Special issue of the 3rd International Conference on Computational and Experimental Science and Engineering (ICCESEN 2016)

Bending of Laminated Composite Beams by a Multi-Layer Finite Element Based on a Higher-Order Theory

V. Kahya* and M. Turan

Karadeniz Technical University, Faculty of Engineering, Department of Civil Engineering, 61080 Trabzon, Turkey

In this study, a layered beam element based on a higher-order theory is presented for bending analysis of laminated composites. This is an N-layer element which contains $(9N+7)$ degrees-of-freedom. The element stiffness matrix is derived by means of the Lagrange equations. Deflections and stresses in laminated beams with different end conditions and stacking order are calculated numerically. The results are compared with those available in the literature to show the accuracy of element.

DOI: 10.12693/APhysPolA.132.473

PACS/topics: laminated composite beams, finite element method, bending

1. Introduction

Composite materials may be preferred for many reasons, for example they are stronger, lighter when compared to traditional materials. There is a vast literature related to the laminated composite beams. Reddy [1] gave analytical and numerical solutions to bending, buckling and free vibration problems of laminated composite beams and plates in a comprehensive manner. Yuan and Miller [2] derived a higher-order multilayered element for bending of laminated beams to accurately predict the transverse shear stress distribution through-the-thickness. Khdeir and Reddy [3] solved bending of cross-ply laminated composite beams with different end conditions by using the state-space approach. Kapuria et al. [4] proposed an efficient zigzag one-dimensional theory of laminated beams. Kahya [5] presented dynamic analysis of laminated beams traversed by moving loads using a multilayered beam element based on the first-order shear deformation theory. Filippi and Carrera [6] proposed one-dimensional layer-wise theories that make use of higher-order zig-zag functions for bending and vibration analyses of laminated beams. Experimental studies have also been presented in the literature related to mechanical behavior and bending analysis of fiber reinforced composite structures [7, 8].

Here, we present bending of laminated composite beams. This N-layer element contains $(9N+7)$ degrees-of-freedom (DOFs). Delamination and slip between the layers are not allowed. Accuracy of the element is validated through the comparisons with available results for normalized maximum deflections, normal stresses and shear stresses of laminated beams with different boundary conditions and lamination scheme.

2. Theory and finite element formulation

According to higher-order shear deformation theory considered for the present work, displacements at any point in the beam (U, W) are assumed as in the following form:

$$U(x,z) = u(x) - z\phi(x) - z^2\beta_1(x) - z^3\beta_2(x),$$
$$W(x,z) = w(x), \tag{1}$$

where u, w and ϕ are the axial and transversal displacements, and cross-sectional rotation, respectively. β_1 and β_2 are higher-order terms arising from the Taylor expansion. All displacement components are measured on the neutral axis.

Figure 1 shows five-noded beam element with four equally spaced nodes and a node at the middle. It has sixteen DOFs including three axial and four transversal displacements, and nine rotations which are measured on the neutral axis of the beam. The nodal displacement vector can thus be given as

$$\boldsymbol{q} = \{u_1 \cdots u_3 w_1 \cdots w_4 \phi_1 \cdots \phi_3 \beta_{11} \cdots \beta_{13} \beta_{21} \cdots \beta_{23}\}^{\mathrm{T}} \tag{2}$$

Assume the solutions to be

$$u(x) = \sum_{i=1}^{3} \varphi_i(x) u_i,$$
$$w(x) = \sum_{i=1}^{4} \psi_i(x) w_i,$$
$$\phi(x) = \sum_{i=1}^{3} \theta_i(x) \phi_i,$$
$$\beta_1(x) = \sum_{i=1}^{3} \varpi_{1i}(x) \beta_{1i},$$
$$\beta_2(x) = \sum_{i=1}^{3} \varpi_{2i}(x) \beta_{2i}, \tag{3}$$

where $\varphi_i(x)$, $\psi_i(x)$, $\theta_i(x)$, $\varpi_{1i}(x)$ and $\varpi_{2i}(x)$ are the shape functions. The governing equations of motion can be obtained by the Lagrange equations which is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \tag{4}$$

where $L = T - U$ is the Langragian functional, q_i denotes the generalized coordinates corresponding to nodal displacements given by Eq. (2). T and U represent the kinetic and strain energy of the beam, respectively. Dot denotes derivative with respect to time. Equation of motion of the single lamina element can be written as

$$\boldsymbol{k}\boldsymbol{u} = \boldsymbol{f}, \quad (5)$$

where $\boldsymbol{k}$ is the stiffness matrix, $\boldsymbol{f}$ is the external force vector.

