central Electoral Commission of the Republic of Lithuania [61] (the website is managed by the Central Electoral Commission of the Republic of Lithuania). Sadly at the time of writing the website works well in Lithuanian language only (some parts remain untranslated). Therefore we have provided a cleaned up version of the data used in our analysis at [62]. The original data sets were downloaded on August 31, 2016.

Let us start the empirical analysis by considering the Lithuanian parliamentary election of 1992. 17 parties competed in that election, but only 4 of them won seats by the popular vote. One of the four parties just barely made past the threshold of 5%, while the other three enjoyed noticeably larger support. As the fourth party introduces distorting effect, which we will discuss a bit later, let us ignore it as well as the other 13 parties, which did not make past the threshold. The statistical properties of the remaining three most popular parties are not significantly distorted by various additional effects, so let us consider their renormalized vote share for the introductory analysis. We use the following party name abbreviations for these parties: SK will stand for "Sąjūdžio koalicija" (en. Sąjūdis coalition), LKDP — "Lietuvos krikščionių demokratų partijos (en. Lithuanian Christian Democratic Party), Lietuvos politinių kalinių ir tremtinių sąjungos ir Lietuvos demokratų partijos jungtinis sąra¨as", LDDP — "Lietuvos demokratinė darbo partija" (en. Democratic Labour Party of Lithuania).

In Fig. 1 we show the vote share PDFs of SK, LKDP, and LDDP fitted by the four distributions commonly used in the literature [3, 22, 25, 27–31]. The respective distribution parameters are given in Table I. As can be easily seen from the figure, the beta and Weibull distributions provide good fits for the empirical PDFs, normal distribution also provides a rather good fit, while log-normal distribution seems to be somewhat off. A formal comparison of the relative quality of these fits can be checked using the Watanabe–Akaike information criterion (abbreviation WAIC) [63]. These values are given alongside parameter values in Table I. See Fig. 2, for a visual comparison between the obtained WAIC values. From the table and the figure it can be seen that the fits provided by the beta and Weibull distributions are of similar quality, with the fit provided by the Weibull distribution being slightly better. The fits provided by the normal and log–normal distributions have noticeably larger WAIC values and thus provide relatively worse fits for the data.

