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On the Current Flow in Superconductors: Universal Trends and Holographic Analysis

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The superconducting state can be destroyed by the increase of temperature, magnetic field or current flow beyond their critical values. The critical current I_c is of special interest as most of the practical applications of superconductors crucially depend on its limiting value. Recent analysis of experimental data in many families of type I and type II superconductors have discovered an interesting universal relation between critical current density j_c, the critical magnetic field H_c and the penetration depth λ. For type II superconductors the role of the thermodynamic critical field H_c is played by the lower critical field H_{c1} and ratio between the relevant dimension of the system d with respect to the penetration depth matters. Thus the effective dimensionality of the system is important and rules the system behaviour. It turns out that the holographic analogy provides an interesting justification of the above findings. We have calculated the temperature dependence of the critical current in the strongly coupled holographic superconductors with the current flow. It has been found that, independently of the symmetry of the order parameter, the critical current depends on temperature in 2d systems as $I_c \propto (T_c - T)^{3/2}$ and agrees with that observed in thin films ($d < \lambda$). Similar calculations for 3d systems ($d > \lambda$) reveal linear T-dependence $I_c \propto (T_c - T)$.

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1. Introduction

From the basic science and practical applications points of view the knowledge of the phase diagram for the substance is of critical importance. The superconducting state of the materials can be destroyed by the increase of the temperature T beyond its critical value known as the critical temperature T_c. Likewise increase of the magnetic field or the critical current above their respective critical values destroy the state [1]. The latter parameters *i.e.* the critical current and the critical field have always been thought to be related to each other. Imagine the superconductor having a shape of long cylinder of diameter a. The current I flowing along its axis produces magnetic field at distance r from the center of the cylindrical wire

$$H(r) = \frac{I}{2\pi r}. \quad (1)$$

Silsbee proposed [2] (this is known as the Silsbee hypothesis[†]) that the critical current I_c is that one, which produces the critical value of the magnetic field H_c on the surface of the wire. Thus $I_c = 2\pi d H_c$ and depends on the size of the wire. This shows that the critical current and the critical current density $j_c = I_c/(\pi d^2) = 2H_c/d$ are not intrinsic properties of the superconductor but depend on the diameter of the wire.

The desire to increase the critical currents has resulted in the huge number of experimental and theoretical works. There exist a number of proposals of understanding the multitude of data, but still we are quite far from consensus. The theoretical ideas make use of London and Ginzburg - Landau theories of superconductors in contact with normal systems or simply superconductors with surfaces. Both theories predict that the magnetic field is not expelled completely near the surface and exponentially vanishes close to the surface $H(r) = H(0)\,\mathrm{e}^{-r/\lambda}$ with characteristic scale given by the penetration depth λ.

In this paper we first review the reaction of the superconductor to the external magnetic field, discuss the factors influencing the critical current, present recent arguments in favour of the universal behaviour of thin films. The important property is the temperature dependence of the critical currents close to T_c. This seem to be different for thin films with thickness d smaller than the penetration depth λ and bulk materials with all dimensions larger than λ. Studying the current currying holographic superconductors we use gauge–gravity duality to obtain the temperature dependence of clean two or three dimensional systems in the limit of strong coupling. Surprisingly as it might be the calculations using the gravity theory indicate that that the critical current depends on temperature in 2d systems as $I_c \propto (T_c - T)^{3/2}$ and agrees with that observed in thin films ($d < \lambda$). Similar calculations for 3d systems ($d > \lambda$) reveal linear T-dependence $I_c \propto (T_c - T)^1$. Such behaviour has been first measured by Onnes [2] and sometimes is referred to as Onnes relation. These findings seem to agree with recent analysis [3, 4] of transport measurements of the critical currents.

In the next Section we review the issue of the critical current in superconductors and various arguments of its rationalisation. Section 3 presents basic facts of holographic approach [5] to study the strongly interacting

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[†]The very good introduction, historical account of the early attempts to understand the critical currents in superconductors as well as the relevant original literature can be found in [2]. Thus in this paper we shall not cite the old, difficult to access sources.

superconductors and present the results. We end with summary and conclusions.

