Application of Intrinsic Quantized Flux of Electrons and Holes in Josephson Junctions

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The aim of this study is to investigate the influence of spin degrees of freedom on the flux quantization in a 2D Josephson junction. One of the most important properties of the Josephson structures is the total quantum flux which can be related to the phase difference across the junction. For example the sign of the phase difference controls the direction of the Josephson current while the magnitude of the phase difference affect the critical current itself. So far in literature to calculate the total quantum flux in the Josephson structures only the flux of the external magnetic field (and hence the external vector potential) has been considered but the intrinsic quantum flux of correlated electrons and holes have not been taken into account. We have recently calculated the intrinsic quantized magnetic flux of electrons and holes. We showed that depending on the spin orientations, the spin contribution to the quantized intrinsic flux of a correlated electron is equal to ($\Phi_{\rm int} = \pm\frac{g^*\Phi_0}{2}$). Here g^* is the effective Landé g-factor and Φ_0 is the unit of flux (fluxoid). In the present study we calculate the above mentioned phase differences across the junction considering the intrinsic quantum flux of electrons and holes. For electrons the additional flux contribution will be: $\Delta\Phi_{\rm int} = \pm\frac{g_e^*\Phi_0}{2}$ and for holes, the related contribution will be: $\Delta\Phi_{\rm int} = \pm\frac{g_h^*\Phi_0}{2}$. We show that, for both charge carriers, the effective Landé g-factors (g_e^*, g_h^*), take only even integer values such as $(0,2,4,\ldots)$. The present calculations can be easily extended to the intrinsic Josephson junctions as well. We found that flux contribution to the total flux due to spin is very important and it is in fact $\pm\Phi_0/2$ depending on the spin up and down cases or the ground state.

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1. Introduction

Layered high-temperature superconducting materials such as $Bi_2Sr_2CaCu_2O_x$ (BSCCO) are composed of superconducting cuprite layers coupled by the Josephson interaction. This system possesses the Josephson effect at the atomic scale and is called the intrinsic Josephson effect [1]. It is believed that superconductivity and the charge transport are mostly confined to the CuO_2 planes. Therefore, we can assume that on each CuO_2 plane we have a two-dimensional (2D) electron system where we have the Landau quantization similar to the quantum Hall effect (QHE) systems. One of the most important properties of the Josephson structure is the total quantum flux [2] which can be related to the phase difference across the junction, the sign of the phase difference controls the direction of the Josephson current while the magnitude of the phase difference affects the critical current itself [3–6]:

$$J_{\rm tot} = J_{\max}\cos\left(\frac{e\Phi_{\rm tot}}{\hbar}\right), \tag{1}$$

where $J_{\max} = 2J_0\sin\delta$ and $\delta = \theta_2-\theta_1$ is the phase difference and $\Phi_{\rm tot}$ is [7]:

$$\Phi_{\rm tot} = \left(l + \tfrac{1}{2} \pm \frac{g^*}{4}\right)\Phi_0. \tag{2}$$

In Eq. (2) the last term is the spin contribution of the quantized intrinsic flux of a correlated electron and hole ($\Phi_{\rm int} = \pm\frac{g^*\Phi_0}{2}$). Here $l=(0,1,2,\ldots)$, g^* is the effective Landé g-factor and Φ_0 is the unit of flux (fluxoid). In all these calculations the starting point is the canonical momentum of the charge carriers (electrons and holes with charge q). For an electron the canonical momentum is given by [8]:

$$(J_c)_z = (\boldsymbol{r}\times\boldsymbol{p}) = \left[\boldsymbol{r}\times m_0\boldsymbol{v} - \frac{e}{c}\boldsymbol{r}\times\boldsymbol{A}\right]_z = J_z - \frac{e}{2\pi c}\Phi. \tag{3}$$

