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Influence of Magnetic Field on Dark States in Transport through Triple Quantum Dots

K. Wrześniewski* and I. Weymann

Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland

We theoretically study the electronic transport through a triple quantum dot system in triangular geometry weakly coupled to external metallic leads. By means of the real-time diagrammatic technique, the current and Fano factor are calculated in the lowest order of perturbation theory. The device parameters are tuned to such transport regime, in which coherent population trapping of electrons in quantum dots due to the formation of dark states occurs. The presence of such states greatly influences transport properties leading to a strong current blockade and enhanced, super-Poissonian shot noise. We consider both one- and two-electron dark states and examine the influence of magnetic field on coherent trapping in aforementioned states. When the system is in one-electron dark state, we observe a small shift of the blockade's region, whereas in the case of two-electron dark state, we show that strong magnetic field can lift the current blockade completely.

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1. Introduction

Coupled quantum dot systems reveal many different quantum phenomena favorable for future applications in nanoelectronics and quantum computation [1, 2]. A prominent class of such systems constitute those built of three quantum dots [3, 4], in which a wide variety of effects, interactions and possible configurations give rise to rich physics and consistently stimulate an extensive theoretical and experimental research. An important example of a quantum-mechanical phenomenon that can emerge in triple quantum dots (TQDs) is associated with the formation of dark states, which lead to coherent electron trapping [5–8]. A dark state forms when destructive interference of electronic wave functions decouples one of the dots from the respective lead and, as a result, the current can no longer flow through the system. This work extends the previous studies on coherent population trapping in TQDs [5–7] by analyzing the role of external magnetic field on the current flowing through a triple quantum dot in triangular geometry in transport regime where dark states form.

2. Model and method

The system is built of three single-level quantum dots (see Fig. 1) arranged in a triangular geometry. The first (second) dot is weakly coupled to the left (right) metallic electrode and the dots are coupled to each other by hopping matrix elements t_{ij}. The total Hamiltonian can be written as $H = H_{\rm Leads} + H_{\rm TQD} + H_{\rm T}$. The first term, $H_{\rm Leads} = \sum_{j=L,R}\sum_{k\sigma}\varepsilon_{jk\sigma}c^{\dagger}_{jk\sigma}c_{jk\sigma}$, describes noninteracting electrons in the leads, where $c^{\dagger}_{jk\sigma}$ is the creation operator of an electron with spin σ, momentum k and energy $\varepsilon_{jk\sigma}$ in the left or right ($j = L, R$) electrode. The TQD Hamiltonian reads

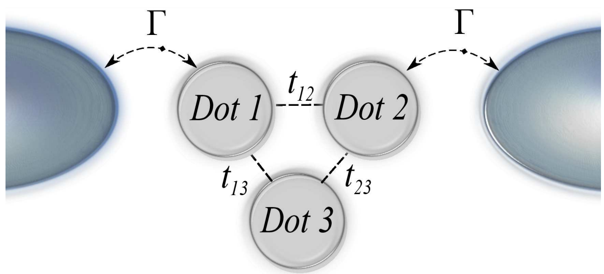

Fig. 1. Schematic of a triple quantum dot system in triangular geometry. Dot 1 (2) is coupled to the left (right) lead with coupling strength Γ. The dots are coupled to each other via hopping matrix elements t_{ij}.

$$H_{\rm TQD} = \sum_{j\sigma}\varepsilon_j n_{j\sigma} + U_j\sum_j n_{j\uparrow}n_{j\downarrow} + \frac{U'_{ij}}{2}\sum_{\langle ij\rangle}\sum_{\sigma\sigma'} n_{i\sigma}n_{j\sigma'} + \sum_{\langle ij\rangle}\frac{t_{ij}}{2}\sum_{\sigma}(d^{\dagger}_{i\sigma}d_{j\sigma} + d^{\dagger}_{j\sigma}d_{i\sigma}) + B\sum_j S_{zj}. \tag{1}$$

The first term describes on-site energy ε_j, with $n_{j\sigma} = d^{\dagger}_{j\sigma}d_{j\sigma}$ and $d^{\dagger}_{j\sigma}$ being the creation operator of an electron with spin σ in j-th quantum dot. The two next terms describe intra- and inter-dot Coulomb interactions, with strength U_j and U'_{ij}, respectively. The fourth term stands for hopping between the dots, while the last one takes into account external magnetic field B in units of $g\mu_B \equiv 1$, with S_{zj} being the z-th component of j-th dot spin.

