

# Elastic Anisotropic and Thermodynamic Properties of Two BC<sub>7</sub> Phases

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The structural, elastic, anisotropic, and thermodynamic properties of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> have been studied in this paper utilizing first-principles calculations. In comparison with the elastic properties of  $Pmm2$ -BC<sub>7</sub>,  $P3m1$ -BC<sub>7</sub> exhibits slightly higher values in bulk modulus and  $B/G$ , with similar values in shear modulus, the Young modulus, and the Poisson ratio. The calculated Pugh modulus ratio ( $B/G$ ) and the Poisson ratio demonstrates  $P3m1$ -BC<sub>7</sub> from brittle to ductile at 93.60 and 93.73 GPa, respectively. Calculations of shear anisotropic factor, universal elastic anisotropy index, shear modulus, the Young modulus, and the Poisson ratio for BC<sub>7</sub> then demonstrate that  $Pmm2$ -BC<sub>7</sub> exhibits a larger elastic anisotropy than  $P3m1$ -BC<sub>7</sub>. Quasi-harmonic Debye model is finally applied to investigate the Debye temperature, the coefficient of thermal expansion, heat capacity and Grüneisen parameter of  $Pmm2$ -BC<sub>7</sub> and  $P3m1$ -BC<sub>7</sub>.

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## 1. Introduction

Potentials for industrial applications of superhard materials are numerous [1], which leads to an exceptional level of scientific interest. Experimental and theoretical efforts to pursue progressive new superhard materials have motivated studies of boron rich systems [2–10]. Recently, there has been considerable interest on B<sub>x</sub>C<sub>y</sub> compounds, mainly due to the discovery of its characteristic superconductivity [11, 12]. Extreme Vickers hardness (71 GPa) was claimed in a successfully synthesized diamond-like BC<sub>5</sub> phase [13]. Xu et al. [14] have predicted two novel low-energy structures of BC<sub>7</sub>: graphite-like  $Amm2$  structure and diamond-like  $P4m2$  structures. Additional findings led to the discovery that diamond-like BC<sub>7</sub> was superconducting with a critical temperature of  $\approx 11.4$  K and theoretical Vickers hardness was 78.0 GPa, indicating a superhard material. Phase transition pressure from graphite-like to diamond-like was determined to be at 2.2 GPa. Zhang et al. [15] have predicted cubic BC<sub>3</sub> by performing a swarm structure search. Calculated hardness and ideal strength results confirm that cubic diamond-like BC<sub>3</sub> is a superhard material (62 GPa) with deformation modes under tensile and shear strains displaying intriguing bond elongation and sequential bond-breaking processes leading to remarkable extended ductility and elastic response. Utilizing first-principles calculations, Zhao and Wang [16] investigated mechanical properties and electronic structure of the recently synthesized diamond-like BC<sub>5</sub> systematically. Liu et al. [17] discovered a potential superhard material (61.9 GPa) and superconductive materials with superconducting critical temperatures reaching  $16.6 \approx 23.4$  K for  $Pmma$ -*a* (space group) phase of diamond-like BC<sub>3</sub>.

Utilizing particle swarm optimization (PSO) methodology for crystal structure prediction, Liu et al. [18] discovered four potential superhard structures of  $P4m2$ ,  $P3m1$ ,  $Pmm2$  and  $R3m$ , with energetically superior traits to the previously proposed  $P43m$  structure. Previous research has not systematically investigated the new  $P3m1$  and  $Pmm2$  phase of BC<sub>7</sub>, including mechanical, elastic anisotropic and thermodynamic properties, thus detailed first-principles calculations of each for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> based on density functional theory (DFT) under high pressure are studied.

## 2. Theoretical method

Structural optimization and property predictions of the BC<sub>7</sub> polymorphs were performed in these calculations utilizing DFT [19, 20] with the generalized gradient approximation (GGA) parameterized by Perdew, Burke and Ernzerhof (PBE) [21] and the local density approximation (LDA) [22, 23] as implemented in the Cambridge Serial Total Energy Package (CASTEP) code [24]. Ultra-soft pseudopotential was applied with an energy cutoff of 700, 700, 700, 580, and 620 eV for  $P3m1$ ,  $Pmm2$ ,  $P6m2$ ,  $P43m$  and  $P4m2$ , respectively, and Monkhorst–Pack  $k$ -point meshes [25] within 0.025, for the Brillouin zone sampling. Interaction between ions and electrons were described utilizing the Vanderbilt ultrasoft pseudopotentials for B and C with  $2s^22p^1$  and  $2s^22p^2$  as valence electrons, respectively. Self-consistent convergence of the total energy is  $5 \times 10^{-6}$  eV/atom, maximum force on the atom is 0.01 eV/Å, maximum ionic displacement is within  $5 \times 10^{-4}$  Å, and the maximum stress is within 0.02 GPa.

