

Josephson Effect in Graphene-Based Junctions

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We study the Josephson effect in graphene based junctions where superconductivity in graphene is induced by the proximity effect from external substrate materials. The electronic properties of the junction is described by the Dirac-Bogoliubov-de-Gennes equations. We consider the junction consisting of two superconductors with different pairing potentials. Using appropriate boundary conditions imposed on the normal region-superconductors interfaces, we calculated the Andreev bound state energy, in the ballistic limit, taking into account two types of reflections namely the retro and specular Andreev reflections.

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1. Introduction

The unusual electronic and spintronic processes in graphene have attracted an intense interest in experimental and theoretical solid state physics [1–4]. The fermionic charge carriers in graphene systems behave as massless chiral relativistic particles, symmetrically located between the valence and conduction bands and forming appropriate conical band forms. The conical conduction band touches the conical valence band at the six Dirac points which are located at the edges of the hexagonal Brillouin zone. Therefore, graphene belongs to gapless semiconductors with a Dirac-like dispersion of massless elementary excitations. Natural graphene does not exhibit any superconducting or ferromagnetic properties. However, long range orders, such as superconductivity or magnetism, can be induced in graphene layer via the proximity effects due to the interactions of Dirac fermions with superconducting [5] or ferromagnetic substrates [6]. Graphene-based junctions in which normal or ferromagnetic regions are connected with a superconductor material have many features which are interesting for application in spintronics as well as intriguing for research in fundamental physics.

In this paper we study the Josephson effect in graphene based junctions [7–13] where superconductivity in graphene is induced by the proximity effect from two different types of superconducting external sheets. The junction considered is described by the Dirac-Bogoliubov-de-Gennes equation with appropriate boundary conditions imposed on the normal metal-superconductor (NS) interface.

In the first step of our investigations we calculate the Andreev bound states in the normal region [10, 14, 15] taking into account two types of reflections with the electron-hole conversion, namely the specular Andreev

and retro Andreev reflections [16]. The Andreev specular (interband) reflection, characteristic feature of graphene, occurs when a conduction electron is converted into a hole in the valence band, while the retro (intraband) Andreev reflection induces intraband process of conversion an electron into a hole. The Andreev retro-reflection appears in conventional materials where the Fermi energy is much greater than a superconducting gap Δ . The specular Andreev reflection can take place only when the Fermi energy is less than a superconducting order parameter. This condition can be easily reached in undoped graphene.

Recently, the Josephson effect in graphene-based junctions was studied by many authors e.g. [3–7, 12, 13]. In this paper, we calculate the Andreev bound state energy in the ballistic limit [6, 10, 12, 13] for the graphene-based junctions in which the induced superconductivity (S_LG , S_RG) is created by two different substrate superconducting materials (S_L , S_R) (Fig. 1).

2. Model and calculations

We discuss the Josephson effect in a graphene-based junction $S_LG/NG/S_RG$ consisting of two superconducting graphene layers (S_LG and S_RG) (see Fig. 1) and a normal graphene layer (NG). The thickness L of the normal region is equal L and the width of the Josephson junction along the y axis is W . The normal region is modeled by a barrier potential which can be created by using either the electric field effect or local chemical doping [1]. We consider the short-junction limit where the values of the thickness should be smaller than the superconducting coherence length given by $\xi = \hbar v_F / \pi \Delta$ [17]. A typical conventional superconductor has a gap of order of 1mV, and thus ξ is order of 1000nm. Similar values are assumed for the proximity induced superconductor in graphene. We assume that $L \ll \xi$, $2\pi/k_F$ (k_F — the Fermi wave vector). The width W of the junction is assumed to be the largest length scale in our model.