Magnetic flux quantization was first recognized by London [9] and Onsager [10] who predicted that the enclosed flux through a superconducting ring is quantized in units of $\Phi_0 = \frac{hc}{e}$. Later, quantization of magnetic flux was experimentally observed in hollow superconducting cylinder [11, 12] and some of the properties in the same system has been theoretically studied [13]. Wan and Saglam [14] calculated the intrinsic magnetic flux associated with the electron's orbital and spin motions. They have obtained three basic magnetic flux quanta: the electron orbital magnetic flux quantum $\Phi_e^{(0)} = \frac{hc}{e}$, the electron spin magnetic flux quantum $\Phi_e^{(s)} = \frac{hc}{2e}$ and the magnetic flux quantum due to supercurrent in a superconducting ring, so-called the Cooper pair magnetic flux quantum $\Phi_e^{(sr)} = \frac{hc}{2e}$ which has a magnitude of 2.07×10^{-15} T m^2, measured by Deaver and Fairbank [11]. Saglam and Sahin [15] have calculated the intrinsic quantized magnetic flux of electron (and positron) in a uniform mag-

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netic field and found the spin-dependent quantum fluxes that an electron and positron carries with itself along the propagation direction even in the absence of the magnetic field. Saglam and Boyacıoglu [16] also calculated the effective Landé g-factor of two-dimensional systems by a simple diagram method which demonstrated that the crossing points correspond to the quantum entanglements of two different Landau states. In literature, the flux of the external magnetic field (and hence the external vector potential) has been considered to calculate the total quantum flux in the Josephson structures but, to our knowledge, no investigation has been made on the intrinsic quantum flux of correlated electrons and holes for the Josephson systems. In this work, we show the application of the quantized intrinsic quantum flux of a correlated electron and a hole in the Josephson junctions.

2. Formalism

In the presence of an external magnetic field $\boldsymbol{B}$ along the z-direction, the one electron Hamiltonian of the 2D system is written as

$$H = H_0 + H_{\text{Zeeman}} + H_{e.l} + H_{e.e} = \frac{p_x^2}{2m} + \frac{(p_y - eBx)^2}{2m} - g\mu_{\text{B}}B\sigma + V_{e.l} + V_{e.e}, \tag{4}$$

where H_0 is kinetic energy operator for free electron. H_{Zeeman} is Zeeman potential. $H_{e.l}$ is electron–lattice interaction and $H_{e.e}$ is electron–electron interaction (covering exchange potential and the Hartree electrostatic potential). In the Zeeman term g is Landé g-factor. $\sigma = \pm\frac{1}{2}$ is the spin number and $\mu_{\text{B}} = \frac{e\hbar}{2m}$. For strong magnetic field $H' = H_{e.l} + H_{e.e}$ can be treated as a small perturbation. The effect of H' can be the lumped in the effective mass (m^*), effective magnetic field (B^*), and the effective Landé g-factor (g^*). In this effective parameters approximation the eigenvalues corresponding to Eq. (4) read [16]:

$$E_{\text{Landau}} = \left[\left(l + \tfrac{1}{2}\right)\hbar\omega_c^* \pm \tfrac{1}{2}g^*\mu_{\text{B}}^*B^*\right] = \left(l + \tfrac{1}{2} \pm \frac{g^*}{4}\right)\hbar\omega_c^*, \tag{5}$$

where μ_{B}^* is the effective Bohr magneton, ω_c^* is the effective cyclotron angular frequency, and $l = (0, 1, 2, \ldots)$ is the Landau index. The quantum flux associated with the above eigenvalues, calculated by Saglam [7], is given by

$$\Phi_{\text{Landau}} = \left(l + \tfrac{1}{2} \pm \frac{g^*}{4}\right)\Phi_0, \tag{6}$$