## 3. Results and discussion

### 3.1. Structural properties

$P3m1$ -BC<sub>7</sub> retains a trigonal symmetry related to the  $P3m1$  space group, and  $Pmm2$ -BC<sub>7</sub> retains an orthorhombic symmetry related to the  $Pmm2$  space group, respectively. Calculated lattice parameters within GGA

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and LDA functionals of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>, combined with previous results are displayed in Table I. Calculated lattice parameters for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are a reasonable comparison [18]. Figure 1 displays the crystal structures of  $P3m1$ - and  $Pmm2$ -BC<sub>7</sub>. The red and black spheres represent B and C atoms, respectively. Pressure dependence of the equilibrium lattice parameters  $a/a_0$ ,  $b/b_0$ ,  $c/c_0$  and  $V/V_0$  (where  $a_0$ ,  $b_0$ ,  $c_0$  and  $V_0$  are the equilibrium conventional lattice constants at zero pressure and temperature, respectively) are illustrated in Fig. 2. Compressibility along  $a$ -axis for  $P3m1$ -BC<sub>7</sub> is remarkably easier than along  $c$ -axis; however, for  $Pmm2$ -BC<sub>7</sub>, compressibility along  $b$ -axis is easier than along  $a$ -axis and  $c$ -axis. The volume compressions  $V/V_0$  as a function of pressure are also plotted in Fig. 2b for comparison of incompressibility for  $P3m1$ -BC<sub>7</sub>,  $Pmm2$ -BC<sub>7</sub>, c-BN, and diamond under pressure. Incompressibility of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are demonstrated as slightly less than diamond and slightly larger than c-BN. Table I also presents the calculated equation of state (EOS) for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> under pressures from 0 to 100 GPa. The bulk modulus  $B$  and its pressure derivative  $B'_0$  are 388.21 (355.82) and 3.70 (4.14) GPa for  $P3m1$ -BC<sub>7</sub> ( $Pmm2$ -BC<sub>7</sub>) respectively fitted to a third-order Birch–Murnaghan EOS equation [26].

TABLE I

Lattice constants  $a$ ,  $b$ ,  $c$  [Å], and cell volume per formula unit  $V_0$  [Å<sup>3</sup>]. EOS fitted bulk modulus  $B_0$  [GPa] and its derivative  $B'_0$  for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are also shown.

Space group	$a$	$b$	$c$	$V$	$B_0$	$B'_0$	
P3m1	GGA <sup>a</sup>	2.5314		8.4996	47.1667	388.21	3.70
	LDA <sup>a</sup>	2.5221		8.3861	46.2001		
Pmm2	GGA <sup>b</sup>	2.5356		8.5188	47.4305		
	GGA <sup>a</sup>	2.5455	5.0925	3.7026	47.9957	355.82	4.14
Pmm2	LDA <sup>a</sup>	2.5148	5.0301	3.6630	46.3375		
	GGA <sup>b</sup>	2.5132	5.1240	3.6870	47.4798		

<sup>a</sup>This work, <sup>b</sup>Ref [18]

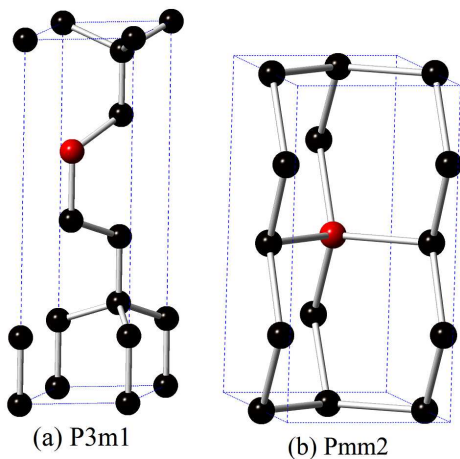


Fig. 1. The crystal structure of  $P3m1$ -BC<sub>7</sub> (a) and  $Pmm2$ -BC<sub>7</sub> (b) (black spheres denote carbon atoms, and blue spheres denote boron atoms).

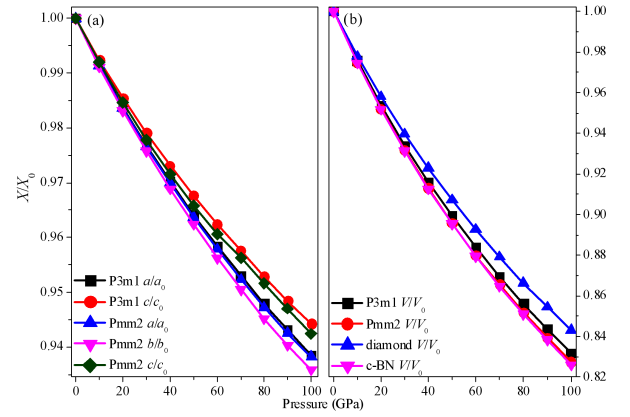


Fig. 2. The lattice constants  $a/a_0$ ,  $b/b_0$ ,  $c/c_0$  compression as functions of pressure for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> (a), and primitive cell volume  $V/V_0$  for  $P3m1$ -BC<sub>7</sub>,  $Pmm2$ -BC<sub>7</sub>, c-BN and diamond (b).