The last term of total Hamiltonian describes tunneling between TQD and the leads and is given by

$$H_{\rm T} = \sum_{j=L,R}\sum_{k\sigma}(V_j c^{\dagger}_{jk\sigma}d_{j\sigma} + {\rm H.c.}), \tag{2}$$

where V_j is the tunnel matrix element between the j-th

*corresponding author; e-mail: wrzesniewski@amu.edu.pl

lead and the corresponding dot. The dot-lead coupling strength is then given by $\Gamma_j = 2\pi|V_j|^2\rho_j$, with ρ_j being the density of states of lead j. It is assumed that $\Gamma_L = \Gamma_R \equiv \Gamma$ and that the system is symmetrically biased: $\mu_L = eV/2$ and $\mu_R = -eV/2$.

We calculate the electronic current, shot-noise and Fano factor using the real-time diagrammatic technique [9, 10]. This technique relies on perturbation expansion of the reduced density matrix and corresponding operators with respect to Γ. In this analysis the weak lead-dot coupling is assumed and only the lowest-order of expansion is considered, which describes sequential tunneling processes. Due to relatively large hopping between the dots, $t_{ij} > \Gamma$, there is a significant overlap of the electrons' wave functions of the dots, which results in the formation of molecular states $|\chi\rangle$. Electronic transport takes places through those states and their occupation probabilities p_χ are found from an appropriate kinetic equation [9, 10]. The current is then calculated from $I = (e/2\hbar)\mathrm{Tr}\{\mathbf{W}^I\mathbf{p}\}$, where $\mathbf{W}^I$ is the self-energy matrix, which takes into account electron tunneling through the system, while $\mathbf{p}$ is the vector containing probabilities p_χ. Finally, we also determine the shot noise S and the corresponding Fano factor $\mathrm{F} = S/(2e|I|)$ [10].

3. Results and discussion

The current as a function of bias voltage, for indicated values of external magnetic field B, is shown in Fig. 2a. The dots' levels are set to $\varepsilon_j = \varepsilon = U/2$, therefore when no bias is applied, the dots levels are unoccupied and the system is empty. For $B = 0$, around $eV/U \approx 0.8$, there is a peak in the current due to the first one-electron state entering the transport window. However, very close to $eV/U \approx 1$, there is a strong drop of the current and transport through the system is blocked, see Fig. 2a. This current blockade is due to the formation of a one-electron dark state, that entered the transport window. The wave function of this dark state has the following form: $|\Psi_{DS1}\rangle = \frac{1}{\sqrt{2}}(|\sigma,0,0\rangle - |0,0,\sigma\rangle)$, where $|\alpha_1,\alpha_2,\alpha_3\rangle$ is a local state of the system, in which dot j is either empty ($\alpha_j = 0$), occupied by spin-σ electron ($\alpha_j = \sigma$) or doubly-occupied ($\alpha_j = d$). In this one-electron dark state the electron occupies the first and third dot with equal probability, however, due to destructive interference, the occupation of the second dot is equal to zero. This effectively decouples the TQD system from the right lead and blocks tunneling processes through the right junction. As can be seen in Fig. 2a, the system remains in the dark state for a significant range of bias voltage, till the current starts flowing through the system again, around $eV/U \gtrsim 2.5$, when two-electron states enter the bias window. When the magnetic field is turned on, one can clearly see in the current dependence that the energy levels of states taking part in transport are shifted toward lower bias voltage and, consequently, the systems becomes trapped in dark state already for smaller voltages, while the size of this blockade remains almost unchanged. This is due to the Zeeman splitting of the TQD levels. The one-electron dark state is spin-split and now the lower-energy dark state enters the transport window earlier, which makes the blockade appear faster. On the other hand, the dark state with opposite spin moves toward higher energies. However, this shift does not increase the size of the blockade, because the two-electron triplet states reach the transport window earlier due to the same mechanism, i.e. Zeeman-splitting-induced shift of their energy.