Stiffness matrix for the multi-layer beam element shown in Fig. 2 can be obtained by adding only nine rotational DOFs for each additional layer. Thus, total number of DOFs becomes $9N+7$. The stiffness matrix of the N-th layer is transferred into a new form involving the DOFs of layer $(N-1)$ plus the nine rotations from the layer N, then, added to the matrix of $(N-1)$-th layer. These combined matrices are then further altered to include the DOFs of layer $(N-2)$, and so on in sequence until the matrices for all layers have been assembled into a single stiffness matrix as

$$\begin{aligned}\boldsymbol{K}^e = {} & \boldsymbol{R}^{(1)\mathrm{T}}\boldsymbol{k}^{(1)}\boldsymbol{R}^{(1)} + \boldsymbol{T}^{(1)\mathrm{T}}(\boldsymbol{R}^{(2)\mathrm{T}}\boldsymbol{k}^{(2)}\boldsymbol{R}^{(2)} + \boldsymbol{T}^{(2)\mathrm{T}} \\ & \times(\boldsymbol{R}^{(3)\mathrm{T}}\boldsymbol{k}^{(3)}\boldsymbol{R}^{(3)} + \ldots + \boldsymbol{T}^{(N-2)\mathrm{T}} \\ & \times(\boldsymbol{R}^{(N-1)\mathrm{T}}\boldsymbol{k}^{(N-1)}\boldsymbol{R}^{(N-1)} \\ & + \boldsymbol{T}^{(N-1)\mathrm{T}}\boldsymbol{k}^{(N)}\boldsymbol{T}^{(N-1)})\boldsymbol{T}^{(N-2)})\ldots)\boldsymbol{T}^{(2)})\boldsymbol{T}^{(1)}. \quad (6)\end{aligned}$$

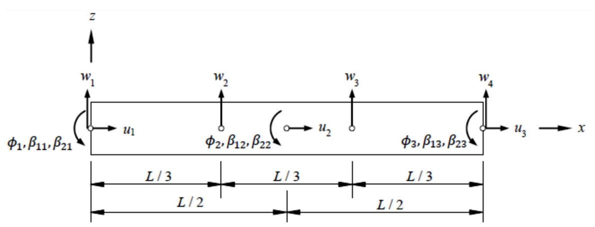

Fig. 1. One-layer beam element with sixteen degrees-of-freedom.

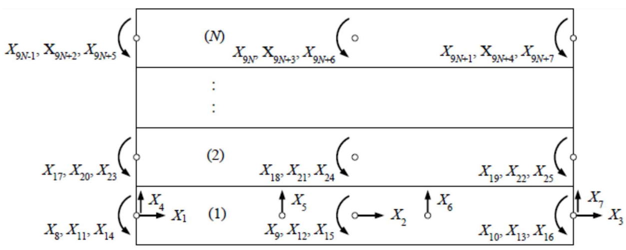

Fig. 2. Multilayered beam element.

For the entire beam with length L, the global matrix equation of motion can be given by

$$\boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}, \quad (7)$$

where $\boldsymbol{K}$ is the global stiffness matrix, $\boldsymbol{X}$ is the nodal displacement vector including unknowns for the N-layer beam.

3. Numerical results

Examples for demonstrating the present element's accuracy to find displacements and stresses of laminated beams with different boundary conditions and lamination scheme are given. Numerical results are obtained by means of a computer code written in FORTRAN language.

Laminated composite beams with different lamina lay-up under center point load are considered. Material properties are selected as $E_1/E_2 = 25$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$ and $\nu_{12} = 0.25$ (Reddy, 1997). Table I shows normalized maximum deflections ($\bar{v} = 100vEAh^2/PL^3$) for different L/h and boundary conditions. As can be seen in the table, results of the present model agree well with the analytical solution based on the first-order shear theory by Reddy (1997).

In Fig. 3, the maximum normal ($\bar{\sigma} = \sigma Ah/PL$) and shear stress ($\bar{\tau} = \tau A/P$) distributions through the thickness of a symmetrically laminated $(0/90)_s$ composite beam with simple supports are given. To avoid the effect of shear, very slender beam ($L/h = 100$) is considered. The present model has a good accuracy for normal and shear stresses with the analytical solution based on the first-order shear theory.

TABLE I

Normalized maximum deflections $\bar{v}$ for laminated composite beams under center point load. Top values — this paper, bottom values —analytical, Ref. [1]; H–H — hinged boundary conditions, C–C — clamped boundary conditions.