where $\Phi_0 = \frac{hc}{e}$ is the flux unit. The last terms in Eq. (5) and Eq. (6) correspond to the spinning motion of an electron (+ sign corresponds to spin-up case and – sign corresponds to spin down case). Next, we define the dimensionless quantity $\nu_l = \frac{E_{\text{Landau}}}{\hbar\omega_c^*}$ which is the conventional filling factor. Thus, when plotting ν_l against $|g^*|$ the crossings occur at the even integer values of $g^*(0, 2, 4, \ldots)$ [16]. At the crossing points two states with opposite spins coincide at the same energy and the flux. Therefore, these points correspond to the quantum entanglements of two different Landau states. When a superconducting ring is placed in a weak magnetic field, the field lines are expelled from the superconductor and the magnetic flux through the ring takes the values $n\Phi_0$, where $\Phi_0 = \frac{hc}{2e}$ and n is a non-zero integer ($\pm1, \pm2, \pm3\ldots$). For classical superconductors, London and London [17] explained the Meissner effect by using a classical model based on the Maxwell equations and minimizing the associated free energy. The calculated penetration depths λ were in the range of 10−100 μm. Therefore when the planar size of the superconductor becomes comparable with λ^2 the above mentioned classical model becomes inapplicable. Because of the growing need to reduce the sizes into sub-micrometers (e.g. quantum computers, superconducting quantum interference devices (SQUID) [17, 18]), we have developed a quantum model which is based on the quantum entanglement of the Landau states of electrons and holes in the system. Because of the additional Zeeman energy term $E_{\text{Zeeman}} = \pm\frac{g^*\hbar\omega_c}{4}$ to the quantized Landau energies [7] given by E_{Landau}, we have entanglements of the states resulting zero total spin. A dimensionless function $f(n, g^*) = (E_{\text{Landau}} + E_{\text{Zeeman}})/\hbar\omega_c$ which takes the form $f(n, g^*) = \left(n + \frac{1}{2} \pm \frac{g^*}{4}\right)$ is defined (here n stands for l) the difference between ν_l and $f(n, g^*)$ is the additional Zeeman energy term. The plots of function $f(n, g^*) \equiv f(l, g^*)$ with respect to $g*$ shows that $g*$ takes only even integer values. Here, g^* is treated as a varying parameter. If we do not consider spin ($g^* = 0$), the Landau orbits in real space will be as shown in Fig. 1a. In the ground state electron will rotate in a circular orbit with radius r_0 which is equal to the magnetic length (λ_{ML}) as

$$r_0 = \lambda_{ML} = \sqrt{\frac{\hbar c}{eB^*}}. \tag{7}$$

Then, the flux corresponding to the ground state orbit will be

$$\Phi_0/2 = \pi\lambda^2 B, \tag{8}$$

which is obtained from Eqs. (2) and (7) by setting $l = 0$. For large l values the cyclotron radius is given by $r_l = \sqrt{2l+1}r_0 = \sqrt{2l+1}\sqrt{\frac{\hbar c}{eB^*}}$. For example, in the ground state because of the spinning motion of an electron we will have an additional, $\Phi_0/2$ flux term, and the orbits in real space will be similar to Fig. 1b which shows Landau orbits for spin-up electrons, for spin-down electrons this additional contribution will be $-\Phi_0/2$.

3. Application of Landau quantization to a 2D superconducting nanoring

Let us consider a superconducting ring of the inner radius a and the outer radius b. To treat the correlated electrons (holes) in quantum mechanical way, several important questions to be answered: At what limit, we treat electrons to be correlated and degenerated, so quantum mechanically considered?

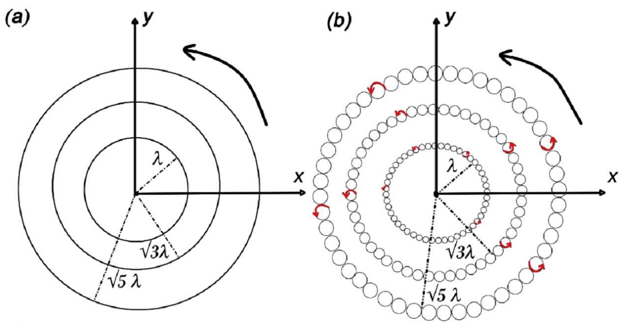

Fig. 1. Landau orbits for electrons: (a) without spin, (b) in real space for spin-up electrons.