### 3.2. Elastic properties and anisotropy

Elastic properties provide information related to stability and stiffness of materials. Trigonal crystals feature six independent calculated elastic constants ( $C_{ij}$ ) for  $P3m1$ -BC<sub>7</sub>; however, orthorhombic crystals feature eight independent calculated elastic constants for  $P3m1$ -BC<sub>7</sub>. Calculated results for elastic constants of BC<sub>7</sub> and other structures of BC<sub>7</sub> are presented in Table II with the elastic constants of diamond are also listed for comparison. Elastic modulus of  $P3m1$ -BC<sub>7</sub>,  $Pmm2$ -BC<sub>7</sub> and diamond were calculated together with previous results for this study as also displayed in Table II. The criteria for mechanical stability of trigonal symmetry group are derived by [27]:

$$C_{ij} > 0, \quad i = j = 1 \div 6, \quad (1)$$

$$C_{11} - C_{12} > 0, \quad (2)$$

$$C_{11}C_{33}C_{44} - C_{14}^2C_{33} - C_{13}^2C_{44} > 0, \quad (3)$$

$$C_{11}C_{33} + C_{12}C_{33} - 2C_{13}^2 > 0, \quad (4)$$

$$C_{44}(C_{11} - C_{12}) - 2C_{14}^2 > 0. \quad (5)$$

Criteria for mechanical stability of orthorhombic symmetry group are derived by [28, 29]:

$$C_{ii} > 0, \quad i = 1 \div 6, \quad (6)$$

$$C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23}) > 0, \quad (7)$$

$$C_{11} + C_{22} - 2C_{12} > 0, \quad (8)$$

$$C_{11} + C_{33} - 2C_{12} > 0, \quad (9)$$

$$C_{22} + C_{33} - 2C_{23} > 0. \quad (10)$$

Elastic constants meet the requirements from Eqs. (1)–(5) and Eqs. (6)–(10), respectively, which indicates that  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are mechanically stable. Phonon spectra are calculated at 0 GPa (Ref. [18]) to ensure stability of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>.  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> exhibit stability at 0 GPa, as there is no imaginary frequency. The elastic constants and elastic modulus versus pressure for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are displayed in Fig. 3. Among this bulk modulus,  $P3m1$ -BC<sub>7</sub> retains

the largest value. The Young modulus  $E$  and the Poisson ratio  $\nu$  are expressed as:  $E = 9BG/(3B + G)$ ,  $\nu = (3B - 2G)/[2(3B + G)]$ . All elastic constants  $C_{ij}$  (except for  $C_{14}$  and  $C_{44}$ ) for  $P3m1$ -BC<sub>7</sub> and elastic modulus increase with different rates under increasing pressure.  $C_{33}$  of  $P3m1$ -BC<sub>7</sub> increases faster than others, and  $C_{66}$  exhibits the slowest growth; however, for  $Pmm2$ -BC<sub>7</sub>,

all elastic constants  $C_{ij}$  and elastic modulus increase under increasing pressure while the difference is  $C_{11}$  of  $Pmm2$ -BC<sub>7</sub> increases faster than others, but  $C_{66}$  still exhibits the slowest growth. Observations of Table II reveal that the Young modulus of  $P3m1$ -BC<sub>7</sub> is smaller than  $Pmm2$ -BC<sub>7</sub>.

TABLE II

Calculated elastic constant  $C_{ij}$ , bulk modulus  $B$  [GPa], shear modulus  $G$  [GPa], the Young modulus  $E$  [GPa] and the Poisson ratio  $\nu$  of BC<sub>7</sub> and diamond.

Space group		$C_{11}$	$C_{12}$	$C_{13}$	$C_{14}$	$C_{22}$	$C_{23}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$B$	$G$	$E$	$\nu$
P3m1	GGA <sup>a</sup>	828	239	67	-41	828	67	1101	382	382	295	389	361	827	0.146
	LDA <sup>a</sup>	893	183	53	-52	893	53	1010	293	293	355	378	347	797	0.149
	GGA <sup>b</sup>	830	215	53				1099	377	293	355	378	368	834	0.133
Pmm2	GGA <sup>a</sup>	936	10	180		925	152	836	470	460	257	375	383	857	0.119
	LDA <sup>a</sup>	906	-3	162		897	129	803	461	454	258	353	379	937	0.105
	GGA <sup>b</sup>	948		168		975	159	812	493	489	330	377	415	911	0.097
$P$ -43m	GGA <sup>a</sup>	312	419						481			384	52		
$P$ -4m2	GGA <sup>a</sup>	905	88	136				908	424		293	382	382	860	0.125
	GGA <sup>c</sup>	830	166	154				884	500		463				
diamond	GGA <sup>a</sup>	1055	122						566			429	520	1111	0.068
	LDA <sup>a</sup>	1100	138						596			459	545	1171	0.075
	exp. <sup>d</sup>	1076	125						577			422			

