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Normal Modes in a Linear Chain of Nanomagnets

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The dynamics of magnetization in low-dimensional systems of micromagnets has been a very interesting subject of scientific studies last years. Excitations in such systems may have forms of normal modes. The frequency of the mode can be determined with the use of analytical methods as well as of micromagnetic numerical techniques. One of the latter is the Hamiltonian dynamical matrix method. The method is an analogue of that used to determine oscillations of atoms in crystal. The advantage of it is the possibility of calculating both of all frequencies and profiles of normal modes. Excitations in a finite system can depend on the size of it, as was noticed by experimentalists. The aim of this study is an analysis of eigenstates of the chain consisting of finite numbers of nanomagnets and dependence of states on orientation of magnetic moments and external magnetic field.

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1. Introduction

The dynamics of magnetization in a low-dimensional system of nanomagnets has been an object of many studies in the last decades. This interest is related to the development in magnetic storage and sensors, especially in the case of thin films of magnetic materials [1]. The magnetic elements used for this purpose have the eigenexcitation frequencies within the range of microwave waves (3–30 GHz). To protect such devices from thermal noise, the study of the dynamics of magnetic moments in them is required [2]. A research is also focused on a system build with the use of nanowires, e.g. one-dimensional systems [3, 4].

The calculation of eigenfrequencies for finite size magnets can be done analytically, if the shape of them is ellipsoidal, since for it the demagnetised factors are known. For other shapes one can use numerical methods. One of the latter is micromagnetic technique called Hamiltonian dynamical matrix method (HDMM) [5, 6]. It is similar to the calculation of vibration in crystal and provides a matrix of complex Hermitian eigenvalue problem. The advantage of this method is a possibility to determine all possible frequencies of harmonic oscillations as well as profiles of normal modes just by one-step calculation.

The aim of the paper is to analyse eigenstates of one-dimensional systems consisting of rectangular cells, containing the same magnetic moment. We use HDMM method for this purpose and look for normal modes of excitation and their dependence on orientation of magnetic moments and a direction of the magnetic field. According to this method of analysis we include in the model only interactions between neighbouring moments and interactions of the moments with the magnetic field. The material constants are assumed to be 1 in appropriate units.

2. HDMM method for one-dimensional system

We consider the row of rectangular cells placed along line, let us say in y direction. Cells are numbered from 1 to N. Each cell contains the reduced magnetic moment $\boldsymbol{m}_k = \boldsymbol{M}_k/M_s$, where $\boldsymbol{M}_s$ is a saturation magnetisation and $k = 1, 2, \ldots N$. In spherical coordinate system (r, θ, ϕ) the vector $\boldsymbol{m}_k$ can be written as

$$\boldsymbol{m}_k(t) = (\sin\theta_k(t)\cos\phi_k(t), \sin\theta_k(t)\sin\phi_k(t), \cos\theta_k(t)). \quad (1)$$

The starting point for the method is determination of an equilibrium state. We assume that this state is found (for example using the micromagnetic methods based on the numerical codes [7]) and described by the set of angles θ_k^0, ϕ_k^0 related to each moment in cell k (see Fig. 1).

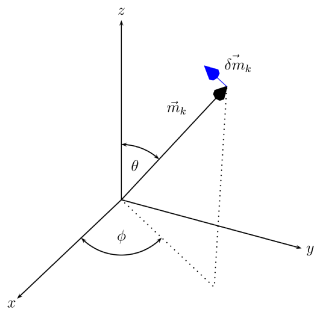

Fig. 1. The magnetisation vector $\boldsymbol{m}_k$. The symbol $\boldsymbol{\delta} m_k$ denotes the small change of the magnetisation being the result of interaction between neighbouring moments and with the magnetic field.

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We restrict ourself to the small excitation from equilibrium state. In this case the change in position of magnetic moments can be described as

$$\theta_k(t) = \theta_k^0 + \delta\theta_k(t), \quad \phi_k(t) = \phi_k^0 + \delta\phi_k(t), \tag{2}$$

where $\delta\theta_k(t)$ and $\delta\phi_k(t)$ are small changes of angles. These changes of positions result in change of energy density E (i.e. energy per unit volume). The energy is a sum of interaction between magnetic moments and the interaction of each moment with an external magnetic field $\boldsymbol{H}$. The former can be defined as

$$E_{exch} = A\sum_{k=1}^{N}\sum_{n=1}^{2}\frac{1-\boldsymbol{m}_k\cdot\boldsymbol{m}_n}{a_{kn}^2}, \tag{3}$$

whereas the latter is given by the formula

$$E_{ext} = -\mu_0 M_s \boldsymbol{H}\cdot\sum_{k=1}^{N}\boldsymbol{m}_k. \tag{4}$$