From quantum statistic to treat electrons quantum mechanically the condition requires that the de Broglie wavelength of electrons, $\lambda_{\text{de-Broglie}}$ must be comparable with the size of the system

$$\lambda_{\text{de-Broglie}} = \frac{h}{p} \approx 2b. \tag{9}$$

Using Eq. (5) and $E = \frac{p^2}{2m}$, one can write

$$p = \sqrt{2l+1}\sqrt{m\hbar\omega_c^*}. \tag{10}$$

By substituting Eq. (10) in Eq. (9) one can find the following relations for $\lambda_{\text{de-Broglie}}$ and b:

$$b \approx \frac{\pi}{\sqrt{2l+1}}\sqrt{\frac{\hbar c}{eB}}. \tag{11}$$

Using Eqs. (5) and (9) we can write

$$\lambda \approx \sqrt{\frac{2l+1}{\pi^2}}b. \tag{12}$$

Since $\lambda < b$ then we get $l \leq 5$ ($l \leq 5$ corresponds to $n \leq 6$ which also agrees with the periodic table elements). That means, in a SQUID, the filling factor cannot exceed 5. So from Eq. (7) and Eq. (11) we can find lower and upper limits of system size.

TABLE I

Ten r_l values ($l = 0, 1, \dots 9$) corresponding to 10 different magnetic field values in the range (0.01 T $\leq B \leq$ 10 T) are listed.

B	r_0	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
0.01	256.50	444.27	573.55	678.64	769.50	850.71	924.82	993.42	1057.58	1118.06
0.1	81.11	140.49	181.38	214.60	243.33	269.04	292.44	314.14	334.42	353.64
1	25.65	44.42	57.35	67.97	76.95	85.08	92.48	99.26	105.75	111.83
2	18.13	31.40	40.53	48.04	54.39	60.13	65.37	70.22	74.75	79.05
3	14.81	25.65	33.12	39.25	44.43	49.12	53.40	57.36	61.06	64.57
4	12.82	22.20	28.67	33.97	38.46	42.52	46.22	49.65	52.86	55.90
5	11.47	19.87	25.65	30.40	34.41	38.05	41.36	44.42	47.29	50.01
6	10.47	18.13	23.41	27.75	31.41	34.72	37.75	40.55	43.16	45.65
8	9.07	15.71	20.50	24.04	27.21	30.09	32.70	35.12	37.40	39.54
10	8.11	14.05	18.13	21.49	24.33	26.90	29.24	31.41	33.44	35.36

In Table I, we have tabulated our calculations for 10 different r_l values corresponding to 10 different magnetic field values in the range 0.01 T $\leq B \leq$ 10 T. Here we can see the superconducting ring of the inner and outer radius, a and b respectively, also the distance between electron and hole charges. Therefore, a superconducting ring of the inner radius a (10 nm $< a <$ 100 nm) and the outer radius b (100 nm $< b <$ 1000 nm) will have the good size (as in SQUIDs) to study the quantum effects for magnetic fields. When it is placed in a magnetic field the charge distribution will be as in Fig. 2a and it can be equivalently replaced with the charge distribution shown in Fig. 2b.

The present model explains the Meissner effect very well and can be extended to the intrinsic quantum Josephson junction as well. In order to calculate the total quantum flux in the Josephson structures only the flux of the external magnetic field (and hence the external vector potential) has been considered in literature. To show

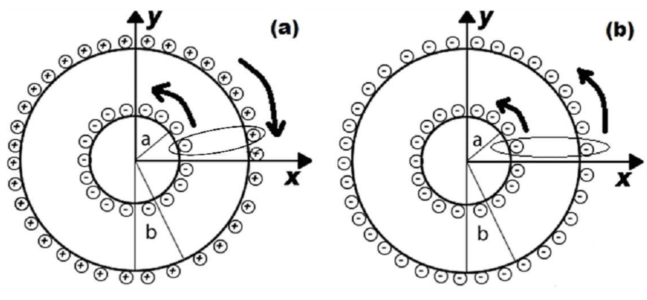

Fig. 2. (a) Electron–hole representation and (b) Cooper pairs.

