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A Study on Optimization of Planetary Gear Trains

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This study presents optimization of planetary gear train in a specific configuration. General characteristics of planetary gear trains are discussed briefly. A compound configuration for planetary gear train is selected and an optimization study is performed for this configuration. For the given input power, motor speed and overall gear ratio, modules, facewidths, teeth numbers of gears are found, satisfying the condition of minimum kinetic energy of the gear trains. In optimization, the objective is set to minimization of kinetic energy. Allowable bending stress and allowable contact stress are considered as design constraints. Minimum teeth number for a given pressure angle, center distance, recommendation on the facewidth, limitations on teeth ratios are considered as geometrical and kinematical constraints. The Matlab® Optimtool optimization toolbox is used. Results for certain operating conditions are obtained and tabulated.

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Nomenclature

m Module
p Circular pitch
e Train value
F Facewidth
Z Number of teeth
D Diameter
R Radius
P Motor power
σ_b Bending stress
σ_c Hertzian contact stress
S_{fb} Corrected bending strength
S'_{fb} Bending strength
S_{fc} Corrected Hertzain contact strength
S'_{fc} Hertzain contact strength
n_b Factor of safety due to bending stress
n_c Factor of safety due to contact stress
W_t Tangential force
K_a Application factor for bending stress
K_m Load distribution factor for bending stress
K_s Size factor for bending stress
K_b Rim-thickness factor for bending stress
K_i Idler factor for bending stress
J Geometry factor for bending strength
K_L Life factor for bending strength
K_T Temperature factor for bending strength
K_R Reliability factor for bending strength
C_p Elastic coefficient for contact stress
C_a Application factor for contact stress
C_m Load distribution factor for contact stress
C_s Size factor for contact stress
C_f Surface condition factor for bending stress
I_g Geometry factor for contact strength
C_L Life factor for contact strength
C_H Hardness ratio factor for contact strength
C_T Temperature factor for contact strength
C_R Reliability factor for contact strength
KE Kinetic energy
I Moment of inertia
w Rotational speed
M Mass
N Rotational speed of motor
N_p Number of planets
v Linear speed
n_F Rotational speed of the first gear
n_L Rotational speed of the last gear
n_a Rotational speed of the arm

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1. Introduction

Planetary gear trains consist of four members, such as sun gear, planet gears, ring gear and arm (planet carrier). Sun gear is located at the center and transmits torque to planet gears orbiting around the sun gear. The planet gears are mounted on an arm or carrier that fixes the planets in an orbit relative to each other. Planetary gear trains (PGT) are often recognized as the compact alternative to standard pinion-and-gear reducers with inherent in-line shafting and cylindrical casing properties. PGTs are being used in many specific areas of application, such as wind turbine gearboxes, automatic transmissions in vehicles, bicycle hubs, electric screwdrivers, machine tool gearboxes and many others.

Planetary gears are found in many variations and arrangements in order to meet a broad range of speed ratios in the design requirements. Different configurations can be easily obtained by re-arranging input member, outer member and stationary member, as described in Table I.

Simple planetary gear trains have a sun gear, a ring, a carrier, and a planet set. Compound planetary gear

TABLE I

Planetary gear train combinations.

Comb. no.	Sun gear	Carrier	Ring gear	Speed	Torque	Direction
1	Input	Output	Held	Maximum reduction	Increase	Same as input
2	Held	Output	Input	Minimum reduction	Increase	Same as input
3	Output	Input	Held	Maximum increase	Reduction	Same as input
4	Held	Input	Output	Minimum increase	Reduction	Same as input
5	Input	Held	Output	Reduction	Increase	Reverse of input
6	Output	Held	Input	Increase	Reduction	Reverse of input
7	When any two members are held together, speed and direction are the same as at input. Direct 1:1 drive occurs.					
8	When no member is held or locked together, output can not occur. The result is neutral condition.					

trains however, involve one or more of the following three types of structures: meshed-planet (there are at least two or more planets in mesh with each other in each planet train), stepped-planet (there exists a shaft connection between two planets in each planet train), and multi-stage structures (the system contains two or more planet sets). Compared to simple planetary gears, compound planetary gears have the advantages of larger reduction ratio, higher torque-to-weight ratio and more flexible configurations. PGT configuration described as combination number 1 in Table I will be considered in this study. It is shown in Fig. 1.