<sup>a</sup>This work, <sup>b</sup>Ref [18], <sup>c</sup>Ref [14], <sup>d</sup>Ref [44]

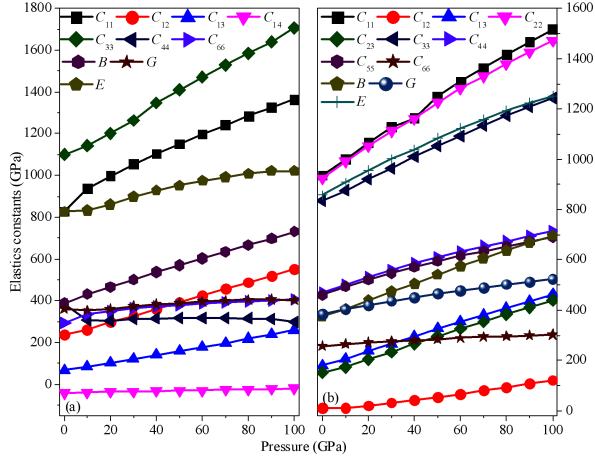


Fig. 3. Elastic constants and elastic modulus of  $P3m1$ -BC<sub>7</sub> (a) and  $Pmm2$ -BC<sub>7</sub> (b) as a function of pressure.

The Poisson ratio  $\nu$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are as follows:  $\nu_{P3m1} > \nu_{Pmm2}$ . The Poisson ratio  $\nu$  refers to a ductile compound, usually exhibiting a large  $\nu$  ( $\nu > 0.26$ ) [30], thus  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are brittle compounds. The  $B/G$  is also consistent with  $\nu$ , referring to a ductile compound usually exhibiting a large  $B/G$  ratio ( $B/G > 1.75$ ). Thus if  $B/G > 1.75$ , the material behaves in a ductile manner [31]. Otherwise, the material behaves in a brittle manner. The  $B/G$  ratio

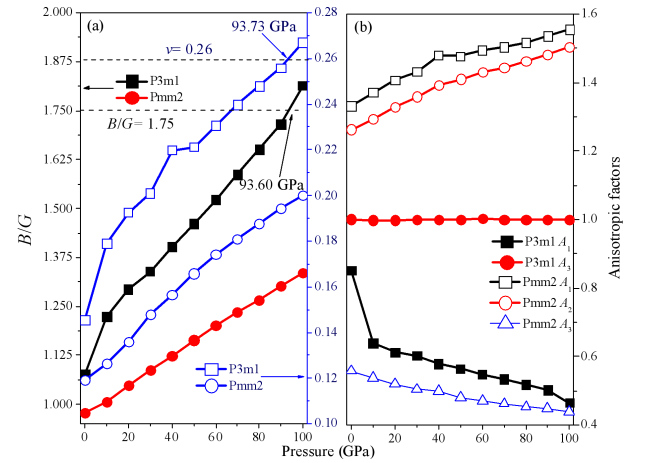


Fig. 4. (a) the Poisson ratio and  $B/G$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> as a function of pressure, (b)  $A_1$ ,  $A_2$ ,  $A_3$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> as a function of pressure.

of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> at  $P = 0$  GPa are 1.08 and 0.979, respectively, while values of  $B/G$  and  $\nu$  for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are less than 1.75 and 0.26 at 0 GPa, respectively. This indicates a brittle state. The  $B/G$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> as a function of pressure are presented in Fig. 4a.  $B/G$  increases under increasing pressure with  $B/G = 1.75$  at 93.60 GPa

for  $P3m1$ -BC<sub>7</sub>. The  $\nu$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> as a function of pressure are also depicted in Fig. 4a with  $\nu$  increases observed with increasing pressure,  $B/G = 1.75$  at 93.60 GPa and  $\nu = 0.26$  at 93.73 GPa for  $P3m1$ -BC<sub>7</sub>.  $P3m1$ -BC<sub>7</sub> is found to turn brittle to ductile in this pressure range, while  $Pmm2$ -BC<sub>7</sub> remains brittle.

Anisotropy of elasticity exerts significant implications in engineering science and crystal physics, thus anisotropy of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> is investigated. Shear anisotropic factors measure a degree of anisotropy in the bonding between atoms in different planes. The shear anisotropic factor for {100} shear planes between  $\langle 011 \rangle$  and  $\langle 010 \rangle$  directions is [32, 33]:

$$A_1 = \frac{4C_{44}}{C_{11} + C_{33} - 2C_{13}}, \quad (11)$$

for the {010} shear planes between  $\langle 101 \rangle$  and  $\langle 001 \rangle$  directions [32, 33];

$$A_2 = \frac{4C_{55}}{C_{22} + C_{33} - 2C_{23}}, \quad (12)$$

for the {001} shear planes between  $\langle 110 \rangle$  and  $\langle 010 \rangle$  directions [32, 33];