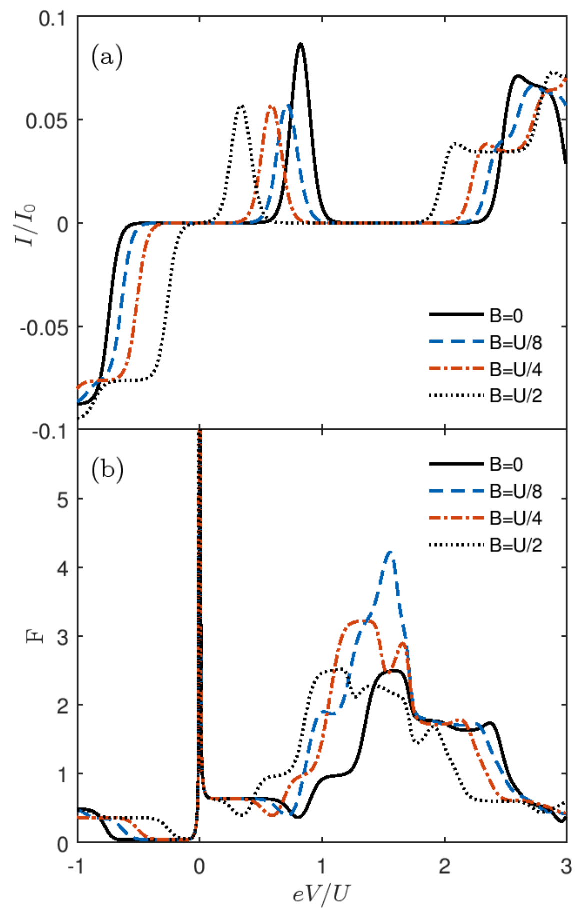

Fig. 2. The bias voltage dependence of (a) the current and (b) the Fano factor for different values of magnetic field B. The parameters are: $U_1 = U_3 = U_{13} = U = 1$, $U_2 = U + \Delta, U_{12} = U_{23} = U - \Delta$, where $\Delta = U/5$. The dots' levels are equal, $\varepsilon_j = 0.5$, $t_{13} = 0.05$, $t_{12} = t_{23} = 0.1$, $\Gamma = 0.01$, and $T = 0.02$ in units of U. The current is plotted in units $I_0 = e\Gamma/\hbar$.

The bias voltage dependence of the Fano factor is presented in Fig. 2b. At zero bias voltage the Fano factor is divergent due to the fact that the current vanishes while the noise is finite due to thermal fluctuations. On the other hand, in the bias range where the system is trapped in the dark state, the shot noise becomes super-Poissonian [7, 8]. For given parameters, the Fano factor reaches $\mathrm{F} \approx 2.5$ for $B = 0$, and $\mathrm{F} >\gtrsim 4$ when the system is in finite magnetic field B.

When the dots' levels are set to $\varepsilon_j = \varepsilon = -U/2$, at zero bias the system is singly occupied. For assumed

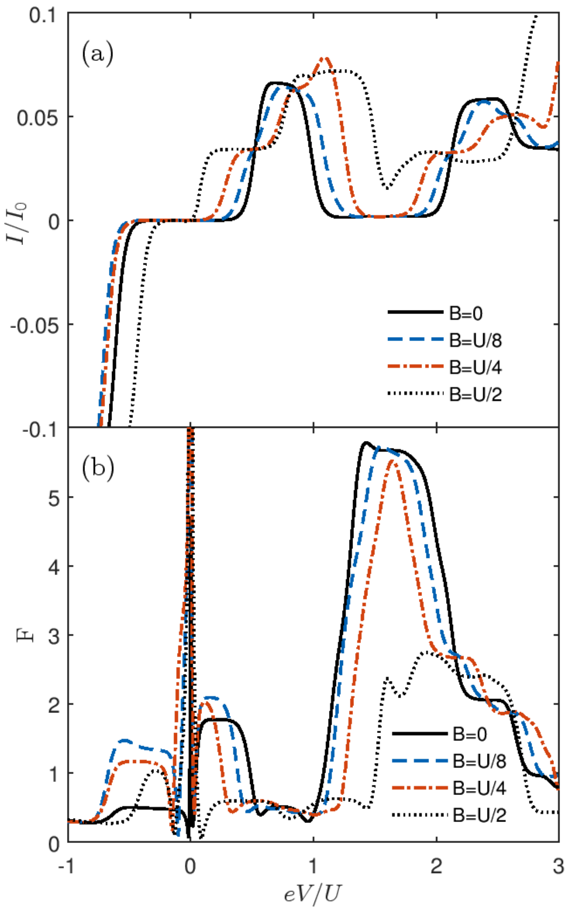

Fig. 3. The same as in Fig. 2 calculated for $\varepsilon_j = \varepsilon = -U/2$.

