Spin Dependent Conductance of a Quantum Dot Side attached to Topological Superconductors as a Probe of Majorana Fermion States

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Spin-polarized transport through a quantum dot side attached to a topological superconductor and coupled to a pair of normal leads is discussed in Coulomb and Kondo regimes. For discussion of Coulomb range equation of motion method with extended Hubbard I approximation is used and Kondo regime is analyzed by Kotliar-Ruckenstein slave boson approach. Apart from the occurrence of zero bias anomaly the presence of Majorana states reflects also in splitting of Coulomb lines. In the region of Coulomb borders the spin dependent negative differential conductance is observed. Due to the low energy scale of Kondo effect this probe allows for detection of Majorana states even for extremely weak coupling with topological wire. In this range no signatures of Majorana states appear in Coulomb blockade dominated transport.

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1. Introduction

Majorana fermions and Majorana bound states (MBSs) have attracted considerable interest in recent years due to their fundamental exotic properties e.g. self-hermicity and associated with this peculiarity non-Abelian statistics \cite{1}. These features make Majorana states potential candidates for the use in fault-tolerant topological quantum computation \cite{2}. MBSs are predicted to exist at the ends of a semiconductor nanowire with strong spin-orbit coupling placed in external magnetic field and brought into proximity of s-wave superconductor \cite{3}. Also several other propositions for the realization of 1d topological superconductor (TS) have been reported e.g. carbon nanotubes with broken chiral symmetry and curvature induced spin-orbit coupling \cite{4}. Various proposals have been made to detect the Majorana states using different hybrid structures based on quantum dots \cite{5,6}. In this report we discuss signatures of Majorana states in transport through quantum dot coupled to normal leads and to a single or a pair of TS wires. Both Coulomb blockade range and Kondo regime are discussed. Due to the helical properties of TS wire its Majorana end-state hybridizes with only one of the dot spin orientations \cite{1,7}, and thus in addition to conductance also polarization of conductance gives information on Majorana states. Present report complements the earlier studies on this topic by detailed analysis of spin polarization. We present the possibility of control of spin transport and we find the spin negative differential conductance, the phenomenon of wide array of potential applications e.g. in spin dependent amplifiers or switching circuits.

2. Model and formalism

T-shape system of Majorana bound state coupled to the quantum dot is presented in Fig. 1a (single side attached TS) and on the inset of Fig. 2a (a pair of TSs). The total Hamiltonian is $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{MBS} + \mathcal{H}_{DM}$. $\mathcal{H}_0$ is the Anderson hamiltonian and it describes the dot and the leads:

$$\mathcal{H}_0 = \sum_{\sigma} E_d n_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{\kappa = L, R, \sigma} \varepsilon_{\kappa \sigma} n_{\kappa \sigma} + \sum_{\kappa \sigma} (\nu_{\kappa \sigma}^\dagger d_{\sigma} + \text{h.c.})$$