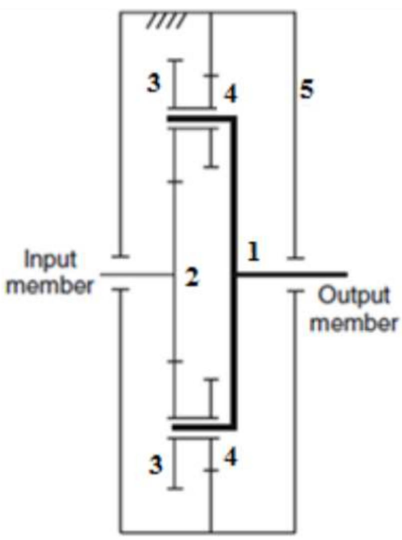

Fig. 1. Selected PGT configuration.

Gear trains used in machines introduce extra inertia into the system which may detrimentally affect the dynamic behavior of the systems. They also require certain amount of extra energy to be input to the system. For these reasons, gear trains have received considerable attention from many reseachers and optimization studies based on different criteria were performed and presented in the literature. Mohan and Seshaiah [1] carried out a study related with the minimization of center distance, weight and tooth deflection of spur gear set by using genetic algorithm. Contact stress and bending stress equations were used as constraint equations and multi-objective function optimization was done. Module, facewidth and number of teeth of gears are parameters to be obtained after optimization process.

Marjanovic et al. [2] focused on the characteristics and problems of optimization of gear trains with spur gears. They provided a description for selection of the optimal concept, based on selection matrix, selection of optimal materials, optimal gear ratio and optimal positions of shaft axes. Definition of mathematical model, with an example of optimization of gear trains with spur gears, based on minimum volume, using original software, were also presented.

Golabi et al. [3] carried out a study to minimize the volume to weight ratio of the gearbox considering one, two and three-stage gear trains. The general form of objective function and design constraints for the volume to weight ratio of a gearbox were written. Practical graphs of the results of the optimization were presented, by choosing different values for the input power, gear ratio and hardness of gears. From the graphs, all necessary parameters of the gearbox, such as number of stages, modules, facewidth of gears, and shaft diameter were obtained.

Thompson et al. [4] presented a generalized optimal design formulation with multiple objectives which is applicable to a gear train of arbitrary complexity. The methodology was applied to the design of two-stage and three-stage spur gear reduction units, subject to identical loading conditions and design criteria. The approach serves to extend traditional design procedures by demonstrating the trade-off between surface fatigue life and minimum volume using a basic multi-objective optimization procedure. This information allows the designer to judge overall trends and, for example, to assess the penalty in surface fatigue lifetime which would occur for a given weight reduction.

Kang and Choi [5] presented a method to optimize the helix angle of a helical gear from the viewpoint of the transmission error, which is the deflection of the teeth due to the transmitted load. The deflection of the gear teeth was calculated by using the bending and shear influence function, which is formulated from the common formula for deflection, obtained from FEM, and the contact influence function based on Hertzian contact theory. Tooth contact analysis was performed to calculate the contact lines of the helical gear, where the deflection of the tooth is measured. The relation between the contact ratio and transmission error was investigated through calculations of the variation in the transmission error with the helix angle.

Tamboli et al. [6] carried out a study to minimize the volume, since the most power transmission systems require low-weight energy-efficient and cost-effective system elements. A helical gear pair of a heavy duty gear reducer was considered for the objective of minimum volume. The various factors for sizing and strength of gears were computed for gear geometry parameters using DIN standard. The solution was attempted by using particle swarm optimization. The results are satisfactory and help the designer to employ minimum material and cost, by fulfilling the strength and performance requirements.