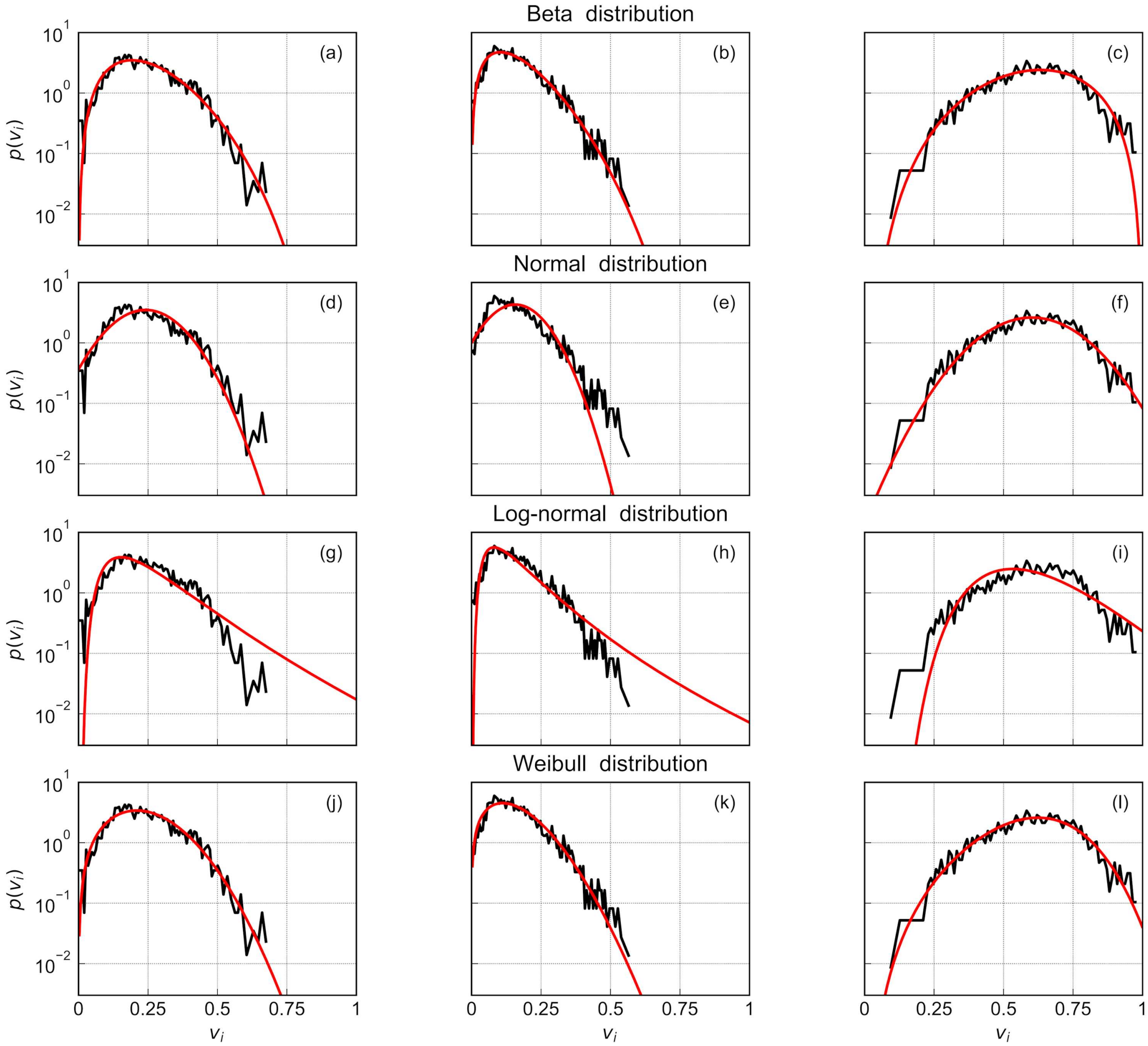

Fig. 1. The empirical vote share PDFs (black curves) fitted by the four commonly used distributions (red curves): (a)–(c) the beta distribution, (d)–(f) the normal distribution, (g)–(i) the log–normal distribution and (j)–(l) the Weibull distribution. Renormalized empirical vote shares of the three most popular parties during the 1992 election were used: SK ((a), (d), (g), (j)), LKDP ((b), (e), (h), (k)) and LDDP ((c), (f), (i), (l)). The respective parameter values are given in Table I.

TABLE I

The selected distribution parameter values, inferred while analyzing the vote shares of the three most popular parties during the 1992 parliamentary election, and the respective WAIC values.

$\mathcal{B}e$	SK	$\alpha = 3.08 \pm 0.18$	$\beta = 9.73 \pm 0.55$	WAIC $= -9560 \pm 120$
	LKDP	$\alpha = 2.32 \pm 0.14$	$\beta = 12.44 \pm 0.75$	
	LDDP	$\alpha = 5.45 \pm 0.35$	$\beta = 3.6 \pm 0.2$	
$\mathcal{N}$	SK	$\mu = 0.241 \pm 0.005$	$\sigma = 0.113 \pm 0.003$	WAIC $= -9020 \pm 120$
	LKDP	$\mu = 0.157 \pm 0.004$	$\sigma = 0.092 \pm 0.002$	
	LDDP	$\mu = 0.602 \pm 0.005$	$\sigma = 0.152 \pm 0.004$	
$\log -\mathcal{N}$	SK	$\mu = -1.556 \pm 0.028$	$\sigma = 0.57 \pm 0.02$	WAIC $= -8480 \pm 170$
	LKDP	$\mu = -2.05 \pm 0.03$	$\sigma = 0.69 \pm 0.02$	
	LDDP	$\mu = -0.547 \pm 0.013$	$\sigma = 0.289 \pm 0.008$	
$\mathcal{W}$	SK	$k = 2.25 \pm 0.07$	$\lambda = 0.272 \pm 0.006$	WAIC $= -9600 \pm 110$
	LKDP	$k = 1.79 \pm 0.05$	$\lambda = 0.177 \pm 0.005$	
	LDDP	$k = 4.53 \pm 0.15$	$\lambda = 0.658 \pm 0.006$	