2. Critical currents in superconductors

There exist two kinds of superconductors which differ by their reaction to the external magnetic field H. Type I superconductors become normal metals if the $H > H_c$, where H_c is the critical magnetic field. Their condensation energy is uniquely defined and equals

$$\Delta G_{ns}(T) = -\frac{1}{2}\mu_0 H_c^2(T). \quad (2)$$

Contrary to that, the application of the external magnetic field to type II superconductors of value larger than so called lower critical field H_{c1} results in the penetration of magnetic field in the form of vortices of radius ξ (known as coherence length) into the material. Except of the core of the vortex which is normal, the rest of the material remains in a superconducting state, up to much higher magnetic field H_{c2}.

Depairing. One way to think about the value of the critical current is to realise that the flow of charge increases the energy of the system. If the increase of energy δE_I exceeds condensation energy (2) the system becomes normal. Denoting the condensed fraction of charges by n_s, the mass m and charge e one calculates δE_I as

$$\delta E_I = n_s \frac{m v_s^2}{2} = \frac{m}{\mu_0 n_s \mathrm{e}^2}(e n_s \boldsymbol{v_s})^2 = \frac{1}{2}\lambda_L \boldsymbol{j}^2, \quad (3)$$

where $\lambda_L = \lambda$ denotes London penetration depth. Comparison with the condensation energy leads to the estimation of the critical current related to the critical field and penetration depth and not geometry of the system

$$\boldsymbol{j}_c^{(d)} = \frac{H_c}{\lambda}. \quad (4)$$

This is known as the depairing current; the maximal current which can flow in a superconductor. The estimated magnitude of $j_c^{(d)} \approx 10^{12}\frac{A}{m^2}$ exceeds the values observed by at least one order of magnitude.

As the validity of equation (1) does not depend on the current distribution it may be applied to *e.g* hollow cylinder. In type I superconductors the Silsbee hypothesis is valid due to the fact that the external magnetic field of magnitude H_c penetrates the whole sample, which enters normal state. This means 1:1 correspondence between the critical field and critical current. However, the actual value of the current or current density depends on the geometry of the system.

Pinning. For $H > H_{c1}$ the vortices enter the type II superconductor. Their cores are normal and in the presence of the current they start to move due to the Lorentz force. This causes the dissipation of energy and thus finite resistivity. In this case Eq. (4) is replaced by

$$\boldsymbol{j}_c^{(d)} = \frac{H_{c1}}{\lambda}. \quad (5)$$

The estimated current densities typically are smaller than measured for such superconductors. One way to understand such behaviour is to argue that in type II superconductors the vortices which enter material at field H_{c1} are fixed up to the higher fields. The pinning of vortices by grain boundaries in poly-crystals and the extended defects with dimensions of the order of the size of the vortex core *i.e* the coherence length ξ is the prevailing explanation of the critical currents in type II superconductors. If the vortices are pinned in the system it means that one needs an extra force to remove them. Thus one expects the magnetic hysteresis. Initial increase of the magnetic field induces the magnetisation $M = -H$ as the system expels it completely. Increasing H beyond H_{c1} introduces some vortices and the magnetisation diminishes until $H = H_{c2}$, when the vortices start to overlap and the whole system becomes normal. If the vortices are pinned than the decrease of magnetic field produces different branch of the magnetisation M and the hysteresis loop forms. It turns out that the width ΔM of the loop at a given field H, according to the Bean model [6] measures the critical current density $j_c = \frac{2\Delta M}{b(1-3a/b)}$, where b is the width of the superconducting film (see Fig. 1). This shows that the magnetsiation measurements ca be used to extract the critical current density.

Self-field. Different approach to understand critical currents have been proposed recently by Talantsev and Tallon [3, 4]. These Authors argue that the critical currents are achieved, when for type I superconductors the magnetic self-field ($H^{(sf)}$ in their notation) at the samples' surface reaches H_c, while in type II materials H_{c1}. The self-field means not the external magnetic field, but the field produced at the surface by the current flowing along the sample.

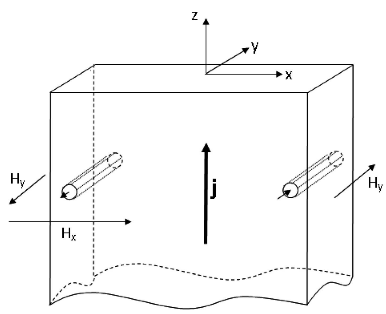

Fig. 1. The type II superconductor in the form of rectangular slab extending along z-axis with cross-section $2a \times 2b$ ($a \gg b$) with the current of density $\boldsymbol{j}$ flowing along it. The current flow produces magnetic field $\boldsymbol{H} = (H_x, H_y, 0)$. The flux lines in y-direction enter the system if $H_y > H_{c1}$. For the system isotropic in (x, y) plane similar vortices parallel to the (x, z) plane would also enter the system.

