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Numerical Computation of the Electrostatic Sheath Thickness

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In this work we have established a one dimensional, stationary and non-magnetized theoretical model that describes the electrostatic sheath formation [M. Moisan, J. Pelletier, *Physique des Plasmas Collisionnels, Application aux Décharges Haute Fréquence*, EDP Sciences, 2006]. The sheath thickness is assessed. For this, we have assumed that all species are described by fluid equations. Dust grains are considered spherical particles with constant radii. Their charge is modeled by the orbit motion limited model [P.K. Shukla, A.A. Mamun, *Introduction to Dusty Plasma Physics*, Institute of Physics, Bristol 2001; A. Bouchoule, *Dusty Plasmas, Physics, Chemistry and Technological Impacts in Plasma Processing*, Wiley, Chichester 1999]. The solution of the obtained set of differential equations is found using the shooting method. The numerical results show that the sheath thickness depends considerably on the solid surface potential, as well as physical parameters, such as particle densities and temperatures, gas pressure, etc. The calculated electrostatic sheath thickness is greater than the thickness predicted by Child-Langmuir law [M.A. Lieberman, A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, Wiley, New York 1994].

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1. Introduction

The interaction of plasma with solid surfaces has received a great attention in the last three decades, as it is related to many physical phenomena, such as surface erosion, plasma processing, ion cyclotron heating, etc. This type of interaction is widely used in many areas of laboratory, industrial and astrophysical plasmas. Indeed, several researchers have investigated analytically and experimentally this subject [1–10]. This interaction gives rise to non-neutral region, called electrostatic sheath, in which a strong localized electric field occurs. Due to the higher mobility of electrons, against the ions mobility, the solid surface, such as electrode in discharge plasmas, acquires a negative potential with respect to the bulk plasma.

The presence of dust grains (in some cases, they are called impurities), ranging from nanometers to micrometers in size, leads to important modifications in the behavior of sheath and therefore remains an important and serious problem in plasma processing of integrated circuits, for example [11, 12]. Thus, they change the plasma parameters and affect the collective processes, leading to the appearance of new modes and new mechanisms of damping and instabilities in such plasma systems.

For both laboratory and industrial plasmas, the electrostatic sheath thickness plays an important role in the bombardment or the erosion of the solid surface by accelerated particles present in the surrounding plasma. Furthermore, the electrostatic sheath thickness plays an important role in the trapping of impurities (dust grains) which constitute a brake of the fusion reaction in tokamaks, such as ITER (International Thermonuclear Experimental Reactor) for example.

Child and Langmuir [13] have computed the electrostatic sheath thickness in a particular case, where the plasma is considered collisionless and in absence of impurities. In recent works [7–9, 14], the electrostatic sheath is determined by the requirement that the electrons density is zero at the electrode (wall) position. In this work, we have proposed a new numerical approach, based on the shooting method, to compute numerically the electrostatic sheath thickness.

2. Theoretical model

We consider a 1D, stationary and unmagnetized dusty low-pressure plasma model. The charged particles in the plasma electrostatic sheath are electrons e, ions i and extremely heavy spherical dust grains d. The electrode, that is a plane solid surface with a negative electrostatic potential, is situated at position $z = L$. The sheath region lies between $z = 0$ and the electrode, where z is the position along the vertical axis, which is in the same direction as gravity $\boldsymbol{g}$. All particles e, i and d are described by fluid equations:

$$\frac{\mathrm{d}\left(n_e v_e\right)}{\mathrm{d}z} = k_i n_e n_n, \tag{1}$$

$$v_e \frac{\mathrm{d}v_e}{\mathrm{d}z} = \frac{e}{m_e}\frac{\mathrm{d}\phi}{\mathrm{d}z} - \frac{1}{m_e n_e}\frac{\mathrm{d}p_e}{\mathrm{d}z} - n_n \sigma_{en} v_e^2, \tag{2}$$

$$\frac{\mathrm{d}\left(n_i v_i\right)}{\mathrm{d}z} = k_i n_e n_n, \tag{3}$$

$$v_i \frac{\mathrm{d}v_i}{\mathrm{d}z} = -\frac{e}{m_i}\frac{\mathrm{d}\phi}{\mathrm{d}z} - \frac{1}{m_i n_i}\frac{\mathrm{d}p_i}{\mathrm{d}z} - n_n \sigma_{in} v_i^2, \tag{4}$$

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$$\frac{\mathrm{d}\,(n_d v_d)}{\mathrm{d}z} = 0, \tag{5}$$

$$m_d v_d \frac{\mathrm{d}v_d}{\mathrm{d}z} = -q_d \frac{\mathrm{d}\phi}{\mathrm{d}z} + m_d g, \tag{6}$$

where ϕ is the electrostatic potential, $m_{e,i}$, $n_{e,i}$, $v_{e,i}$, and $p_{e,i}$ are the mass, the density, the fluid velocity and the pressure of electrons and ions respectively, n_n is the neutral gas density, n_d, v_d, m_d and q_d are the density, the fluid velocity, the mass and the charge of dust grains respectively, k_i is the electronic impact ionization rate [13] and $\sigma_{ei,in}$ are the electron-neutral and ion-neutral diffusion cross section [15], e being the elementary charge.