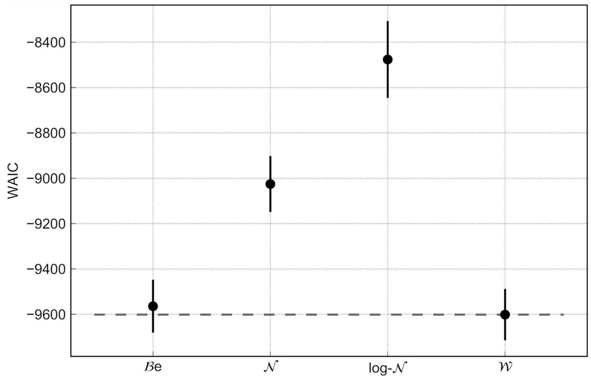

Fig. 2. Comparison of the suitability of the four commonly used distributions as reflected by WAIC in case of the empirical data from the 1992 parliamentary election. The dashed line represents best (lowest) WAIC value (the Weibull distribution WAIC).

In the literature the log–normal distribution was used to model the vote shares of individual politicians on the open party lists [22]. We consider different data so it is not strange at all that the log–normal distribution provides the worst fit. Another difference from the approach taken in [22] is that we use different normalization procedure. Namely we renormalize a sum of the vote shares of the considered parties to be equal to 1 in all polling stations, instead of dividing the vote share by the mean for the respective party, while the normal distribution was quite successfully used for the data gathered during various elections held in well-established democracies [25, 27]. We believe that normal distribution seems to provide a good fit in those cases due to similar vote shares received by the competing parties (the mean vote shares are of similar magnitude) as well as smaller variability of the vote share between the polling stations (smaller standard deviation). This can be also captured by the beta distribution assuming that α and β are large, which would reconcile these results with our approach and the approaches found in [29, 31]. So let us confirm this intuition by generating surrogate data, distributed according to the Dirichlet distribution (the multivariate beta distribution) with $\boldsymbol{\alpha} = \{10, 10, 10\}$. As can be seem from Fig. 3, the normal distribution provides a relatively good fit for the generated surrogate data, while the beta distribution and the Weibull distribution also provide a relatively good fits. In the context of the model defined in the previous section, this would mean that in countries with older democratic traditions self-induced transitions are significantly more common than peer-induced transitions.

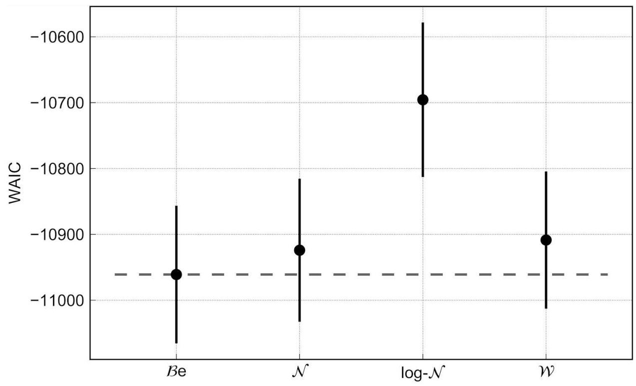

Fig. 3. Comparison of the suitability of the four commonly used distributions as reflected by WAIC in case of the surrogate data (true distribution is the multivariate beta distribution). The dashed line represents best (lowest) WAIC value of the four (the beta distribution WAIC).

Now let us extend the initial analysis by including the fourth party which passed the 5% threshold, "Lietuvos socialdemokratų partija" (abbreviation LSDP). At this point we would also like to stop using the normal and log–normal distributions as they do not seem to provide good fits for the reasons discussed previously. As you can see from Fig. 4 and Table II both the beta and Weibull distributions provide similarly good fits for the data. Note that parameter values β (for the beta distribution) and λ (for the Weilbull distribution) inferred for the LSDP are of somewhat different magnitude than the rest of parameter values. In the context of the model defined in the previous section we could see this as voters choosing to vote for another "like-able" party, because of lack of belief that the preferred party could win enough votes in the election. LDDP was another left-wing party in the 1992 election, which the most likely attracted a significant share of LSDP voters.