The symbol A denotes the exchange coupling constant and a_{kn} is related to the distance between moments $\boldsymbol{m}_k$ and $\boldsymbol{m}_n$. The second sum in Eq. (3) runs over nearest neighbours of $\boldsymbol{m}_k$ (two of them in a linear chain). Choosing the change in angles $\delta\theta_k, \delta\phi_k$ as generalized coordinates and accompanying them with appropriate generalised momenta, we can write the Hamilton equation for each cell. Assuming that deviation from equilibrium can be described using the exponential form $\exp(\mathrm{i}\omega t)$, we can finally end up with the set of equations forming the following eigenvalue problem [6]:

$$C\nu = \tilde{\lambda}\nu, \tag{5}$$

where C is a matrix with non-zero elements defined as follows:

$$C_{2k-1,2l-1} = -\frac{E_{\theta_k\phi_l}}{\sin\theta_k}, \quad C_{2k-1,2l} = -\frac{E_{\theta_k\theta_l}}{\sin\theta_k}$$
$$C_{2k,2l-1} = \frac{E_{\phi_k\phi_l}}{\sin\theta_k}, \quad C_{2k,2l} = \frac{E_{\phi_k\theta_l}}{\sin\theta_k}. \tag{6}$$

ν is an eigenvector containing changes of angles $\nu = (\delta\phi_1, \delta\theta_1,\ \delta\phi_2, \delta\theta_2, \ldots \delta\phi_N, \delta\theta_N)^T$. The symbol $E_{\alpha_k\beta_n}$ denotes derivatives of a total energy density respect to angles α and β, taken for the value defined in cell k and n, respectively. Indices k and l belong to the set $\{1, 2, \ldots, N\}$. The eigenvalue $\tilde{\lambda}$ is related to the eigenfrequency by the formula $\tilde{\lambda} = \mathrm{i}\frac{M_s}{\gamma}\omega$, where γ is a gyromagnetic ratio.

The solution of eigenvalue problem (5) provides the eigenfrequency and the corresponding profile of a normal mode. The energy of the excitation is given by formula $E = E_0 + \frac{1}{2}\sum_{n=1}^{N}\sum_{l=1}^{N}(E_{\phi_n\phi_l}\delta\phi_n\delta\phi_l + 2E_{\phi_n\theta_l}\delta\phi_n\delta\theta_l + E_{\theta_n\theta_l}\delta\theta_n\delta\theta_l)$.

3. Normal modes for a finite chain

We focus our attention on the chain of nanomagnets with finite size and additionally assume periodic boundary conditions. The derivatives of interaction energies (3) and (4) for such a system can be represented by appropriate derivatives of magnetic moments $\boldsymbol{m}_k$. The latter are given by the formulae

$$\frac{\partial^2 E_{exch}}{\partial\alpha_k\partial\beta_l} = \begin{cases} \dfrac{-2A}{a^2}\displaystyle\sum_{n=1}^{2}\frac{\partial^2\boldsymbol{m}_k}{\partial\alpha_k\partial\beta_l}\cdot\boldsymbol{m}_n, & l = k \\ \dfrac{-2A}{a^2}\dfrac{\partial\boldsymbol{m}_k}{\partial\alpha_k}\cdot\dfrac{\partial\boldsymbol{m}_l}{\partial\beta_l}, & \begin{array}{l} l,k \text{ denote} \\ \text{nearest} \\ \text{neigh-} \\ \text{bours} \end{array} \\ 0 & \begin{array}{l} k \neq l \text{ and} \\ \text{do not de-} \\ \text{note near-} \\ \text{est neigh-} \\ \text{bours} \end{array} \end{cases}, \tag{7}$$

$$\frac{\partial^2 E_{ext}}{\partial\alpha_k\partial\beta_l} = \begin{cases} -\mu_o M_s\boldsymbol{H}\cdot\dfrac{\partial^2\boldsymbol{m}_k}{\partial\alpha_k\partial\beta_l} & l = k, \\ 0 & l \neq k. \end{cases}. \tag{8}$$

The calculation was done for two types of orientations of magnetics moments $\boldsymbol{m}_k$. The first type corresponds to the system with the same orientation of all magnetic moments given by the angles θ and ϕ. The another one is related to the case, where half of the chain has one orientation θ_j^1, ϕ_j^1, $j = 1, 2, \ldots N'$, and the rest of it has another orientation θ_j^2, ϕ_j^2, $j = N', N'+1, \ldots N$.