$$A_3 = \frac{4C_{66}}{C_{11} + C_{22} - 2C_{12}}. \quad (13)$$

The factors  $A_1$ ,  $A_2$ , and  $A_3$  must be one for isotropic crystal, while any value smaller or greater than one is a measure of the elastic anisotropy possessed by the crystal. The anisotropy factors of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> as a function of pressure are displayed in Fig. 4b.  $A_1$  of  $P3m1$ -BC<sub>7</sub> obviously decreases as  $C_{44}$  decreases 24.84% from 0 GPa to 10 GPa and  $A_1$  of  $P3m1$ -BC<sub>7</sub> decreases at  $P = 10$  GPa while  $A_3$  of  $P3m1$ -BC<sub>7</sub> remains nearly constant. Elastic anisotropic of  $Pmm2$ -BC<sub>7</sub> is greater than  $P3m1$ -BC<sub>7</sub> (Fig. 4b). Generally, the universal elastic anisotropy index ( $A^U$ ) is derived from the  $B$  and  $G$  of the Voigt and Reuss bounds as the formula  $A^U = 5(G_V/G_R) + (B_V/B_R) - 6$ . The  $A^U$  of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> were calculated as a function with pressure.  $A^U(P3m1) = 0.269$ ,  $A^U(Pmm2) = 0.300$  at 0 GPa, and  $A^U(P3m1) = 0.554$ ,  $A^U(Pmm2) = 0.678$  at 100 GPa, respectively. Moderate deviations of the index from standard value ( $A^U = 0$ ) imply considerable elastic anisotropy of  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> while elastic properties comparisons indicate that  $Pmm2$ -BC<sub>7</sub> exhibits slightly higher values in  $A^U$ .

Utilizing ELAM [34], the calculated Poisson ratio, shear modulus, and the Young modulus along different directions, including projections in different planes, may be obtained in Fig. 5 and Fig. 6. Figure 5a–c displays the 2D representation of the Poisson ratio in the  $xy$ ,  $xz$ , and  $yz$  planes for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>, respectively. Relatively large anisotropy in the Poisson ratio is found as indicated in the figures. Shear modulus for all directions of shear strain were also calculated to quantify the anisotropy. 2D representation of shear modulus in the  $xy$ ,  $xz$ , and  $yz$  planes for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are displayed in Fig. 5d–f, respectively. The calculated maximum result of shear modulus for  $P3m1$ -

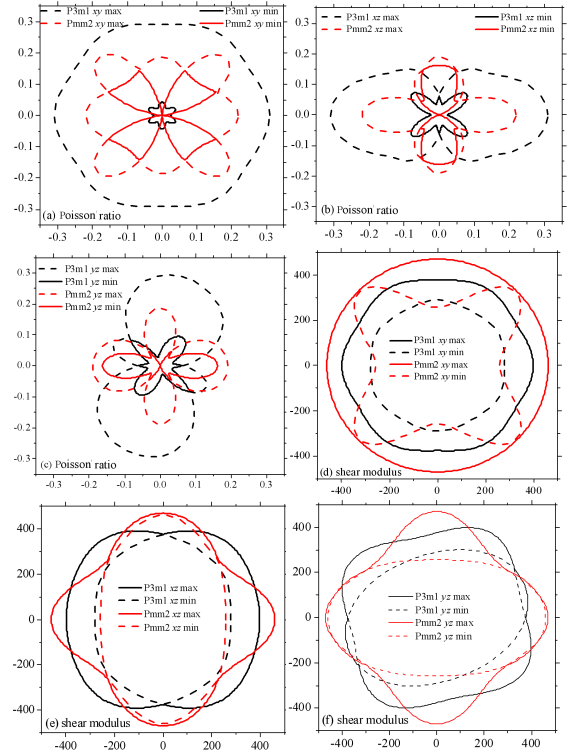


Fig. 5. 2D representation of the Poisson ratio and shear modulus in the  $xy$  plane (a,d),  $xz$  plane (b,e) and  $yz$  plane (c,f) for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>, respectively.

BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are 440 GPa and 470 GPa, respectively, while the calculated minimal result of shear modulus for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> is 279 GPa and 257 GPa, respectively. Ratios  $G_{\max}/G_{\min}(P3m1) = 1.58$  and  $G_{\max}/G_{\min}(Pmm2) = 1.83$  also indicate  $Pmm2$ -BC<sub>7</sub> exhibits a large elastic anisotropy. The directional dependence of the Young modulus for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub> are presented in Fig. 6a and c, respectively. Figure 6b and d demonstrates 2D representation of shear modulus in the  $xy$ ,  $xz$ , and  $yz$  planes for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>. The  $704 \text{ GPa} < E_{P3m1} < 1093 \text{ GPa}$ ,  $648 \text{ GPa} < E_{Pmm2} < 989 \text{ GPa}$ , and the average value of all directions is 876 GPa and 847 GPa for  $P3m1$ -BC<sub>7</sub> and  $Pmm2$ -BC<sub>7</sub>, respectively. The Young modulus 3D figure for the  $P3m1$ -BC<sub>7</sub> exhibits more anisotropic characteristics along the  $z$ -axis than that along the  $x$ -axis.