The superconductivity in graphene are induced due to the proximity effects by a superconducting material, deposited on the top of the junction. For simplicity, we

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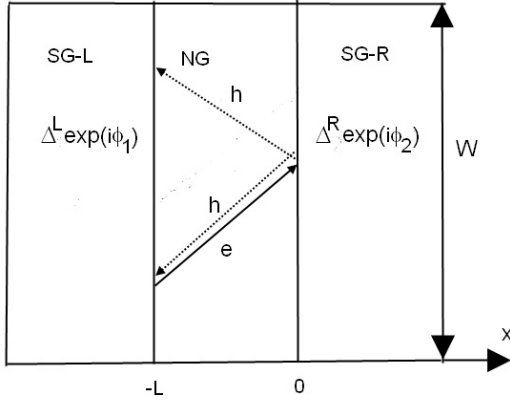


Fig. 1. The schematic graphene-based junction $S_L G : N G : S_R G$. S_L and S_R stand for left and right substrate superconducting materials, respectively. N is a non-superconducting substrate.

consider the s -wave superconductors only. We analyze the behavior of the different scattering probabilities. In particular, we have concentrated on dependence of the Andreev bound state in the normal-superconductor interface on the phase difference between the superconductors. In our analysis we use the Dirac-Bogolubov-de Gennes (DBdG) equations [16] in the following form

$$\begin{pmatrix} H_{\alpha\sigma} - E_F \hat{1} & \Delta(\mathbf{r}) \\ \Delta^\dagger(\mathbf{r}) & -[H_{\alpha\sigma} - E_F \hat{1}] \end{pmatrix} \Psi_\alpha = E \Psi_\alpha, \quad (1)$$

where $\Psi_\alpha = (\Psi_{A\alpha\sigma}, \Psi_{B\alpha\sigma}, \Psi_{A\alpha'\sigma}^*, \Psi_{B\alpha'\sigma}^*)^T$, stands for four-component wave function and T is the transpose; The subscripts have the following meaning: A and B denote the two sublattices of graphene while $\alpha(\alpha')$ indicates the valley $K(K')$ in the Brillouin zone. Index $\sigma = 1(\uparrow)$ denotes the spin-up electron and the spin-down hole, whereas $\sigma = -1(\downarrow)$ labels the spin-down electron and the spin-up hole. The quasiparticle energy E is measured from the Fermi energy E_F , and around each of the Dirac points, low energy electrons and holes have linear, Dirac-like, dispersion. Thus, we get the conical-like conduction and valence bands which touch each other at the Dirac points. This is the origin of a graphene based gapless semiconductor with a relativistic-like dispersion relation. We can easily extend our model for the situation when the normal region is replaced by the ferromagnetic one. Then we can adopt the Stoner model with the exchange fields $h(x) = h_0$ for $x < 0$ and $x > -L$.

The single-particle Hamiltonian $H_{\alpha\sigma}$ for K valley has the form $H_{\alpha\sigma} = -i\hbar v_F[\sigma_x \partial_x + \sigma_y \partial_y] + U(x) - \sigma h(x)$, where $U(x)$ is the electrostatic potential which can be adjusted via a gate voltage or doping. We assume $U(x) = U_0$ for $x < 0$ and $x > -L$.

The wave function, describing the quasiparticle propagation across the junction, is obtained from DBdG equations. The solution of Eq.(1), for all regions of the junction and for the injection of an electron with the energy E and the angle of incidence θ_e , can be written in three

following forms:

in the left superconducting region ($x < -L$):

$$\begin{aligned} \Psi_{S_L G}(x) = & \mathbf{r}_L^e [u_L^e, u_L^e e^{i(\pi-\theta_e)}, v_L^e e^{-i\phi_1}, v_L^e e^{i(\pi-\theta_e-\phi_1)}]^T \\ & \times e^{-iq_L^e \cos \theta_e x} \\ & + \mathbf{r}_L^h [v_L^h, v_L^h e^{i\theta_h}, u_L^h e^{-i\phi_1}, u_L^h e^{i(\theta_h-\phi_1)}]^T \\ & \times e^{iq_L^h \cos \theta_h x}, \end{aligned} \quad (2)$$

in the normal graphene region ($-L < x < 0$):

$$\begin{aligned} \Psi_{N G}(x) = & \mathbf{a} [1, e^{i\theta}, 0, 0]^T e^{ip_e \cos \theta x} \\ & + \mathbf{b} [1, -e^{-i\theta}, 0, 0]^T e^{-ip_e \cos \theta x} \\ & + \mathbf{c} [0, 0, 1, e^{-i\theta_A}]^T e^{-ip_h \cos \theta_A x} \\ & + \mathbf{d} [0, 0, 1, -e^{i\theta_A}]^T e^{ip_h \cos \theta_A x}, \end{aligned} \quad (3)$$

