Enhancement of the Hong–Mandel Higher-Order Squeezing and Amplitude Odd-Power Squeezing in Even Coherent State by its Superposition with Vacuum State

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We study the Hong–Mandel $2n$ th-order squeezing of the Hermitian operator, $X_\theta \equiv X_1 \cos\theta + X_2 \sin\theta$ and amplitude n th-power squeezing of the Hermitian operator, $Y_\theta^{(n)} \equiv Y_1^{(n)} \cos\theta + Y_2^{(n)} \sin\theta$ in superposed state $|\psi\rangle = K[|\alpha,+\rangle + r\,\mathrm{e}^{\mathrm{i}\varphi}|0\rangle]$, of vacuum state and even coherent state defined by $|\alpha,+\rangle = K_+[|\alpha\rangle + |-\alpha\rangle]$. Here operators $X_{1,2}$ are defined by $X_1 + \mathrm{i}\,X_2 = a$, operators $Y_{1,2}^{(n)}$ are defined by $Y_1^{(n)} + \mathrm{i}\,Y_2^{(n)} = a^n$, a is the annihilation operator, α, θ, r and φ are arbitrary and the only restriction on these is the normalization condition of the superposed state. We show that the Hong–Mandel $2n$ th-order squeezing and amplitude odd-power squeezing exhibited by even coherent state enhance in its superposition with vacuum state. Variations of these higher-orders squeezing with different parameters near its maxima have also been discussed.

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1. Introduction

In quantum optics, much attention is being paid to non-classical features [1] of a state, which cannot be explained on the basis of classical probability concepts. The non-classical features of a quantum state can be manifested in different ways like squeezing, anti-bunching, sub-Poissonian photon statistics and various kinds of squeezing etc. Earlier study of such non-classical features was largely in academic interest [2, 3], but now their applications in quantum information theory such as communication [4], quantum teleportation [5], dense coding [6] and quantum cryptography [7] are well realized. It has demonstrated that non-classicality is the necessary input for entangled state [8].

Squeezing, a well-known non-classical effect, is a phenomenon in which variance in one of the quadrature components becomes less than that in vacuum state or coherent state [9] at the cost of increased fluctuations in the other quadrature component. This definition of squeezing has been generalized to case of several variables [10–13]. Hong and Mandel [10] introduced the concept of higher-order squeezing by considering the $2n$ th order moments of the quadrature component and defined a state to be $2n$ th order squeezed if the expectation value of the $2n$ th power of the difference between a field quadrature and its average value is less than what it would be in a coherent state. According to the Hong and Mandel definition [10], a state $|\psi\rangle$ is said to be $2n$ th-order squeezed for the operator,

$$X_\theta = X_1 \cos\theta + X_2 \sin\theta, \tag{1}$$

if the $2n$ th-order moment of X_θ,

$$\langle\psi|\,(\Delta X_\theta)^{2n}\,|\psi\rangle < (2n-1)!!/2^{2n}. \tag{2}$$

Here Hermitian operators $X_{1,2}$ are defined by $X_1 + \mathrm{i}\,X_2 = a$, a is the annihilation operator, θ is an arbitrary angle, $\Delta X_\theta = X_\theta - \langle\psi|\,X_\theta\,|\psi\rangle$ and $(2n-1)!!$ is product of factors, starting with $(2n-1)$ and decreasing in steps of 2 and ending at 1.

Another form of higher-order squeezing in terms of real and imaginary parts of square of the amplitude, the so-called "amplitude-squared squeezing", has been proposed by Hillery [11] by considering the operators Y_1 and Y_2, such that $Y_1 + \mathrm{i}\,Y_2 = a^2$, a is annihilation operator. Hillery introduced another type of higher-order squeezing, called sum squeezing and difference squeezing [11] by considering two mode systems and using sum and differences of various bilinear combinations constructed from the creation and annihilation operators. Zhang et al. [12] generalized amplitude-squared squeezing defined by Hillery [11] to amplitude n th-power squeezing. According to Zhang et al. definition [12], a state $|\psi\rangle$ is said to be amplitude n th-power squeezed for the operator,

$$Y_\theta^{(n)} \equiv Y_1^{(n)} \cos\theta + Y_2^{(n)} \sin\theta, \tag{3}$$

if the n th-order moment of $Y_\theta^{(n)}$,

$$\langle\psi|\,(\Delta Y_\theta^{(n)})^2\,|\psi\rangle < \tfrac{1}{4}\left|\langle\psi|\,[Y_\theta^{(n)}, Y_{\theta+\pi/2}^{(n)}]\,|\psi\rangle\right|. \tag{4}$$