### 3.3. Hardness

The formula for the Knoop hardness [35] (in GPa) was utilized to calculate hardness  $H_v$ :

$$H_v = \frac{423.8}{V} n \sqrt[n]{\prod_{k=1}^n N_k X_k e^{-2.7f_k} - 3.4}, \quad (14)$$

where  $V$  is the volume of the unit cell and  $N_k$  is the number of bonds of the type  $k$  in the unit cell.  $X_k$  and  $f_k$  represent electron-holding energy of the bond  $k$  and the iconicity indicator, defined as in the original work of Li et al. [36]:

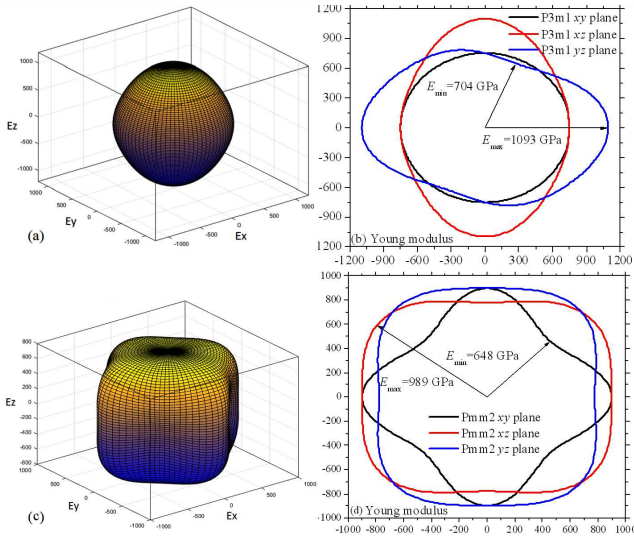


Fig. 6. The directional dependence of the Young modulus for  $P3m1$ - $BC_7$  (a) and  $Pmm2$ - $BC_7$  (c), 2D representation of the Young modulus for  $P3m1$ - $BC_7$  (b) and  $Pmm2$ - $BC_7$  (d).

$$X_k = \sqrt{\frac{\chi_i^k \chi_j^k}{CN_i^k CN_j^k}}, \quad (15)$$

$$f_k = \frac{|\chi_i^k - \chi_j^k|}{4\sqrt{\chi_i^k \chi_j^k}}, \quad (16)$$

$\chi_i^k, \chi_j^k$  denote the electronegativity of atoms  $i, j$  in bonds.  $CN_i^k, CN_j^k$  are the coordination numbers of atoms  $i$  and  $j$ . Bond valence ( $s$ ) is computed utilizing the classical Brown bond valence model [37]:

$$s_i^k = \frac{v_i \exp(-\Delta_k/0.37)}{\sum_{k'} \exp(-\Delta_{k'}/0.37)}. \quad (17)$$

Denoted by  $n$  are the number of different bond types in the unit cell, as labeled with the index  $k = [1 \div n]$ . The model [36] computes the electronegativity of each  $i$ -th atom as  $\chi = 0.481n_i/R_i$  where  $n_i$  and  $R_i$  represent the number of valence electrons and univalent covalent radius of the atom, respectively. The sum applies to all bonds  $k'$  where atom  $i$  participates, and  $\Delta$  represents deviation from reference covalent bond length. The definition satisfies the sum rule exactly:

$$\sum_{k'} s_i^{k'} = v_i. \quad (18)$$

Use of the multicolor graph theory is the most defining feature of the Lyakhov–Oganov approach, i.e. bond topology is utilized, in addition to bond strengths, for computing hardness. The weak bonds for molecular, chain and layered structures, defining crystal hardness, also maintain three dimensionality.

The output file contains the following information: length symbolizes bond modulus, Delta ( $\Delta$ ) is a deviation from bond length defined by the reference univalent covalent radius,  $N^k$  is number of type  $k$  bonds in the unit cell, and  $s_i$  and  $s_j$  represent bond valence.