in the right superconducting region ($x > 0$):

$$\begin{aligned} \Psi_{S_R G}(x) = & \mathbf{r}_R^e [u_R^e, u_R^e e^{i\theta_e}, v_R^e e^{-i\phi_2}, v_R^e e^{i(\theta_e-\phi_2)}]^T \\ & \times e^{iq_R^e \cos \theta_e x} \\ & + \mathbf{r}_R^h [v_R^h, v_R^h e^{i(\pi-\theta_h)}, u_R^h e^{-i\phi_2}, u_R^h e^{i(\pi-\theta_h-\phi_2)}]^T \\ & \times e^{-iq_R^h \cos \theta_h x}, \end{aligned} \quad (4)$$

where

$$p_{e(h)\sigma} = (E + (-)E_F + \sigma h)/\hbar v_F,$$

$$q_{L(R)}^{e(h)} = (E_F^S + (-)\sqrt{E^2 - \Delta_{L(R)}^2})/\hbar v_F;$$

u and v are the BCS coherence factors given by:

$$u_{L(R)}^{e(h)} = \sqrt{\frac{1}{2}(1 + \sqrt{E^2 - |\Delta_{L(R)}(\Theta^{+(-)})|^2}/E)}, \quad v_{L(R)}^{e(h)} = \sqrt{\frac{1}{2}(1 - \sqrt{E^2 - |\Delta_{L(R)}(\Theta^{+(-)})|^2}/E)},$$

and $\Phi = \phi_2 - \phi_1$ is the superconducting phase difference between the left and right superconducting regions. These phases are associated with the broken $U(1)$ gauge symmetry in the superconducting states. The angles θ^A , θ_e , θ_h are related to the angle of incidence θ and can be determined from the assumed momentum conservation in the y direction where the system is translationally invariant: $q_{L(R)}^i \sin \theta_i = p_e \sin \theta$ $i = e, h$, $p_h \sin \theta_A = p_e \sin \theta$ and $p_e = E + E_F$, $p_h = E - E_F$.

From the conservation of momentum, we find a critical incidence angle θ_c , which is defined as that for which the angle of Andreev reflection $\theta_A = \pi/2$. Consequently, the critical angle for the graphene based junction is given by the equation:

$$\theta_c = \arcsin \frac{E - E_F}{E + E_F} \quad (5)$$

For angles greater than θ_c , the wave functions describing the reflection processes become evanescent, and thus, these processes do not contribute to any charge transport.

The probability amplitudes, for the all transport processes in graphene-based junctions, are determined from the appropriate boundary conditions, imposing only continuity of the wave functions at all the interfaces:

$$\Psi_{NG}(x = -L) = \Psi_{SLG}(x = -L)$$

$$\Psi_{SRG}(x = 0) = \Psi_{NG}(x = 0) \quad (6)$$

The first stage of our investigation is to calculate energy of the Andreev bound state which determines the Josephson current. From the boundary conditions (see Eq.(6)) we obtain a set of 8 linear equations. The condition for non-zero solutions leads to the Andreev bound state energies $E(\Phi)$. In the numerical calculations we assume the width junction limit ($W \gg L$) and we neglected a magnetic order for $-L < x < 0$. We also assume that there is no Fermi level mismatch. It means the same value of the Fermi level in the normal and superconducting regions ($E_F = E_F^S$). The last assumption yields only one Andreev bound state energy for each value of the phase difference Φ (Fig. 2). The Josephson current can be obtained from the following expression [14, 17, 18]:

$$I(\Phi) = \frac{4e}{\hbar} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \frac{dE(\Phi)}{d\Phi} f(E(\Phi)/(k_B T)), \quad (7)$$

where the factor 4 accounts for the twofold spin and valley degeneracies.

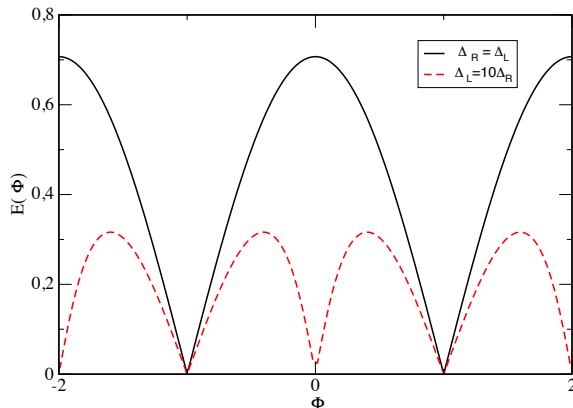


Fig. 2. Andreev bound state energy in $1/\sqrt{\Delta_L \Delta_R}$ units vs. phase difference divided by π .