The dust grain charge is due mainly to the electron and the ion fluxes, arriving to the dust grain,

$$v_d \frac{\mathrm{d}q_d}{\mathrm{d}z} = n_i e a_i - n_e e a_e. \tag{7}$$

In the case of the orbit motion limited model [16–18], the attachment rates a_i and a_e are given by:

$$a_e = \begin{cases} -\pi r_d^2 \left(8T_e/\pi m_e\right)^{1/2} \exp\left(e q_d/r_d T_e\right) & \text{if } q_d \le 0, \\ -\pi r_d^2 \left(8T_e/\pi m_e\right)^{1/2} \left(1 + e q_d/r_d T_e\right) & \text{if } q_d > 0, \end{cases} \tag{8}$$

$$a_i = \begin{cases} \pi r_d^2 v_i \left(1 - 2e q_d/r_d m_i v_i^2\right) & \text{if } q_d \le 0, \\ \pi r_d^2 v_i \exp\left(2e q_d/r_d m_i v_i^2\right) & \text{if } q_d > 0, \end{cases} \tag{9}$$

where r_d is the dust grain radius and T_e is the electron temperature.

Finally, the electrostatic potential is determined by the Poisson's equation,

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}z^2} = -\frac{1}{\varepsilon_0}\left(n_i e - n_e e + n_d q_d\right), \tag{10}$$

where ε_0 is the vacuum permittivity.

3. Numerical results and discussion

The main focus of this paper is the numerical computation of the electrostatic sheath thickness by using the shooting method. For this, argon is considered to be the background gas and the physical parameters used are: $T_e = 2$ eV, $T_i = 0.05$ eV, $n_{i0} = 10^9$ cm^{-3}, $n_n = 10^{14}$ cm^{-3}, $\sigma_{in} = 5 \times 10^{-15}$ cm^2, $\sigma_{en} = 5 \times 10^{-16}$ cm^2, $u_{i0} = v_{i0}/c_{is} = 1.5$, $u_{d0} = v_{d0}/c_{ds} = 2.5$, $r_d = 1$ μm, $\rho_d = 2$ g/cm^3 is the dust grains mass density, $c_{is} = (T_e/m_i)^{1/2}$ is the ion sound velocity, $c_{ds} = (z_c T_e/m_d)^{1/2}$ is the dust sound velocity and $z_c = r_d T_e/e^2$, where the subscript "0" denotes the equilibrium quantities ($\phi = 0$).

The set of Figs. 1a–d shows the effect of the solid surface (electrode) potential on the electrostatic sheath parameters and its thickness. Figure 1b and c shows that the increase of the electrode potential makes the electron and ion density profiles stiffer and the sheath thickness becomes more important. Indeed, the increase of the solid surface potential increases the Coulomb repulsion, as well as the charge separation and consequently the sheath thickness.

In Fig. 1d, we have plotted the normalized sheath thickness d/λ_{D_i} versus the normalized solid surface potential $\Phi_0 = -e\phi_0/T_e$, where λ_{D_i} is the ion Debye length. Unlike the Child-Langmuir law, which states that the electrostatic sheath thickness depends only on the electrode potential and the electrons temperature, i.e., $\left(d/\lambda_{D_i} \approx \Phi_0^{3/4}\right)$, the numerical results show that the electrostatic sheath thickness depends considerably on the solid surface potential and all other physical parameters, such as particle densities and temperature, gas pressure, etc. Furthermore, the computed electrostatic sheath thickness is significantly higher than that, derived by Child-Langmuir law. Finally, for practical uses, we have computed a numerical fit given by, $d/\lambda_{\mathrm{D}_i} = 35.08\Phi_0^{0.18} + 3.43\Phi_0^{0.76}$.

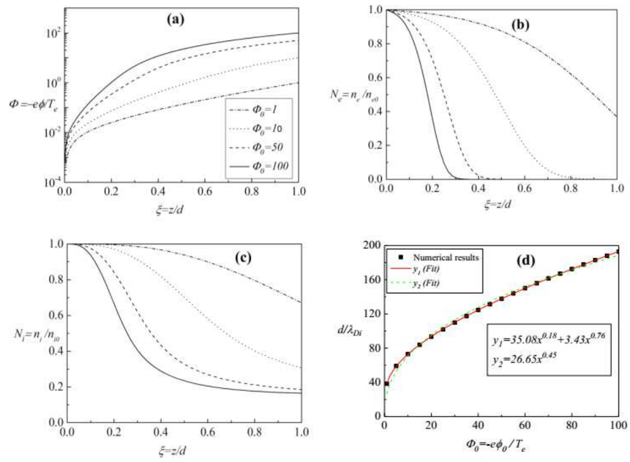

Fig. 1. Normalized electrostatic potential (a), normalized electrons density (b) and normalized ions density (c) versus the normalized spatial variable for different values of the normalized electrode potential. Normalized sheath thickness versus the normalized electrode potential (d).

4. Conclusions

The interaction of plasma with a solid surface (in our case, a plane electrode) and the formation of an electrostatic sheath is analysed analytically and numerically by using a fluid model. The electrostatic sheath properties, such as electrostatic potential and particle densities are computed and interpreted. In particular, the sheath thickness, that is a serious and important problem in the field of plasma-solid surface interaction, is computed exactly by using a shooting method. In contrast with the Child-Langmuir law, which indicates that the sheath thickness depends only on the electrode potential and electron temperature, it is found that in our case it depends strongly on all physical parameters and its value is higher than that of Child-Langmuir. For practical uses, the obtained results are summarized in a numerical fit.

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