One possible geometry is illustrated in Fig. 1. The rectangular sample with cross-section $2a \times 2b$ with $a \gg b$ extends along z-axis. The current density $\boldsymbol{j}$ flows in the z-direction. It produces magnetic field $\boldsymbol{H} = (H_x, H_y, 0)$. The flux lines in y-direction enter the system if $H_y > H_{c1}$. For the system isotropic in (x, y) plane similar vortices parallel to the (x, z) plane would also enter the system complicating the behaviour close to the sample edges.

The current density $\boldsymbol{j}$ is obtained by dividing the total current along the sample by its crossection: $4ab$ for the shown geometry.

Using London or Landau-Ginzburg theory the authors [3, 4] derived the expressions for current densities of thin films with $b \leqslant \lambda$ in the self-field approach

$$j_c^{(\mathrm{I};sf)} = \frac{H_c}{\lambda} = \frac{\phi_0 \kappa(T)}{2\sqrt{2}\mu_0 \lambda^3(T)} \tag{6}$$

$$j_c^{(\mathrm{II};sf)} = \frac{H_{c1}}{\lambda} = \frac{\phi_0}{4\pi\mu_0 \lambda^3(T)}(\ln \kappa(T) + 0.5), \tag{7}$$

where $\phi_0 = \frac{\hbar}{2e}$ is the flux quantum. μ_0 is the permeability of free space and $\kappa = \frac{\lambda}{\xi}$ is the Ginzburg-Landau parameter. For thick films with dimension $b > \lambda$ the geometric factor λ/b has to be introduced as calculated by London [7], who found the correction to the critical field of the film $H_{c;f}$ of thickness d with respect to the bulk values H_c as

$$H_{c;f} = H_c \left[1 - \frac{\lambda}{d} \tanh \frac{d}{\lambda}\right]. \tag{8}$$

Let us note that this correction indicates large increase of the film critical field $H_{c;f} = \sqrt{3}\frac{\lambda}{d}H_c$ very early verified experimentally [8]. Thin films accordingly can curry higher currents than bulk superconductors.

The same correction ($\approx \lambda/b$) has to be introduced to Eqs. (6) and (7) for thick samples. As a result and in view of the weak changes of Ginzburg - Landau parameter κ with temperature one finds [3, 4] that the critical current of thin type II superconducting film is characterised by

$$j_c \propto \lambda^{-3}(T), \tag{9}$$

while bulk materials display quadratic

$$j_c \propto \lambda^{-2}(T) \tag{10}$$

dependence on the penetration depth.

From the last equations and the Ginzburg-Landau dependence $\lambda(T) \propto (1 - T/T_c)$ valid for temperatures close to T_c one expects $j_c^{\mathrm{II}} \propto (1 - T/T_c)^{3/2}$ for thin films (or essentially 2 dimensional systems) and $j_c^{\mathrm{II}} \propto (1 - T/T_c)$ for thick films (3d materials). In the next section we shall obtain the same temperature dependence of the critical currents of holographic superconductors on temperature in two-dimensional and three-dimensional systems.

3. Holographic approach and the results

The idea behind using gravity theory to study strongly coupled matter stems from the Maldacena conjecture stating that the partition function of the gravity theory of $(d + 1)$-dimensional system is equivalent to the partition function of the field theory on the boundary *i.e.* of the d-dimensional system. Maldacena have shown this equivalence for two special fields, and the hypothesis has latter been extended to other models and is being tested extensively *via* applications to study various condensed matter systems. Due to the fact that the coupling constant λ_g on the gravity side corresponds to the coupling $\lambda_{FT} = 1/\lambda_g$ in the boundary field theory the duality provides access to strong coupling regime.

This weak-strong coupling duality provides the most attractive feature of AdS/CFT or gauge/gravity duality also known as holographic approach. In short one starts with appropriate fields representing the condensed matter system coupled to the gravity fields, analyses their equation of motion in the $(d+1)$-dimensional gravity theory and extracts the asymptotic *i.e.* $r \to \infty$ behaviour of the fields and extracts appropriate characteristics of the system at the boundary.