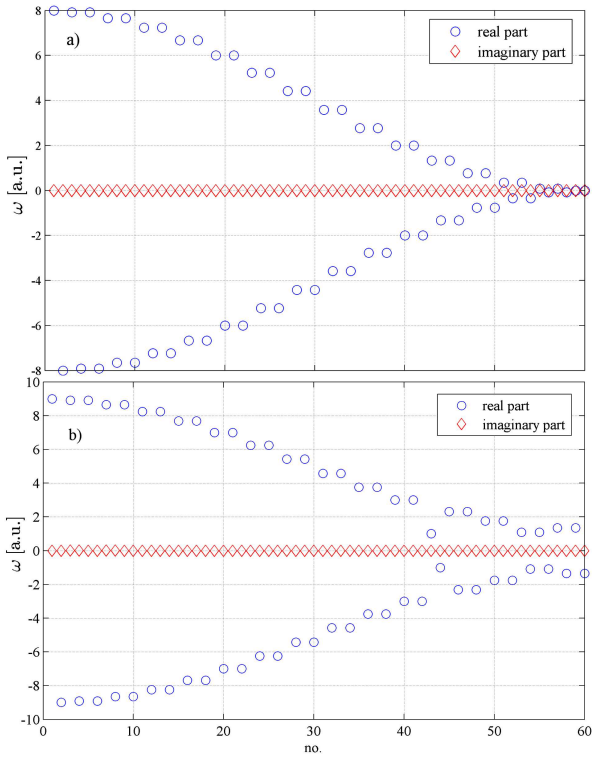

Fig. 2. The eigenfrequencies ω of normal modes for a chain of nanomagnets with (b) and without (a) magnetic field. All magnetic moments $\boldsymbol{m}_k$ are parallel. The label "no." denotes the number of solution of eigenvalue problem.

In Fig. 2 we present the frequencies of normal modes for chain of $N = 30$ nanomagnets with all magnetic mo-

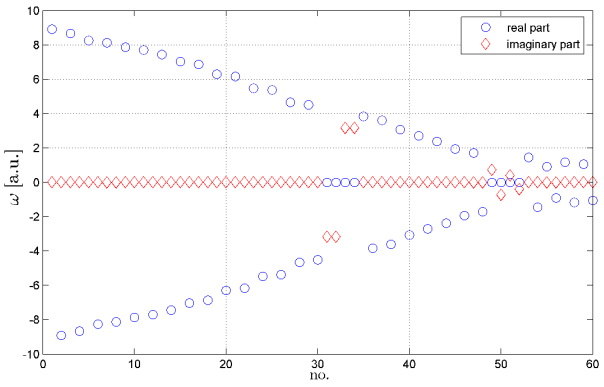

Fig. 3. The eigenfrequencies ω of normal modes for the chain of nanomagnets with two different orientations of magnetic moments (half of the chain with one orientation and half with another one). The label "no." denotes the number of solution of eigenvalue problem.

ments parallel to each other (the angles are given as follows: $\theta_k = \pi/2$, $\phi_k = \pi/2$, $k = 1, 2, \ldots N$). Part (a) represents the case without magnetic field, whereas part (b) corresponds to presence of the magnetic field, with direction given by angles $\theta_B = \pi/2$, $\phi_B = \pi/2$. The strength of the magnetic field is taken as 1 in an appropriate unit. The main difference between the presence and the absence of the magnetic field is lack of zero frequency mode in the former case.

The allowed frequencies for two different orientations of magnetic moments along a chain are presented in Fig. 3. Half of the chain has orientation given by angles $\theta_k = \pi/2$, $\phi_k = \pi/2$, $k = 1, 2, \ldots N'$, whereas for another half angles are $\theta_k = \pi/7$, $\phi_k = \pi/7$, $k = N', N'+1, \ldots N$. We observe for this case, in the presence of the magnetic field (with orientation the same as in a homogeneous chain), that there are frequencies with imaginary part different from zero. They correspond to localised excitations, which vanish along the chain. It was proven that the values of these frequencies depend on the orientation of magnetic moments within two parts of the chain.

4. Conclusions

The HDMM method allows us to calculate all possible frequencies for given orientation of a magnetic field and magnetic moments as well as to determine the profile of excitations. The frequencies for the homogeneous chain (the same orientation of magnetic moments) have real values which correspond to waves travelling across system.

In the chain consisting of two sub-chains, with different orientations of $\boldsymbol{m}_k$, we observe other solutions. There exist modes with complex values of frequencies ω, which are related to localised excitations. This way, by manipulation of magnetic moment orientations, it is possible to extinguish selected frequency mode.

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