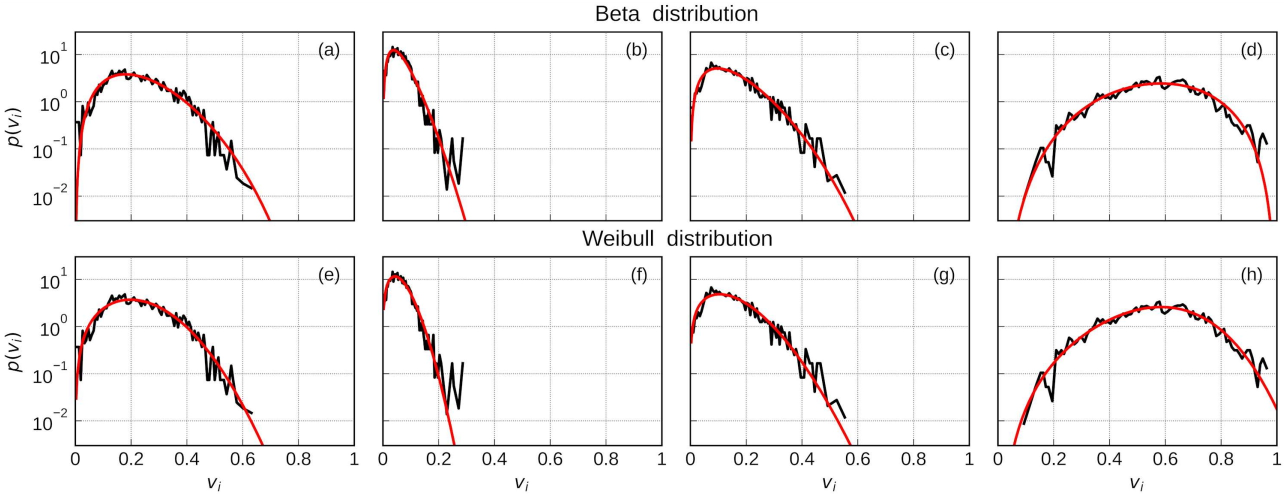

Fig. 4. The empirical vote share PDFs (black curves) fitted by the (a)–(d) beta and (e)–(h) Weibull distributions (red curves). Renormalized empirical vote shares of the four most popular parties during the 1992 election were used: SK ((a) and (e)), LSDP ((b) and (f)), LKDP ((c) and (g)) and LDDP ((d) and (h)). The respective parameter values are given in Table II.

TABLE II

The selected distribution parameter values, inferred while analyzing the vote shares of the four most popular parties during the 1992 parliamentary election, and the respective WAIC values.

$\mathcal{B}e$	SK	$\alpha = 3.25 \pm 0.19$	$\beta = 9.73 \pm 0.55$	$WAIC = -18400 \pm 170$
	LSDP	$\alpha = 2.42 \pm 0.14$	$\beta = 37.1 \pm 2.4$	
	LKDP	$\alpha = 2.37 \pm 0.13$	$\beta = 13.7 \pm 0.7$	
	LDDP	$\alpha = 5.42 \pm 0.31$	$\beta = 4.15 \pm 0.24$	
$\mathcal{W}$	SK	$k = 2.28 \pm 0.09$	$\lambda = 0.254 \pm 0.005$	$WAIC = -18450 \pm 170$
	LSDP	$k = 1.76 \pm 0.06$	$\lambda = 0.069 \pm 0.002$	
	LKDP	$k = 1.78 \pm 0.06$	$\lambda = 0.166 \pm 0.005$	
	LDDP	$k = 4.24 \pm 0.14$	$\lambda = 0.622 \pm 0.007$	