TABLE III

Calculated hardness of  $BC_7$  and diamond.

	bond	length	$\Delta_k$	$N_k$	$s_i$	$s_j$	$X_k e_k^{-2.7f}$	$Hv$
$P3m1$	B-C	1.607	0.007	4	0.750	0.970	1.571	51.4
	C-C	1.473	-0.047	1	1.167	1.098	0.739	
	C-C	1.543	0.023	11	1.005	0.982	6.809	
$Pmm2$	B-C	1.611	0.011	4	0.750	0.996	1.589	59.8
	C-C	1.538	0.018	12	1.001	1.000	7.508	
$P-4m2$	B-C	1.648	0.048	4	0.956	0.750	1.524	56.8
	C-C	1.546	0.026	12	1.010	1.005	7.518	
$P-43m$	B-C	1.601	0.001	4	1.072	0.750	1.658	58.9
	C-C	1.556	0.036	12	0.996	0.980	7.329	
diamond	C-C	1.544	0.024	16	1.000	1.000	9.966	89.7, 91.2 <sup>a</sup> , 90 <sup>b</sup>

<sup>a</sup>Ref [35], <sup>b</sup>Exp., Ref [38]

Calculated hardness of  $P3m1$ - $BC_7$ ,  $Pmm2$ - $BC_7$ ,  $P-4m2$ - $BC_7$ ,  $P-43m$ - $BC_7$  and diamond via the approach is listed in Table III. The calculated hardness of diamond is 89.7 GPa, in alignment with the reported experimental result (90 GPa [38]).  $Pmm2$ - $BC_7$  retains superior hardness over other  $BC_7$  in the  $BC_7$  system with order of hardness (for the considered  $BC_7$  compounds) as  $Pmm2 > P-43m > P-4m2 > P3m1$ .

### 3.4. Thermodynamic properties

Thermodynamic properties of solids at high pressure and at high temperature are noteworthy, within condensed matter physics. Thermodynamics calculations are necessary to acquire the corresponding properties as functions of temperature and pressure in terms of the quasi-harmonic Debye model [39–43], as applied to estimate thermodynamic properties of various materials [29, 43]. The coefficient of thermal expansion  $\alpha$  demonstrates the relationship between object size and temperature change. The coefficients of thermal expansion for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  are plotted as a function of pressure and temperature (Fig. 7a and b) and are found to increase with increase of temperature and decrease of pressure. Additionally, the coefficient of thermal expansion versus pressure exhibits a near linear relation at a given lower temperature ( $T < 500$  K). Two-dimensional contour plots of the Grüneisen parameter  $\gamma$ , versus pressure and temperature for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  are displayed in both Fig. 7c and d. The Grüneisen parameter  $\gamma$  describes the effect on vibrational properties of changing the crystal lattice volume and, as a consequence, the effect of changing temperature on the size or dynamics of the lattice. Effect of the temperature  $T$  on the coefficient of thermal expansion is more significant than pressure  $P$  (Fig. 7), and in contrast with the Grüneisen parameter effect, pressure  $P$  is greater.

Two-dimensional contour plots illustrating dependence of the Debye temperature on pressure and temperature for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  are displayed in Fig. 8. The Debye temperature is affected by both pressure and temperature and the effect of increase of temperature on  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  is the same as decrease of pressure. The Debye temperature decreases quickly at a given temperature with increase of pressure and typically,

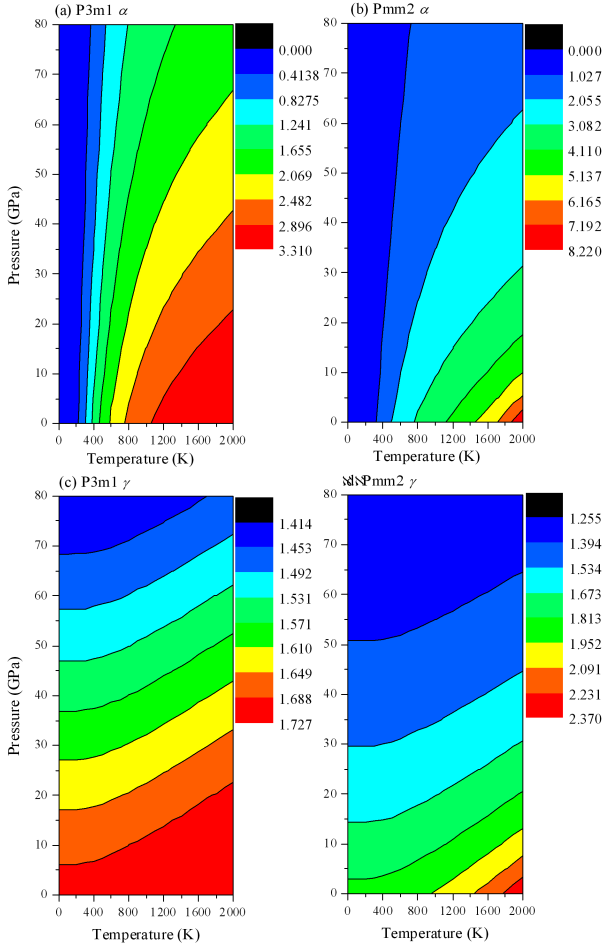


Fig. 7. Two-dimensional contour plots of coefficient of thermal expansion and Grüneisen parameter versus pressure and temperature for  $P3m1$ - $BC_7$  (a, c) and  $Pmm2$ - $BC_7$  (b, d).

the higher Debye temperature, the greater material microhardness. The resulting Debye temperature is 1797 K and 1878 K for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$ , which aligns with the previous description.