the spin contribution to the total flux we take the results of Saglam [7] and Saglam and Boyacıoglu [19]; depending on the spin orientations, to the quantized intrinsic flux of a correlated electron (or hole) is equal to

$$\Phi_{\text{int}} = \pm\frac{g^*\Phi_0}{2}. \tag{13}$$

In the present study, considering the intrinsic quantum flux of electrons and holes, we have calculated the above mentioned phase differences across the Josephson junction. For electrons the additional flux contribution takes the value

$$\Delta\Phi_{\text{int}} = \pm\frac{g_e^*\Phi_0}{2} \tag{14}$$

and for holes, the related contribution is

$$\Delta\Phi_{\text{int}} = \pm\frac{g_h^*\Phi_0}{2}. \tag{15}$$

4. Conclusions

We have calculated the quantized intrinsic flux of a correlated electron and hole and showed that, for both charge carriers, the effective Landé g-factors (g_e^*, g_h^*) takes only even integer values. We have calculated ten different cyclotron radii for large Landau orbits l in a magnetic field ranging from 0.01 T to 10 T. We have presented the application of intrinsic quantized flux of electron and hole in the Josephson structure. Our calculations also demonstrated that Φ_{tot} is as a function of $B*, g^*$ (effective Landé g-factor) and the spin orientation.

Appendix: Calculation of the magnetic flux for a spinning electron (hole)

Spin magnetic moment of a free electron is given by

$$\boldsymbol{\mu} = g\mu_{\text{B}}\boldsymbol{S}, \tag{A.1}$$

where $\hbar\boldsymbol{S}$ is the spin angular momentum of the electron. When we introduce the magnetic field $\boldsymbol{B} = B\boldsymbol{z}$, the z component of the magnetic moments becomes

$$\mu_z = \pm\mu_{\text{B}} = \pm\frac{e\hbar}{2mc}. \tag{A.2}$$

Following Saglam and Boyacıoglu [19] we assume that spin angular momentum of the electron (hole) is produced by the fictitious point charge $\pm e$ rotating in a circular orbit with the angular frequency ω_s and radius R in $x{-}y$ plane. As it is shown in [19] as far as the magnetic flux is concerned the radius are is a phenomenal concept whose detailed calculation in terms of electron (hole) radius is not important here. When we put spinning electron (hole) in an external magnetic field B, the field will not change the electron's intrinsic angular velocity ω_s (because $\omega_s \gg \omega_c = eB/mc$). But it will apply a tork of $\boldsymbol{\mu}\times\boldsymbol{B}$ which becomes zero when the spin is either parallel or antiparallel to the magnetic field. In this case z-component of this magnetic moment for spin-down electron will be

$$\mu_z = -\frac{IA}{c} = \frac{e\omega_s A}{2\pi c}, \tag{A.3}$$

where $A = \pi R^2$ is the area of the above mentioned circular loop. If we compare (A.2) and (A.3) we find

$$A = -\frac{h}{2m\omega_s}. \tag{A.4}$$

Now we want to calculate the flux for spin-down electron during the cyclotron period T_c.

It is worth to note that during the cyclotron period T_c electron will complete ω_s/ω_c turns about itself. So the total flux during the cyclotron period will be

$$\phi(\downarrow) = \frac{\omega_s}{\omega_c}AB. \tag{A.5}$$

Substitution of (A.4) and $\omega_c = eB/mc$ in (A.5) gives

$$\Delta\Phi_{\text{int}}^e = \pm\frac{g_e^*\Phi_0}{2}, \quad \Delta\Phi_{\text{int}}^h = \mp\frac{g_h^*\Phi_0}{2}$$

$$\Delta\Phi_{\text{int}} = \pm\frac{g_e^*\Phi_0}{2}, \quad \phi(\downarrow) = \frac{hc}{2e} = \frac{\phi_0}{2}. \tag{A.6}$$

Similarly the flux for the spin-up electron will take the form

$$\phi(\uparrow) = \frac{hc}{2e} = -\frac{\phi_0}{2}. \tag{A.7}$$

Now if we follow similar procedure for spin-up and spin-down holes we get

$$\phi(\downarrow) = -\frac{\phi_0}{2} \tag{A.8}$$

and

$$\phi(\uparrow) = +\frac{\phi_0}{2}. \tag{A.9}$$

From the above equations the change of the flux in the flip will be

$$\Delta\phi = \pm\phi_0. \tag{10}$$

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