Bo et al. [7] carried out a study in which differential equations which govern the behavior of the gear transmission system of a 1.5 MW wind turbine were established, the external excitation caused by wind speed changes was discussed and the internal excitation due to time-varying mesh stiffness and comprehensive error was also analyzed. Based on the presented models, the expression of the application factor and dynamic load factor were obtained. Optimization design model with minimal volume was described by considering the equal-strength principle and reliability. The results demonstrate that the presented method is an effective way to enhance system reliability and to reduce the weight and the volume.

Tripathy and Cauhan [8] presented a study related with the multi-objective optimization of multi-stage planetary gear train. Minimization of surface fatigue life factor and minimization of volume of gear box are two conflicting objective functions under consideration. Two methods, one classical (SQP) and other non-traditional (NSGA-II) have been used for analysis, to satisfy strength and other geometric criteria. This work is an extension of an earlier work in a sense that planetary gear train with reduced speed involves more geometric constraints.

Roos and Spigelberg [9] derived the equations for the minimum gear sizes, necessary to drive a given load. The equations are based on the Swedish standards for spur gear dimensioning: SS1863 and SS1871. Minimum size equations for both the spur gear pairs and the three-wheel planetary gears were presented. Furthermore, expressions for the gear weight and inertia function of gear ratio, load torque and gear shape were derived. The results indicate that the Hertzian flank pressure limits the gear size in most cases. The teeth root bending stress is only limiting for very hard steels. The sizes, weights and inertia are shown to be smaller for planetary gears than for the equivalent pinion and gear configuration. Both these results are consistent with state of practice; planetary gears are commonly known to be compact and to have low inertia.

In the previous studies, gear trains were optimized based on minimum volume, minimum weight, minimum volume-to-weight ratio or minimum inertia. In transferring power from motor to the other unit of a machine the kinetic energy spent in the gear box should be kept minimum in order to have efficient systems. Kinetic energy is due to inertia and speeds of the gears and mutual effects of inertia and speed would be evaluated by minimizing kinetic energy. This topic was studied by Salgado and Alonso [10]. For two PGT configurations, optimization study was performed and 24 different gear boxes were tested and results were tabulated. Factor of safety for bending and surface failures were not taken into consideration in their study.

In this study, PGT examples with different input data are selected as case studies and factor of safety is included in the design calculations. Effects of power input, reduction ratio, motor speed and strength factor of safety are clearly indicated. Only spur gears are used in the train and the kinematical and geometrical constraints are written for the configuration given in Fig. 1. The optimization parameters are module, facewidth, number of teeth of the sun gear, planet gears and ring gear.

2. Constraint equations in PGT design

Constraints are treated in two groups. Constraints related to geometry and kinematics of gears are considered in one group and facewidth, bending and surface strengths are treated as the second group. First group of constraints is dependent on the configuration and may change for different configurations. Facewidth and gear strength considerations are valid for all spur gear pairs. In writing constraint equations, AGMA [11] standards and recommendations are taken into consideration.

2.1 Constraints involving gear size and geometry

The first constrain equation is related with gear sizing. The recommendation for the facewidth is given with respect to module in Eq. (1) below,

$$3p_i \le F_i \le 5p_i \text{ and } p_i = \pi m_i, \tag{1}$$

where $i = 2, 3, 4$ and 5 for sun gear, planet 1, planet 2 and ring gear, respectively. The second limitation is related with the gear tooth ratio. There are different limitations for external gear pairs and internal gear pairs suggested by AGMA [11] and they are expressed in Eqs. (2) and (3) respectively,

$$0.2 < \frac{Z_2}{Z_3} < 5.0, \tag{2}$$

$$-7.0 < \frac{Z_5}{Z_4} < -2.2. \tag{3}$$

The minus sign comes from the internal gear meshing of planet 2 and ring gear. There is another limitation on the ratio of pitch diameters of planet gears, recommended by AGMA [11]. This limitation is written in Eq. (4),

$$\frac{1}{3} < \frac{D_3}{D_4} < 3.0, \tag{4}$$

where diameters can be expressed in terms of module and number of teeth as $D_i = m_i Z_i$.