Laminate	L/h	H–H	C–C
0	10	1.588	0.826
		1.600	0.850
	20	1.148	0.397
		1.150	0.400
	100	1.005	0.256
		1.001	0.256
90	10	25.596	6.843
		26.500	7.750
	20	25.149	6.398
		25.375	6.625
	100	25.006	6.256
		25.015	6.265
$(0/90)_s$	10	1.706	0.848
		1.991	1.141
	20	1.280	0.426
		1.348	0.498
	100	1.142	0.290
		1.143	0.292

4. Conclusion

A multilayered higher-order finite element for bending analysis of laminated composite is presented. Slip and delamination between the layers are not allowed. Element matrices are derived through Lagrange's equations. According to results of the study, the present element can predict deflections and stresses of thin and thick composite beams with different number of layers and lamination scheme in a good accuracy.

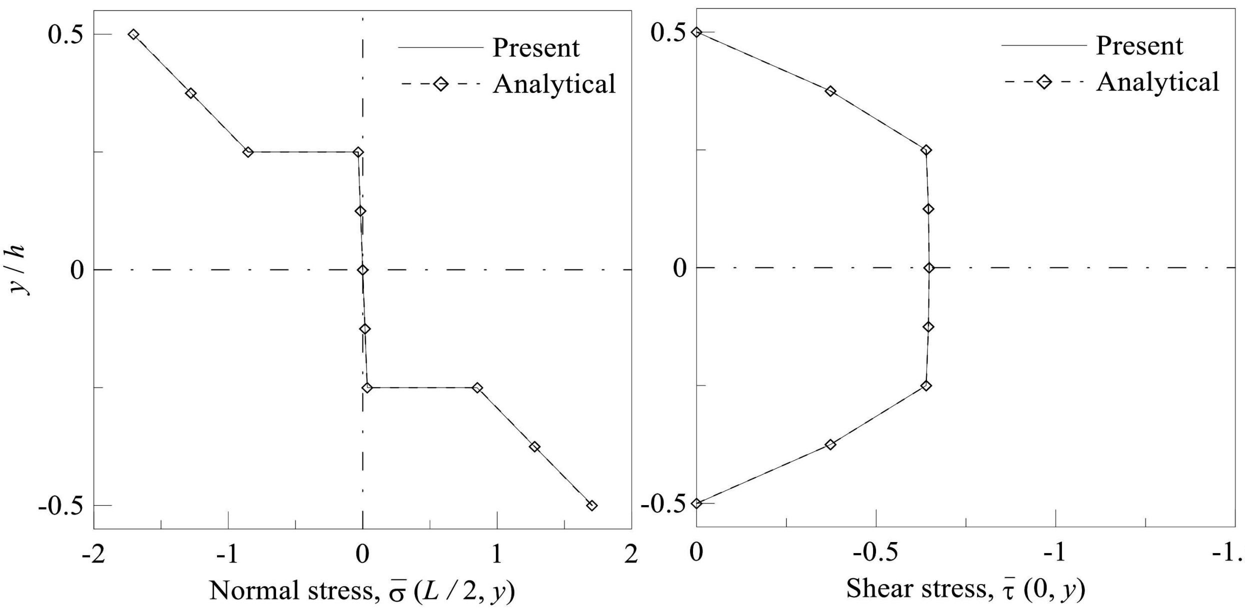

Fig. 3. Normal and shear stresses through the thickness of a simply supported $(0/90)_s$ laminated beam.

References

[1] J.N. Reddy, *Mechanics of Laminated Composite Plates: Theory and Analysis*, CRC Press, Boca Raton 1997.

[2] F.G. Yuan, R.E. Miller, *Comput. Struct.* **14**, 125 (1990).

[3] A.A. Khdeir, J.N. Reddy, *Composite Struct.* **37**, 195 (1997).

[4] S. Kapuria, P.C. Dumir, N.K. Jain, *Composite Struct.* **64**, 317 (2004).

[5] V. Kahya, *Struct. Eng. Mech.* **42**, 729 (2012).

[6] M. Filippi, E. Carrera, *Composites Part B* **98**, 269 (2016).

[7] N. Ozsoy, M. Ozsoy, A. Mimaroglu, *Acta Phys. Pol. A* **130**, 297 (2016).

[8] S. Ekşi, K. Genel, *Acta Phys. Pol. A* **128**, B-59 (2015).