Heat capacity, or thermal capacity, is a measurable physical quantity equal to the ratio of heat added to (or subtracted from) an object to the resulting temperature change. The two-dimensional contour plots indicating dependence of heat capacity on pressure and temperature for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  are displayed in Fig. 9, with heat capacity as a function of temperature at various pressures. Heat capacity at constant pressure ( $C_P$ ) and at constant volume ( $C_V$ ) follows the law of  $T^3$  as expected in the range from 0 K to 800 K, considered as low temperatures with respect to the large Debye temperature of  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$ .  $C_V$  and  $C_P$  then expand with increase of temperature at a given pressure and decrease with increase of pressure at a given temperature. The  $C_V$  of  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  increases slowly at high temperature up to 1600 K, due to the high Debye temperature, they tend to achieve limitations in accordance with the Dulong-Petit law.

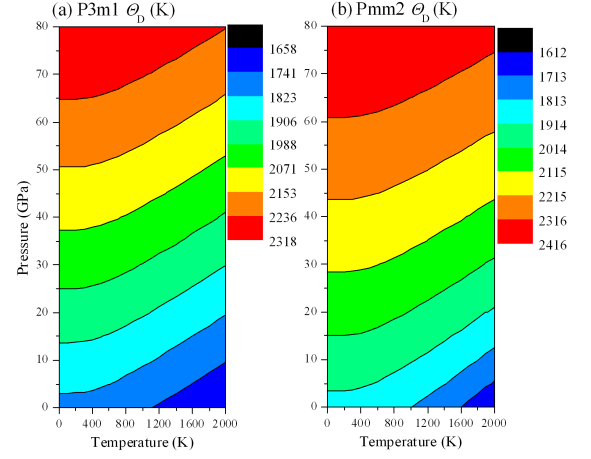


Fig. 8. Two-dimensional contour plots of the Debye temperature versus pressure and temperature for  $P3m1$ - $BC_7$  (a) and  $Pmm2$ - $BC_7$  (b).

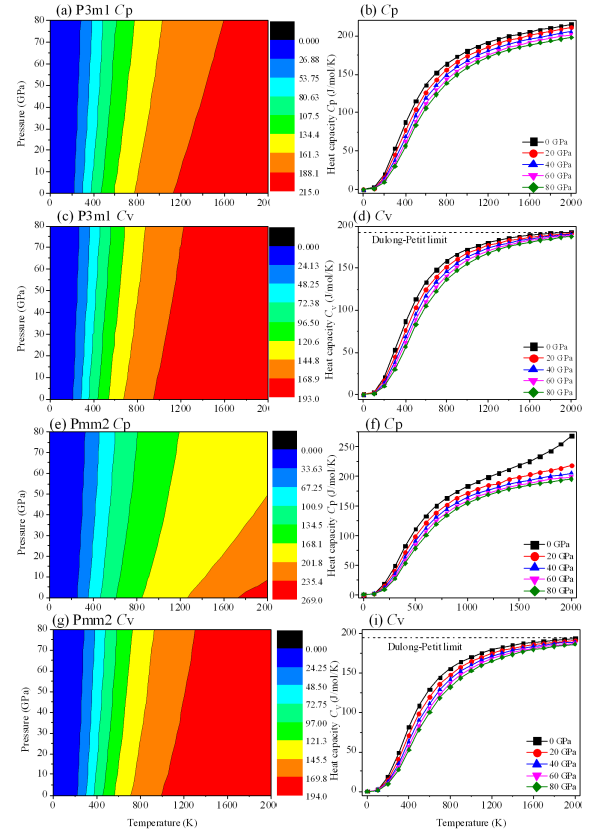


Fig. 9. Calculated specific volume  $C_V$  and pressure heat capacity  $C_P$  as a function of pressure for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  at different temperature:  $C_P$  contours (a, e),  $C_P$ - $T$  (b, f),  $C_V$  contours (c, g), and  $C_V$ - $T$  (d, i), respectively.

#### 4. Conclusions

A detailed investigation of the structural, elastic, and anisotropic properties of  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  utilizing the DFT within the ultrasoft pseudopotential scheme in the frame of GGA and LDA was conducted in this study. The elastic anisotropy of  $P3m1$ - $BC_7$  and

$Pmm2$ - $BC_7$  are discussed in detail under pressure from 0 GPa to 100 GPa. The calculated Pugh modulus ratio ( $B/G$ ) and the Poisson ratio indicate transition of  $P3m1$ - $BC_7$  from brittle to ductile at approximately 93.68 GPa. The  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A^U$ , the shear modulus, the Young modulus, and the Poisson ratio for  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$  reveal that  $Pmm2$ - $BC_7$  exhibits a larger elastic anisotropy than  $P3m1$ - $BC_7$ . Thermodynamic properties of  $P3m1$ - $BC_7$  and  $Pmm2$ - $BC_7$ , including the Debye temperature, heat capacity, and thermal expansion and the Grüneisen parameters are researched through application of the quasi-harmonic Debye model.

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