3. Final remarks

In this paper we consider the Josephson junction where superconductivity in graphene is induced by two different superconducting substrates. The charge transport in the graphene-based Josephson junctions is determined by the Andreev reflection processes. For junctions consisting of two different superconducting materials (S_L and S_R) both the magnitude and period of oscillations of the Andreev bound energy, as a function of the phase difference Φ , strongly depends on $\sqrt{(\Delta_L)\Delta_R}$. The following important questions should be answered in our further studies:

- 1) How to distinguish the contributions to the Josephson current coming from the retro and specular Andreev reflections respectively, for mid-gap incident energies?
- 2) How the chiral nature of the quasiparticle influences the Josephson supercurrent in superconducting graphene based junctions?

In our preliminary calculations we consider only conventional s -wave superconductivity. It should be expected, however, that the oscillating nature of the Andreev bound states, and thus the oscillating character of the induced Josephson current in graphene, will depend not only on the magnitude of the gap functions and the phase difference, but on the anisotropy of the order parameter in the case of unconventional pairing superconducting graphene structures.

References

- [1] M.I. Katsnelson, K.S. Novoselov, A.K. Geim, *Nat. Phys.* **2**, 620, (2006).
- [2] S.Y. Zhou, G.H. Gweon, J. Graf, A.V. Fedorov, C.D. Spataru, R.D. Diehl, Y. Kopelevich, D.-H. Lee, S.G. Louie, A. Lanzara, *Nat. Phys.* **2**, 595 (2006).
- [3] B. Huard, J.A. Sulpizio, N. Stander, K. Todd, B. Yang, D. Goldhaber-Gordon, *Phys. Rev. Lett.* **98**, 236803 (2007).
- [4] V. E. Calado, et al., *Nature Nanotechnology* **10**, 761 (2015).
- [5] M. Ben Shalom, M.J. Zhu, V.I. Fal'ko, A. Mishchenko, A.V. Kretinin, K.S. Novoselov, C.R. Woods, K. Watanabe, T. Taniguchi, A.K. Geim, J.R. Prance, [arXiv:1504.03286](https://arxiv.org/abs/1504.03286), (2015).
- [6] Zhiyong Wang, Chi Tang, R. Sachs, Y. Barlas, Jing Shi, *Phys. Rev. Lett.* **114**, 016603 (2015).
- [7] Xiaowei Li, *Solid St. Commun.* **151**, 1976 (2011).
- [8] D. Bolmatov, Chung-Yu Mou, *Physica B* **405**, 2896 (2010).
- [9] M. Maiti, K. Sengupta, *Phys. Rev. B* **76**, 054513 (2007).
- [10] M. Titov, C.W.J. Beenakker, *Phys. Rev. B* **74**, 041401(R) (2006).
- [11] A.G. Moghaddam, M. Zareyan, *Phys. Rev. B* **74**, 241403(R) (2006).
- [12] Kun-Hua Zhang, Zhen-Gang Zhu, Zheng-Chuan Wang, Quing-Rong Zheng, *Physics Letters A* **378**, 3131 (2014).
- [13] N. Mizuno, B. Nielson, Xu Du, *Nature Communications* **4**, 2716 (2013).
- [14] C.W.J. Beenakker, H. van Houten, *Phys. Rev. Lett.* **66**, 3056 (1991).
- [15] T. Löfwander, V.S. Shumeiko, G. Wendin, *Supercond Sci. Technol.* **14**, R53 (2001)
- [16] C.W.J. Beenakker, *Phys. Rev. Lett.* **97**, 067007 (2006).
- [17] J. Linder, T. Yokoyama, D. Huertas-Hernando, A. Sudbø, *Phys. Rev. Lett.* **100**, 187004 (2008).
- [18] R.A. Reidel, P.F. Bagwell, *Phys. Rev.* **57**, 6085 (1998).