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Temperature Dependence of Spin Hall Effect in $\boldsymbol{k}$–Cubed Rashba Model

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Within the Matsubara Green function formalism and linear response theory we considered theoretically the temperature dependences of the spin Hall effect for a two-dimensional gas with an isotropic $\boldsymbol{k}$–cubed form of the Rashba interaction. We utilize a standard model for treating spin-orbit phenomena in p-doped semiconductor heterostructures and also for an electron gas formed at perovskite oxides interfaces.

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1. Introduction

The spin Hall effect [1] is one of the hallmarks in the field of spintronics offering the opportunity to generate and control spin currents in a pure electrical way [2–7]. The detectors based on the inverse spin Hall effect have become a standard tool for sensing spin currents and were instrumental in the discovery of the spin Seebeck phenomenon [8–10]. Recenty, it was also shown that the spin currents generated via spin Hall effect can be large enough to produce sizable spin torque effects (see e.g. [11–14]).

The underlying physics of the spin Hall effect is very rich depending on the nature of the intrinsic spin-orbit coupling of the host material. Impurities and structural defects are further sources of spin-orbit couplings that is of prime imporatnce for spin-dependent scattering phenomena (for review see: [15–17] and references therein). A proper theoretical description of the fundamental physics responsible for the SHE in materials that are potentially important for spintronics is crucial when it comes to the control of spin currents driven by spin Hall effect. Another important issue is the thermal behavior of the spin Hall effect. The role of finite temperatures in the theoretical description of spin-orbital phenomena has been studied in few papers in the context of 2D $\boldsymbol{k}$–linear Rashba gas (for example [18–23]). However there is still lack of a consistent theory of SHE which takes into account all microscopic mechanisms and describes the spin Hall effects at high temperatures.

The spin Hall effect at zero-temperature is fairly well understood in n-doped semiconductor heterostructures where the simple model describing 2D electron gas with $\boldsymbol{k}$–linear Rashba and Dresselhaus spin-orbit interaction seems to be valid approximation allowing an analytical description of spin-orbital phenomena. For n-doped semiconductors the Luttinger Hamiltonian is a suitable framework. Recently, a large Rashba spin-orbit coupling has been experimentally observed at the interface of $LaAlO_3/SrTiO_3$ (LAO/STO) [25]. The interfaces of oxide perovskites such as LAO/STO, where the 2D electron gas has been recently discovered are attracting a great attention both experimentally and theoretically. The transition metal oxides heterostructures show interesting physical properties such as two-dimensional metallic conductivity [26] metal-insulator transition [27, 28], low-temperature superconductivity and ferromagnetism as well as its coexistence [29, 30], or large negative magnetoresistance [31]. The physics for the formation of the electron gas is still not settled for all compounds. One of proposed theoretical models of Rashba 2D gas in STO based heterostructures utilizes the effective $\boldsymbol{k}$-cubed Rashba Hamiltonian [32, 33] which is well known from the case of heavy-hole model for semiconductor heterostructures [34].

In this paper we discuss the temperature dependences of the spin Hall effect for the $\boldsymbol{k}$–cubed Rashba Hamiltonian. Our results may be suitable for both p-doped semiconductor heterostructures and oxide perovskites thin films or interfaces.

2. Model and methods

The effective Hamiltonian describing electrons or holes in isotropic $\boldsymbol{k}$–cubed Rashba gas can be written in the matrix form [35]:

$$\mathcal{H} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} & \mathrm{i}\lambda k_-^3 \\ -\mathrm{i}\lambda k_+^3 & \frac{\hbar^2 k^2}{2m} \end{pmatrix}, \tag{1}$$

where $k^2 = k_x^2 + k_y^2$, $k_\pm = k_x \pm \mathrm{i} k_y$, and m is the effective mass of the particle defined by Luttinger parameters. λ is the Rashba spin-orbital coupling parameters which depends on Luttinger parameters, width and the potential strength of the quantum well - see Ref. [35].