Let us now consider another less popular party named "Lietuvos lenkų sąjunga" (abbreviation LLS). This party is interesting to us as its vote share distribution exhibits another interesting effect — vote segregation effect. We would like to claim that this is related to the fact that LLS was mainly supported by the ethnic minorities, which are geographically segregated (most of the ethnic minorities living in major cities and Vilnius County). This creates a need to use a mixture distribution, because distribution parameter values will be different for those regions in which ethnic minorities make up a significant part of population and for those regions in which representatives of ethnic minorities are few. In Fig. 5 we have compared the vote share and rank-size distributions of the two parties from the 1992 parliamentary election: one with pronounced segregation effect (LLS) and one without pronounced segregation effect (LSDP). LSDP vote share distribution seems to be rather well fitted by the beta distribution, while LLS vote share distribution is fitted using a mixture of the two beta distributions.

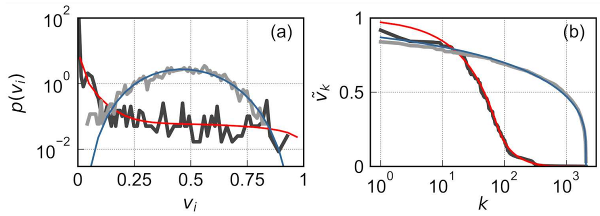

Fig. 5. Comparison between vote share PDFs (a) and rank-size distribution (b) of LLS (pronounced vote segregation effect) and LDDP (without pronounced vote segregation effect) during the 1992 parliamentary election. Empirical data are shown as wide gray curves (dark — LLS, light — LDDP), while best fits are shown as narrow colored curves. Best fits are provided by a mixture of beta distributions (red curve), $0.95\mathcal{B}e\,(0.08, 10) + 0.05\mathcal{B}e\,(1.22, 1.37)$, and the beta distribution (blue curve), $\mathcal{B}e\,(5.7, 6.5)$.

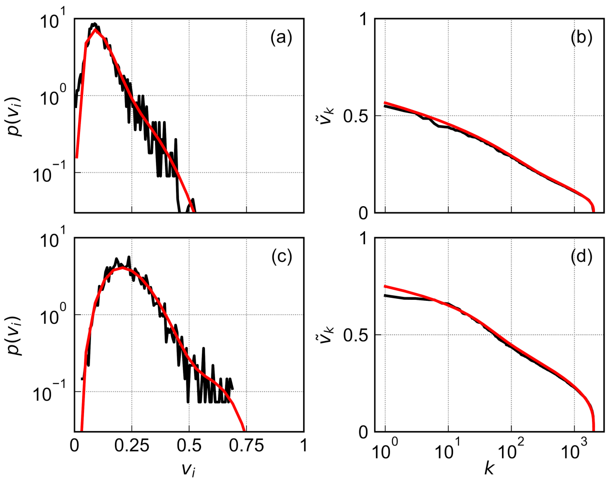

Fig. 6. The vote share PDFs ((a) and (c)) and the rank-size distribution ((b) and (d)) of LSDP during the 2008 election ((a) and (b)) and "Darbo partija" during the 2012 election ((c) and (d)). Empirical statistical properties are shown as black curves, while fits using a mixture of the beta distributions are shown in red. Parameter values of the fitting distributions were set as follows: $0.85\mathcal{B}e\,(3.9, 31.7)+0.15\mathcal{B}e(4.3, 12.9)$ (for LSDP), $0.97\mathcal{B}e\,(4.5, 14.5) + 0.03\mathcal{B}e\,(15.3, 11.1)$ (for "Darbo partija").