where $E_d$ is QD level and $\varepsilon_{\kappa \sigma}$ are energies of conduction electrons. The term parameterized by $U$ describes Coulomb interaction and $V$ represents hopping between the dot and normal leads. $\mathcal{H}_{MBS} = \sum_{l} i \delta \gamma_{1l}^\dagger \gamma_{2l}$ describes coupling between the two Majorana states $\gamma_{1l}, \gamma_{2l}$ in the $l$-th TS wire ($l = 1, 2$). Coupling between Majorana modes can be neglected in the case when TS wire is much longer than the coherence length ($L \gg \xi$). In the opposite limit (dirty topological superconductors) $\delta \neq 0$ ($\delta \sim e^{-L/\xi}$). The Majorana operators can be expressed by fermionic operators: $\gamma_{1l} = (f_l + f_l^\dagger)/\sqrt{2}$, $\gamma_{2l} = i(f_l - f_l^\dagger)/\sqrt{2}$. Due to helical properties of TS wire Majorana states are spin polarized \cite{7}. The dot-Majorana coupling term is $\mathcal{H}_{DM} = -t_1 \gamma_{1l}^\dagger (-d_l + d_l^\dagger) + i t_2 \gamma_{2l}^\dagger (d_l + d_l^\dagger)$, where we assumed configuration in which spin polarizations of Majorana states $\gamma_{1l}^\dagger$ and $\gamma_{2l}^\dagger$ are opposite and thus electrons of one spin direction, say up, are coupled with MBS from the lower wire and spin down electrons with MBS from the upper wire. To find QD retarded Green’s functions $G^R(t) = \langle \langle d_{\sigma}(t); d_{\sigma}^\dagger(0) \rangle \rangle$ in Coulomb...
blockade regime we use extended Hubbard I approximation. In Hubbard I approach two-particle Green’s function $\langle d_i^\dagger d_j^\dagger d_k d_l \rangle$ are decoupled as follows $\langle n_i \rangle \langle d_i^\dagger d_i \rangle$, and in the extension we use they are treated exactly. Kondo regime is discussed within finite $U$ slave boson approach (K-R) [8], which maps the problem into the slave boson regime with occupation numbers on it. Conductance map with occupation numbers. White and black circles symbolize two Majorana regions for spin down orientation. e,f) Plots of polarizations of conductance for $E_d = -U/2$ plotted vs. $t_1$ for several fixed values of $t_1 = t_2^*$ (right axis) or vs. $t_1$ for fixed values of $t_2 = t_2^*$ (left axis), $t_1^{*2} = 0.005$ (solid gray line), $t_1^{*2} = 0.1$ (dotted blue), $t_1^{*2} = 0.5$ (solid red) and $t_1^{*2} = 1$ (dashed black). Lower inset presents a scheme of quantum dot coupled to two TS wires and upper inset shows total conductance map for $t_1 = 0.5$ and $t_2 = 0.25$ ($I = 0.05$). b,c) Differential conductance for $\delta_1 = \delta_2 = 0$ (b) and $\delta_1 = 0, \delta_2 = 0.2$ (c). d) Polarization of conductance in the low bias region for $\delta_1 = 0, \delta_2 = 0.2$ ($I = 0.25, t_1 = t_1 = 0.5$).

3. Results

First we discuss the impact of single Majorana state on Coulomb blockade considering QD with only one side attached TS wire ($t_2 \to 0$) (inset of Fig. 1a). Figs. 1c, d show the spin resolved differential conductance maps. Zero bias anomaly (ZBA) with conductance reaching $(1/2)e^2/h$ is only observed for spin-up channel. Another signature of Majorana-dot coupling is the observed splitting of Coulomb lines. Apart from typical Coulomb blockade diamonds also additional lines with nonlinear bias and gate voltage dependencies appear. The former lines are dominated by spin down electrons and the latter
by tunneling of spin up carriers. Whereas spin up contribution is directly modified by Majorana-dot coupling, spin down is influenced only indirectly by the change of occupation. The splitting between ordinary Coulomb blockade point and excited lines at two degeneracies points \( (E_d = 0, E_a = -U) \) is of order of \( t \). Interesting observation is the occurrence of spin dependent negative differential conductance \( dI/dV < 0 \) (SNDC) occurring for both spin orientations (blue regions at Figs. 1c, d). SNDC occurs on Coulomb blockade straight lines for spin up and on the curved Coulomb lines for spin down. To visualize this effect more clearly we present also spin polarization maps, where in the regions of SNDC, according to the definition of polarization of conductance, \( PC > +1 \) occurs for spin up NDC (red regions on Figs. 1d, e) and \( PC < -1 \) for spin down (blue regions). SNDC increases with weakening of the coupling to the normal electrodes.

![Fig. 3](image_url)

Fig. 3. a) Comparison of conductance in the Kondo (K) and Coulomb blockade regimes (C) for different values of coupling of the dot with TS wire \( t_1 \) \((t_2=0)\). b) Conductances for the SU(2) Kondo (dotted black line, \( t_1 = 0 \)) for SU(2) Majorana-Kondo effect (solid green, \( \delta = (3/2) \)) and for intermediate case (black line). The solid blue and dashed red lines represent the polarization of conductance for \( t_1 = 10^{-4} \) and \( t_1 = 0.01 \) \((U = 3, \Gamma = 0.05)\).

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### References