The casual Green function corresponding to the Hamiltonian (1) can be written in the form:

$$G_{\boldsymbol{k}}(\varepsilon) = G_{\boldsymbol{k}0}\sigma_0 + G_{\boldsymbol{k}x}\sigma_x + G_{\boldsymbol{k}y}\sigma_y \tag{2}$$

with coefficients:

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$$G_{\boldsymbol{k}0} = \tfrac{1}{2}(G_{k+} + G_{k-}) \tag{3}$$

$$G_{\boldsymbol{k}x} = \sin(3\phi)(G_{k+} - G_{k-}) \tag{4}$$

$$G_{\boldsymbol{k}y} = -\cos(3\phi)(G_{k+} - G_{k-}) \tag{5}$$

where $G_{k\pm} = [\varepsilon + \mu - E_\pm + \mathrm{i}\delta\mathrm{sgn}(\varepsilon)]^{-1}$ and $E_\pm = \frac{\hbar^2 k^2}{2m} \pm \lambda k^3$ denote the energy eigenvalues; σ_0 and σ_α ($\alpha = x, y, z$) are unit and Pauli matrices in spin space.

To obtain the spin Hall conductivity and discuss its temeprature dependence we have used the Matsubara-Green function formalism [36]. In the regime of linear response to the external electromagnetic field the spin Hall current derives from the expression

$$\sigma_{xy}^{s_z}(\mathrm{i}\omega_m) = \frac{1}{\beta}\sum_{\boldsymbol{k},n}\mathrm{Tr}\left\{\hat{j}_x^{s_z} G_{\boldsymbol{k}}(\mathrm{i}\varepsilon_n + \mathrm{i}\omega_m)\hat{H}_{\boldsymbol{A}}^E(\mathrm{i}\omega_m) G_{\boldsymbol{k}}(\mathrm{i}\varepsilon_n)\right\}, \tag{6}$$

where $\beta = 1/k_{\mathrm{B}}T$ and $\hat{H}_{\boldsymbol{A}}^E(\mathrm{i}\omega_m) = -\hat{j}_y A_y(\mathrm{i}\omega_m)$ is the perturbation Hamiltonian defined by the y-component of the current density operator $\hat{j}_y = e\hat{v}_y$ and the y-component of electromagnetic vector potential $A_y(\mathrm{i}\omega_m)$. Operator of the spin current density is defined as an anticommutator of the velocity operator and the z-th component of spin operator ($s_z = 3\hbar\sigma_z/2$), that is $\hat{j}_x^{s_z} = [\hat{v}_x, \hat{s}_z]_+/2$ (note that the velocity operators are defined as $\hat{v}_{x,y} = \partial\hat{H}/\partial k_{x,y}$) and $G_{\boldsymbol{k}}(\mathrm{i}\epsilon_n)$ denotes Matsubara Green function. The equation above leads finally to the frequency-dependent spin Hall conductivity in the form [21]:

$$\sigma_{xy}^{s_z}(\omega) = -\frac{e\hbar}{\omega}\mathrm{Tr}\int\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\int\frac{\mathrm{d}\varepsilon}{2\pi}f(\varepsilon) \times\Big[\hat{j}_x^{s_z} G_{\boldsymbol{k}}^R(\varepsilon+\omega)\hat{v}_y[G_{\boldsymbol{k}}^R(\varepsilon) - G_{\boldsymbol{k}}^A(\varepsilon)] + \hat{j}_x^{s_z}[G_{\boldsymbol{k}}^R(\varepsilon) - G_{\boldsymbol{k}}^A(\varepsilon)]\hat{v}_y G_{\boldsymbol{k}}^A(\varepsilon - \omega)\Big]. \tag{7}$$

Upon tracing and integrating over ε we obtain an expression for the dc-limit

$$\sigma_{xy}^{s_z} = e\frac{9}{16\pi}\frac{\hbar^2}{m}\left[\int \mathrm{d}k k\frac{f'(E_+) + f'(E_-)}{1 + (\lambda k^3/\Gamma)^2} - \int\frac{\mathrm{d}k}{\lambda k^2}[f(E_+) - f(E_-)]\right] \tag{8}$$

This is our general result for spin Hall conductivity. Here we should comment that the above expression is obtained in a single-loop approximation which corresponds to the quasi-ballistic limit. Assuming randomly distributed point-like impurities we may find that (i) the relaxation rate Γ is the same for both subbands [35], and (ii) the impurities vertex correction in the ladder approximation vanishes [35, 37].