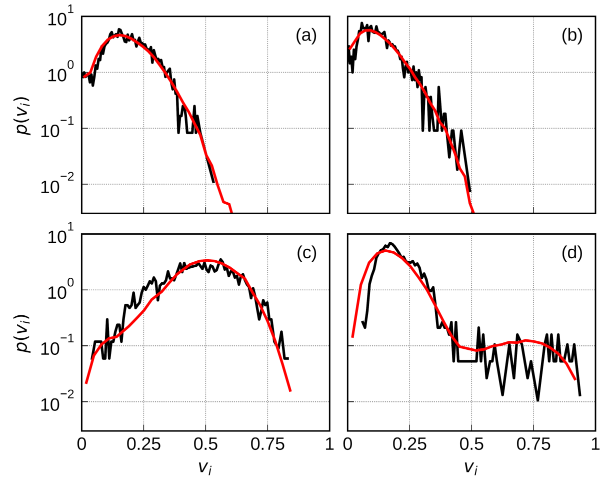

Fig. 7. Reproducing the vote share PDFs of (a) SK, (b) LKDP, (c) LDDP and (d) O observed during the 1992 parliamentary election. Black curves show the empirical PDFs, while red curves were obtained by numerically simulating the agent-based model discussed in Sect. 2. The used model parameter values are given in Eq. (14).

While analyzing the data from the later elections, we find that the other parties also start to exhibit vote segregation patterns. For a more recent examples of this pattern see Fig. 6. This could be a sign that a party is finding its political niche, taking over a segment of electorate (e.g., by promoting policies favoring certain sociodemographic groups). Also this could be a sign that a party is taking over municipalities (e.g., by pressuring public sector workers to vote for certain party with a threat to lose their jobs).

Let us now use the insights from the previous paragraphs to select parameters for the model proposed in previous section. Our aim is to reproduce the 1992 parliamentary election vote share distributions. We consider SK, LKDP, and LDDP as well as the "Other" party (abbreviation O), which is composed of the other 14 parties which participated in that election. The "Other" party includes LSDP, so we will have to violate the simplifying assumptions, and LLS, so we will have to pick two different parameter sets: one for the polling stations in which LLS was weak ($\approx 95\%$ of polling stations) and the other for the polling stations in which LLS was strong ($\approx 5\%$ of polling stations). Initial parameter values) were estimated using Bayesian inference and later adjusted to obtain a better fit between the model and the empirical data (see Fig. 7), after the fitting procedure we arrived at the following parameter set:

$$\varepsilon^{(95\%)} = \begin{pmatrix} 0 & 2.6 & 9.3 & 4.7 \\ 3.8 & 0 & 9.3 & 4.7 \\ 3.8 & 2.6 & 0 & 4.7 \\ 3.8 & 2.6 & 15.3 & 0 \end{pmatrix},$$

$$\varepsilon^{(5\%)} = \begin{pmatrix} 0 & 0.35 & 2.5 & 7 \\ 0.65 & 0 & 2.5 & 7 \\ 0.65 & 0.35 & 0 & 7 \\ 0.65 & 0.35 & 2.5 & 0 \end{pmatrix}. \qquad (14)$$

4. Conclusions

In this paper we have considered the vote share distributions observed in the Lithuanian parliamentary election data. Most of the attention was given to thorough analysis of the 1992 parliamentary election data set. We have compared four competing distributions often used to fit various election data sets from around the world [3, 22, 25, 27–31] and found that the beta and Weibull distributions seem to provide good fit, while normal and log–normal distributions are ill-suited if peer influence is strong in comparison to the independent voter behavior. We have also shown that as democratic tradition takes root parties start to take over electoral segments. As these segments are usually differentiated based on socioeconomic properties, the voters belonging to different segments are segregated and thus vote share distributions also become segregated. Segregated vote share distributions are well fitted by a mixture of the beta distributions (although they could be also well fitted by a mixture of Weibull distributions [30]).

To provide sound argument for the use of the beta distributions we have formulated a multi-state agent-based model for the voting behavior. This model, under certain simplifying assumptions, produces the Dirichlet distributed vote share (marginal distributions of which are distributed according to the beta distribution). One could violate these assumptions to add some flexibility to the model. Note that proposed model is very different from the psychologically motivated models for the opinion dynamics or voting behavior, e.g., bounded confidence model [41], thus we would like to raise the idea that vote share distribution could reflect some other processes in addition to (or instead of) opinion dynamics. A similar idea was already raised in [27]. This suggests a possible future development for both the agent-based modeling and empirical analysis — to consider spatio-temporal modeling of the Lithuania parliamentary elections.

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