parameters, the TQD can easily reach a two-electron dark state. The current and Fano factor for this case are shown in Fig. 3. For $B = 0$, the current exhibits a plateau for $0.5 \lesssim eV/U \lesssim 1$, which is related to two-electron state entering the transport window, which is not a dark one. Similarly to the previously analyzed case, the dark state has higher energy than the first state participating in transport. Therefore, in both cases we are able to observe a strong peak in the current, followed by sudden current suppression due to trapping the system in a dark state. As can be seen in Fig. 3a, the two-electron dark state enters the transport window around $eV/U \approx 1$ and has the following wave function: $|\Psi_{DS2}\rangle = \frac{1}{2}(|d,0,0\rangle + |\uparrow,0,\downarrow\rangle - |\downarrow,0,\uparrow\rangle + |0,0,d\rangle)$. The occupation of the second dot is again equal to zero, therefore the electrons are not able to reach the right electrode and the current is blocked. However, the behavior of this dark state in magnetic field is quite different from the previous case. The difference results from the fact that the two-electron dark state is formed by two particles of opposing spin, i.e. it is a singlet state. Consequently, this state is not split by external magnetic field. However, the one-electron and three-electron states that are close in energy to this state are under strong influence of magnetic field, and because of the Zeeman splitting of those levels the current blockade shrinks with increasing B. This is clearly visible in Fig. 3a. The bias voltage range where the two-electron dark state is present gets narrower as the magnetic field becomes stronger. Eventually, for the large enough field ($B = U/2$) the blockade is lifted, however, a significant drop in current is still visible.

The Fano factor dependence on V shown in Fig. 3b again shows strong enhancement of shot-noise within the blockade. The Fano factor for the two-electron dark state is even higher than in previously discussed one-electron case and reaches values F $>\gtrsim 5$. When the magnetic field is present, due to the reduced size of the blockade, the peak in the Fano factor also gets narrower. When the magnetic field is strong enough to lift the blockade, the Fano factor significantly drops as well (F < 3), see Fig. 3b.

To conclude, we have studied the bias voltage dependence of the current and the Fano factor for TQD system in triangular geometry. We have focused on regimes where the transport is blocked due to the presence of one and two-electron dark states, and analyzed how the external magnetic field influences transport in those regimes. We have shown that the blockade's position gets shifted in the case of one-electron dark state, whereas in the two-electron dark state regime, the presence of strong magnetic field can lift the current blockade.

Acknowledgments

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References

[1] L.P. Kouwenhoven, D.G. Austing, S. Tarucha, *Rep. Prog. Phys.* **64**, 701 (2001).

[2] D. Loss, D.P. DiVincenzo, *Phys. Rev. A* **57**, 120 (1998).

[3] C.-Y. Hsieh, Y.-P. Shim, M. Korkusinski, P. Hawrylak, *Rep. Prog. Phys.* **75**, 114501 (2012).

[4] K. Wrześniewski, I. Weymann, *Phys. Rev. B* **92**, 045407 (2015).

[5] C. Emary, *Phys. Rev. B* **76**, 245319 (2007).

[6] C. Poltl, C. Emary, T. Brandes, *Phys. Rev. B* **80**, 115313 (2009).

[7] I. Weymann, B.R. Bułka, J. Barnaś, *Phys. Rev. B* **83**, 195302 (2011).

[8] K. Wrześniewski, I. Weymann, *Acta Phys. Pol. A* **127**, 460 (2015).

[9] H. Schoeller, G. Schön, *Phys. Rev. B* **50**, 18436 (1994); J. König, J. Schmid, H. Schoeller, G. Schön, *Phys. Rev. B* **54**, 16820 (1996).

[10] A. Thielmann, M.H. Hettler, J. König, G. Schön, *Phys. Rev. Lett.* **95**, 146806 (2005).