3. Results and discussion

In the low temperature limit we may find simple analytical expressions in the form

$$\sigma_{xy}^{s_z} = -\frac{9}{16\pi}e\frac{\hbar^2}{m\lambda}\frac{k_{\mathrm{F}+} - k_{\mathrm{F}-}}{k_{\mathrm{F}+}k_{\mathrm{F}-}} - \frac{9}{16\pi}e\frac{\hbar^2}{m}\left(\frac{\nu_+}{1 + (\lambda k_{\mathrm{F}+}^3/\Gamma)^2} + \frac{\nu_-}{1 + (\lambda k_{\mathrm{F}-}^3/\Gamma)^2}\right) \tag{9}$$

where $k_{\mathrm{F}\pm}$ and $\nu_\pm$ are the Fermi vawevectors and density of states related to the $E_\pm$ subband respectively. Assuming $\Gamma \ll \lambda k_{\mathrm{F}\pm}^3$ we obtain

$$\sigma_{xy}^{s_z} = -\frac{9}{16\pi}e\frac{\hbar^2}{m\lambda}\frac{k_{\mathrm{F}+} - k_{\mathrm{F}-}}{k_{\mathrm{F}+}k_{\mathrm{F}-}}. \tag{10}$$

This result is consistent with result in Ref. [38].

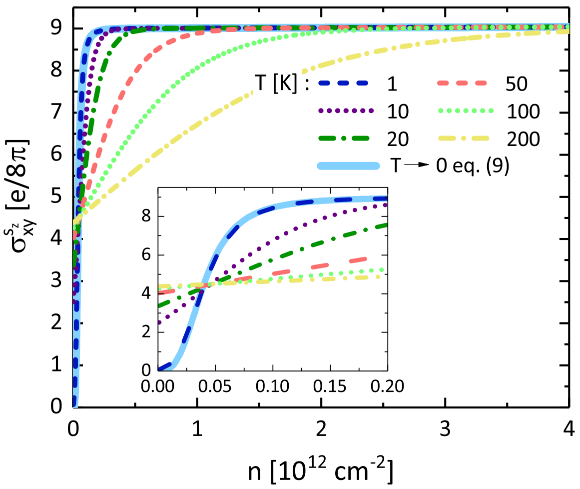

Fig. 1. Spin Hall conductivity as a function of carrier density. Different curves correspond to indicated values of temperature. The solid blue line corresponds to the analytical expression (Eq. (10)) obtained in the zero-temperature limit. In numerical calculations it was assumed that $\lambda = 3.96\cdot 10^{-29}$ eV m^3, $\Gamma = 5\cdot 10^{-6}$ eV.

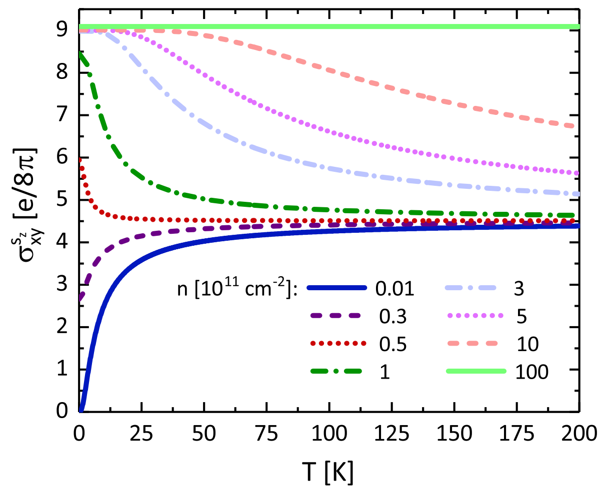

Fig. 2. Spin Hall conductivity as a function of temperature for fixed values of carrier density. Other parameters are as in Fig. 1.

Figures 1 and 2 present some numerical results for spin Hall conductivity obtained based on Eq.(8) and expressed in terms of carrier density n. In case of the model under consideration the relations between $k_{\mathrm{F}\pm}$, n and chemical potential are simple and can be found e.g. in [38].

In Fig. 1 we show the spin Hall conductivity plotted as a function of carrier density for fixed values of temperature. The analytical result (for $T \to 0$) is also indicated. Fig. 2 presents the spin Hall conductivity as a function of the temperature for different charge concentrations.

To conclude, we considered theoretically the temperature dependence of Spin Hall effect in 2D electron gas with isotropic $\boldsymbol{k}$–cubed Rashba interaction that formed at the perovskite oxides interfaces. The model under consideration is a basic model suitable not only for some group of perovskite oxides but also for p-doped semiconductor heterostructures.

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