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Here $Y_1^{(n)} + \mathrm{i}Y_2^{(n)} = a^n$, $\Delta Y_\theta^{(n)} = Y_\theta^{(n)} - \langle\psi| Y_\theta^{(n)} |\psi\rangle$, $[Y_\theta^{(n)}, Y_{\theta+\pi/2}^{(n)}] = \sum_{r=1}^{n} ({}^nC_r)^2 r! a^{\dagger(n-r)} a^{(n-r)}$ and θ is an arbitrary angle.

The Hong and Mandel higher-order squeezing [10] is quite distinct from ordinary and higher-order squeezing defined by authors [11, 12] because such squeezing does not require that the uncertainty product be a minimum and therefore both quadratures of the field can have higher-order squeezing simultaneously [14]. In other words, states exist for which product of higher-order fluctuations of both quadrature $\langle\psi| (\Delta X_\theta)^n |\psi\rangle \langle\psi| (\Delta X_{\theta+\pi/2})^n |\psi\rangle$, takes a value less than that for a coherent state. Lynch et al. [15] studied the minimization of product of higher-order fluctuations of both quadratures numerically. Recently we have studied [16] simultaneous occurrence of the Hong–Mandel higher-order squeezing of both quadrature components in orthogonal even coherent state.

Many different schemes of generating such non-classical states have been proposed [17–21] such as four-wave mixing, resonance fluorescence, the use of free electron laser, cavities, harmonic generation, parametric amplification, and JC model. Other possibilities for generating such effect have been proposed by superposition of two or more coherent states. It has been realized that a coherent state does not exhibit non-classical effects but the superposition of two or more coherent state exhibit [22–29] various non-classical effects like squeezing, antibunching, higher-order squeezing and higher-order sub-Poissonian statistics etc. Buzek et al. [22] and Xia and Guo [25] studied such effects in the superposition of two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ and reported that the even coherent state $\sim (|\alpha\rangle + |-\alpha\rangle)$ exhibits squeezing but not sub-Poissonian statistics while the odd coherent state $\sim (|\alpha\rangle - |-\alpha\rangle)$ exhibits sub-Poissonian statistics but not squeezing. Xia and Guo also studied [25] such effects in the displaced even and odd coherent state. Schleich et al. [23] studied such effects in the superposition of two coherent states, $|\alpha\rangle$ and $|\alpha^*\rangle$, and reported that such superposition can exhibit both squeezing and sub-Poissonian statistics when $|\alpha|^2 \gg 1$. Recently we reported [27–29] several non-classical features in superposition of two arbitrary coherent states. In practice, the superposition of coherent states can be generated in interaction of coherent state with nonlinear media [30] and in quantum non-demolition techniques [31].

In this paper we study the Hong–Mandel $2n$ th-order squeezing of the operator X_θ and amplitude n th-power squeezing of the operator $Y_\theta^{(n)}$ in the superposed state,

$$|\psi\rangle = K[|\alpha, +\rangle + r\,\mathrm{e}^{-\mathrm{i}\varphi}\,|0\rangle];$$

$$K = [1 + r^2 + 4rK_+ \cos\varphi\, \mathrm{e}^{-|\alpha|^2/2}]^{-1/2} \quad (5)$$

of vacuum state and even coherent state defined by $|\alpha, +\rangle = K_+[|\alpha\rangle + |-\alpha\rangle]$; $K_+ = [2(1 + \mathrm{e}^{-2|\alpha|^2})]^{-1/2}$. Here parameters α, θ, r and φ are arbitrary and the only restriction on these is the normalization condition of the superposed state. We show that the Hong–Mandel $2n$ th-order squeezing and amplitude odd-power squeezing exhibited by even coherent state enhance in its superposition with vacuum state. Variations of these higher-orders squeezing with different parameters near its maxima have also been discussed.