The appropriate action in d-dimensional spacetime is a sum of gravity and matter parts $S = S_g + S_m$. The minimal gravitational action in d-dimensional space-time reads $S_g = \int \sqrt{-g}\,\mathrm{d}^d x \frac{1}{2\kappa^2}(R - 2\Lambda)$, where $\kappa^2 = 8\pi G_d$ is an d-dimensional gravitational constant. The cosmological constant is given by $\Lambda = -\frac{(d-1)(d-2)}{2L^2}$, where L is the radius of the AdS space-time. The matter action contains the Abelian-Higgs sector and is given by $S_m = \int \sqrt{-g}\,\mathrm{d}^d x(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\psi))$. The scalar field potential representing the superconductor satisfies $V(\psi) = m^2|\psi|^2 + \frac{\lambda_\psi}{4}|\psi|^4$. m is known as a mass of the field ψ. $F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$ stands for the ordinary Maxwell field strength tensor. Moreover λ_ψ, q represent the coupling constant and charge related to the scalar field ψ, respectively. Analysis of the three-dimensional superconducting systems living in a $3+1$ dimensional space-time, requires $d = 5$ dimensional gravity theory.

s-wave superconductor. To describe 3-dimensional superconductor at finite temperature one assumes existence of the black hole which corresponds to the classic gravity configuration characterised by temperature T. The line element reads $\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \frac{r^2}{L^2}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$, where $f(r) = r^2/L^2 - r_+^4/r^2L^2$. In what follows, without loss of generality we set $L = 1$. The Hawking temperature for the black hole is equal to $T_{BH} = r_+/\pi$. We assume that the non-zero components of the Maxwell fields are given by $A_t(r) = \phi(r),\ A_y(r)$.

p-wave superconductor. There exist at least three possible ways to construct holographic superconductor with p-wave symmetry of its order parameter [9]. Here we restrict our attention to one of them, namely the $SU(2)$ model. Again to describe 3-dimensional holographic superconductor at non-zero temperature one takes into account five-dimensional AdS black hole background and the following gauge fields components $A = \phi(r)\tau^3\,\mathrm{d}t + A_y(r)\tau^3\,\mathrm{d}y + w(r)\,\tau^1\,\mathrm{d}x$, where $w^{(0)}$ is the x-component of condensing field (its finite value on the boundary defines holographic p-wave superconductor).

For both symmetries of 3d supercondutors one obtains the same linear dependence of J_y on temperature T close to T_c. The analogous calculations for s-wave and p-wave two-dimensional superconductors [10–12] again reveal the symmetry independent behaviour $J_y \propto (\mathrm{T} - T_c)^{3/2}$.

4. Discussion and conclusions

We have been mainly interested in the temperatures close to the superconducting transition temperature. The holographic analogy addressing strong coupling systems show that the temperature dependence of the critical current does not depend on the symmetry of the superconducting system. Both for s-wave and *p*-wave holographic superconductors one finds that $I_c \propto (1 - T/T_c)^{3/2}$ for two dimensional materials and $I_c \propto (1 - T/T_c)$ for 3 dimensional superconductors. These findings agree with calculations based on Ginzburg-Landau theory for thin films and Onnes relation, respectively. The recent analysis of many experimental findings seem to indicate that the critical current is an intrinsic property defined by self-field, *i.e.* the field produced by the current flow, equal to the critical field H_c for type I superconductor or lower critical field H_{c1} for type II material.

Presented here theoretical analyses basing on holographic analogy is limited to temperatures close to T_c and does not show any dependence on the symmetry (s-wave or *p*-wave) of the order parameter contrary to phenomenological approach of Talantsev and Tallon [3] which have found the behaviour of s-wave superconductors slightly different from *d*-wave. On the other hand the *p*-wave symmetry superconductors have not been analysed [4] and the results of such measurements are not known to the authors. Thus as a check of the present ideas we propose the experimental transport measurements of the temperature dependence of the critical currents [13] in bulk *p*-wave superconductor Sr_2RuO_4. We also mention here that the holographic analyses of *d*-wave superconductors is far from trivial and we are not aware of the calculations similar to those discussed above.

It has to be added that the Talantsev and Tallon analysis [3, 4] of the existing transport (not magnetisation) data shows that the relations (9) and (10) have been shown to be valid over the whole temperature range. Our holographic analysis is limited to temperatures close to T_c, as we have studied it using analytic tools. It would be interesting to extend this analysis to wider range of temperatures and check if the "self-field" hypthesis is valid for strongly interacting systems there.

The studies of superconductors with the current flow may be also interesting from basic physics point of view as it has been shown recently [14].

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