Another relationship must be satisfied as a consequence of the geometry of the PGTs for this configuration. In this particular case, such relationship is the equality of center distances between mating gears. It is expressed as follows,

$$R_2 + R_3 = R_5 - R_4. \quad (5)$$

The gear ratio, which is considered as another constraint, may be obtained from the train value of this particular configuration as follows [12],

$$e = \frac{n_L - n_a}{n_F - n_a}, \quad (6)$$

where $n_L = 0$ due to the fact that the ring gear is the last gear and it is a stationary member.

$$e = \frac{-n_a}{n_F - n_a} \rightarrow en_F = en_a - n_a \rightarrow \frac{n_F}{n_a} = \frac{e-1}{e}. \quad (7)$$

For this configuration, input member and output member rotate in reverse directions. Therefore train value has a negative sign. The equation becomes as follows,

$$\text{GearRatio} = \frac{n_{\text{input}}}{n_{\text{output}}} = \frac{n_F}{n_a} = \frac{-e-1}{-e} = 1 + \frac{1}{e}. \quad (8)$$

Train value may be also calculated by using number of teeth of the gear in mesh.

$$e = \frac{Z_2 Z_4}{Z_3 Z_5}. \quad (9)$$

Gear ratio is calculated as follows;

$$\text{GearRatio} = 1 + \frac{Z_3 Z_5}{Z_2 Z_4}. \quad (10)$$

Minimum number of teeth is another constraint related with gear size and geometry. To avoid undercut and interference for the normal pressure angle of 20°, minimum teeth number is required to be equal or greater than 18. That is,

$$Z_{\min} \geq 18. \quad (11)$$

2.2 Constraints involving gear strength

Gear strength requirements consist of bending stress constraints and contact stress constraints. These equations are given based on AGMA norm. These equations will be applied for each gear in the configuration. The bending stress constraint is written in Eq. (12) as,

$$\sigma_{bi} < \frac{Sf_{bi}}{n_{bi}}, \quad (12)$$

where

$$\sigma_b = \frac{W_t}{FmJ} \frac{K_a K_m K_s K_b K_i}{K_v}, \quad (13)$$

$$S_{fb} = S'_{fb} \frac{K_L}{K_T K_R}. \quad (14)$$

Similarly, the contact stress constraint equation is given in Eq. (15) below,

$$\sigma_{ci} < \frac{Sf_{ci}}{n_{ci}}, \quad (15)$$

where

$$\sigma_c = C_p \sqrt{\frac{W_t}{FI_g D} \frac{C_a C_m C_s C_f}{C_v}}, \quad (16)$$

$$S_{fc} = S'_{fc} \frac{C_L C_H}{C_T C_R}. \quad (17)$$

3. Optimum planetary gear train design

In this study, kinetic energy is minimized for the configuration given in Fig. 1. Therefore, objective function is the kinetic energy equation of the gear train, which is written in Eq. (18) as,

$$\text{KE} = \frac{1}{2} I_2 w_2^2 + \frac{1}{2} N_P (M_3 + M_4) v_3^2 + \frac{1}{2} N_p (I_3 + I_4) w_3^2. \quad (18)$$

In this equation, first term is the kinetic energy of the sun gear due to rotational motion, the second term is the kinetic energy due to movement of planet gears inside the ring gear and last term is the kinetic energy due to rotational motion of planets around their axes. The kinetic energy of the arm is negligible compared to other members. Equation (18) is rearranged in terms of module, facewidth and number of teeth as follows,

$$\text{KE} = \frac{1}{64} \pi \rho F_2 m_2^4 Z_2^4 w_2^2 + \frac{1}{8} N_P \pi \rho v_3^2 (m_3^2 Z_3^2 F_3 + m_4^2 Z_4^2 F_4) + \frac{1}{64} \pi \rho w_3^2 (m_3^4 Z_3^4 F_3 + m_4^4 Z_4^4 F_4). \quad (19)$$

Thus, the objective function and constraint equations are written in terms of gear design parameters of spur gear trains which are module, facewidth and number of teeth. The optimization variables are given in the Table II. Gears 2 and 3 are in a mesh with each other, so they have the same module and facewidth. Similarly, Gears 4 and 5 are working together. Therefore, they also have the same module and facewidth.

TABLE II

Optimization variables.

Parameter	Variable
$m_2 = m_3$	$x(1)$
$m_4 = m_5$	$x(2)$
$F_2 = F_3$	$x(3)$
$F_4 = F_5$	$x(4)$
Z_2	$x(5)$
Z_3	$x(6)$
Z_4	$x(7)$
Z_5	$x(8)$

4. Application of optimization tool

The optimization tool was created using Matlab® software. The optimization consist of the following steps,

- Choosing a solver
- Writing objective function in an m-file
- Writing constraint equations in an m-file
- Setting options
- Recording results

In this study, the objective function is the kinetic energy. Constraint equations are related to stresses, gear size and geometry, as stated previously. Some constraint equations are linear and some are non-linear inequalities. There are also many 4th order terms in the objective function. Considering these points, fmincon solver is decided to be used in this study, because of the non linear constraints and minimization operation. In order to use fmincon solver, it is necessary to write objective function and constraint equations into an m-file in terms of optimization variables. Then, it is necessary to state lower and upper bounds of the optimization variables.

As given in Table II, there are eight different parameters to be optimized. Therefore, optimization settings must be done properly in order to obtain accurate results. Maximum iteration number and maximum function evaluation value are the critical parameters in these options. In the following step, optimum parameters can be obtained by modifying the input parameters. All constraint equations and objective function are only valid for the configuration given in Fig. 1. However, they can be applicable to other configurations by making modifications in the objective function and in the constraining equations.

5. Results and discussion

Case studies are carried out in order to illustrate the effects of power, motor speed, gear ratio and factor of safety for both the bending and the surface failures of the gears. Same factor of safety is suggested for both failures, but if required, the designer may assign different values for bending and surface failures. The gear material is steel (with $S_y = 360$ MPa and $\rho = 7850$ kg/m^3). Module, facewidth and number of teeth are the output parameters. The results are tabulated in Table III, where minimum kinetic energy for each case is given in the last column of the Table.

TABLE III

Case study.

Power [kW]	Motor speed [rpm]	Gear ratio	Factor of safety	$m_2 = m_3$ [mm]	$m_4 = m_5$ [mm]	$F_2 = F_3$ [mm]	$F_4 = F_5$ [mm]	Z_2	Z_3	Z_4	Z_5	KE [kJ]
5.0	2000	3.0	1.0	3.39	5.39	53.22	84.56	31	22	18	51	10.09
5.0	2000	3.0	1.5	4.52	7.20	71.00	113.06	31	22	18	51	42.70
5.0	2000	3.0	2.0	5.55	8.86	87.20	139.07	31	22	18	51	119.4
5.0	2000	4.0	1.0	4.58	6.10	71.87	95.77	19	21	18	48	16.45
5.0	2000	4.0	1.5	6.10	8.13	95.80	127.77	19	21	18	48	69.37
5.0	2000	4.0	2.0	7.49	9.99	117.58	156.92	19	21	18	48	193.41
5.0	2000	5.0	1.0	5.50	7.07	51.88	103.90	18	25	18	52	41.75
5.0	2000	5.0	1.5	7.30	9.13	68.83	143.50	18	25	18	52	165.97
5.0	2000	5.0	2.0	11.08	5.00	104.50	78.45	18	30	77	184	1840
5.0	3000	3.0	1.0	3.01	4.80	47.34	75.37	31	22	18	51	12.65
5.0	3000	3.0	1.5	4.02	6.43	63.25	100.95	31	22	18	51	53.95
5.0	3000	3.0	2.0	4.95	7.91	77.77	124.30	31	22	18	51	151.64
5.0	4000	3.0	1.0	2.77	4.42	43.60	69.537	31	22	18	51	14.92
5.0	4000	3.0	1.5	3.71	5.93	58.33	93.23	31	22	18	51	63.98
5.0	4000	3.0	2.0	4.57	7.31	71.77	114.89	31	22	18	51	180.55
7.5	2000	3.0	1.0	3.91	6.22	61.45	97.76	31	22	18	51	20.74
7.5	2000	3.0	1.5	5.22	8.32	82.06	130.82	31	22	18	51	88.09
7.5	2000	3.0	2.0	6.42	10.25	100.84	161.00	31	22	18	51	246.95
10.0	2000	3.0	1.0	4.33	6.90	68.09	108.38	31	22	18	51	34.61
10.0	2000	3.0	1.5	7.27	6.18	68.60	68.68	25	23	48	105	291.80
10.0	2000	3.0	2.0	8.62	13.08	81.31	123.30	30	22	20	54	639.77

In case studies, gear hardness is taken as 149 HB, expected life is 1×10^7 cycles and the temperature is 20 °C. Uniform shock is assumed in the system. The optimization results show that kinetic energy increases with increasing speed, power, gear ratio and factor safety, as expected. The effects of motor speed, motor power, overall gear ratio and factor of safety on kinetic energy of PGT are shown in Fig. 2.

As shown in Fig. 2, factor of safety is more influential on minimum kinetic energy with respect to other input parameters. Program gives the optimum design parameters for each specific operating condition. As noticed, modules calculated in optimization are not standard values. If necessary, for any case, modules may be replaced by the nearest standard values and kinetic energy may be recalculated.

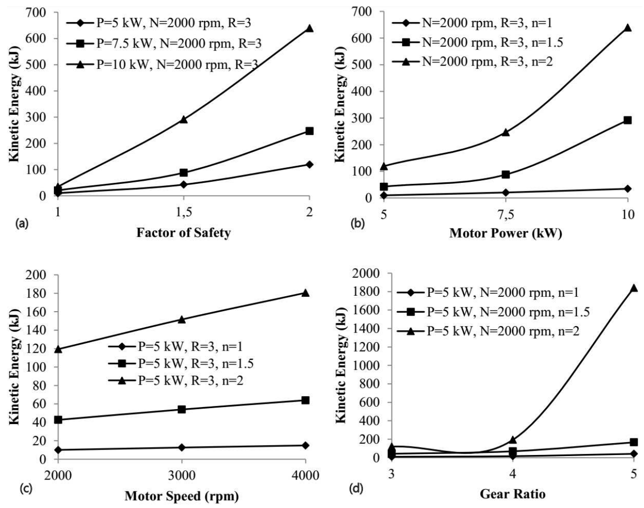

Fig. 2. The effects of alteration of safety factor, motor power, motor speed and overall gear ratio on kinetic energy of PGT.

6. Conclusions

For a specific PGT configuration, design parameters of the gears involved are determined by going through the optimization study. Objective was to minimize kinetic energy of gear trains which would satisfy geometric and kinematic constraints, together with constraints on the failure of gear teeth by bending and surface contact. Design parameters were selected as the module, facewidth and teeth numbers of the gears. Objective function and all constraint equations are written in terms of these parameters. These functions are transferred to the Matlab® software via m-files. The fmincon solver is selected due to non linear constraint equations and minimization operation. Then, after adjusting program settings, optimum design variables are obtained under certain operating conditions. The concept of kinetic energy provides us with a tool to see the mutual effects of both inertia and speed of the gears. Figure 2 shows that considerable attention must be given on deciding the factor of safety, required in gear applications.

As shown in Table I, modules may not have the standard values. If required, a nearest larger value may be assigned and kinetic energy may be recalculated. In this respect, this study may be considered to have interactive nature and would be beneficial to gear manufacturers and designers involved in gear designs. In addition, this optimization program may be integrated with an interface in order to obtain a software which can provide the optimum results under different working conditions and application areas.

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