2. The Hong–Mandel higher-order squeezing of X_θ in the superposed state $|\psi\rangle$

A single mode coherent state $|\alpha\rangle$ defined by $a\,|\alpha\rangle = \alpha\,|\alpha\rangle$ can be written as

$$|\alpha\rangle = \exp(-1/2\,|\alpha|^2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle = D(\alpha)\,|0\rangle\,. \quad (6)$$

Here $\alpha = A\,\mathrm{e}^{-\mathrm{i}\theta_\alpha}$, $|n\rangle$ is the occupation number and $D(\alpha) = \exp(\alpha a^+ - \alpha^* a)$ is the displacement operator. Since $\mathrm{e}^{-\mathrm{i}\theta N} a\,\mathrm{e}^{-\mathrm{i}\theta N} = a\,\mathrm{e}^{-\mathrm{i}\theta}$ and $|\alpha\rangle = \mathrm{e}^{-\mathrm{i}\theta_\alpha N}\,|A\rangle$, where $N = a^+a$, we have

$$\langle\psi|\,(\Delta X_{\theta,|\psi\rangle})^{2n}\,|\psi\rangle = \langle\psi_1|\,(\Delta X_{\delta,|\psi_1\rangle}. \quad (7)$$

Here $|\psi\rangle = \mathrm{e}^{-\mathrm{i}\theta_\alpha N}\,|\psi_1\rangle$, $|\psi_1\rangle = K[|A, +\rangle + r\,\mathrm{e}^{-\mathrm{i}\phi}|0\rangle]$, $|A, +\rangle = K_+[|A\rangle + |-A\rangle]$, $\Delta X_{\theta,|\psi\rangle} = X_\theta - \langle\psi|\,X_\theta\,|\psi\rangle$, $\Delta X_{\delta,|\psi\rangle\psi_1\rangle} = X_\delta - \langle\psi_1|X_\delta|\psi_1\rangle$ and $\delta = \theta_\alpha - \theta$. Now we have

$$a\,|A, +\rangle = KA_+\,|A, -\rangle\,; \quad a^2\,|A, +\rangle = A^2\,|A, +\rangle\,;$$

$$\langle A, +|\,a^+ a\,|A, +\rangle = \tanh A^2. \quad (8)$$

Here $|A, -\rangle = K_-[|A\rangle - |-A\rangle]$, $K_- = [2(1 - \mathrm{e}^{-2A^2})]^{-1/2}$, $A_+ = A(1 - \mathrm{e}^{-2A^2})^{-1/2}$ and $K = [1 - \mathrm{e}^{-4A^2}]^{-1/2}$.

For even m, straightforward calculations lead to

$$\langle A, +|\,a^m\,|A, +\rangle = A^m;$$

$$\langle A, +|\,a^{\dagger m} a^m\,|A, +\rangle = A^{2m}, \quad (9)$$

and

$$\langle 0|\,a^m\,|A, +\rangle = 2K_+ A^m\,\mathrm{e}^{-A^2/2};$$

$$\langle A, +|\,a^m\,|0\rangle = 0. \quad (10)$$

Similarly for odd m, we have

$$\langle A, +|\,a^m\,|A, +\rangle = 0;$$

$$\langle A, +|\,a^{\dagger m} a^m\,|A, +\rangle = A^{2m}\tanh A^2, \quad (11)$$

and

$$\langle 0|\,a^m\,|A, +\rangle = 0; \quad \langle A, +|\,a^m\,|0\rangle = 0. \quad (12)$$

We have also

$$\langle 0|\,a^{\dagger m} a^m\,|A, +\rangle = 0; \quad \langle A, +|\,a^{\dagger m} a^m\,|0\rangle = 0. \quad (13)$$

Hence we finally get, for even m

$$\langle\psi_1|\,a^m\,|\psi_1\rangle = K^2 A^m \left[2rK_+\,\mathrm{e}^{-A^2/2}\,\mathrm{e}^{\mathrm{i}\phi} + r^2\right], \quad (14)$$

$$\langle\psi_1|\,a^{\dagger m} a^m\,|\psi_1\rangle = K^2 r^2 A^{2m}, \quad (15)$$

and for odd m,

$$\langle\psi_1|\,a^m\,|\psi_1\rangle = 0, \quad (16)$$

$$\langle\psi_1|\,a^{\dagger m} a^m\,|\psi_1\rangle = K^2 r^2 A^{2m}\tanh A^2. \quad (17)$$

Now, the $2n$ th-order moment of X_δ can be written as

$$\langle\psi_1|\,(\Delta X_\delta)\,|\psi_1\rangle = \sum_{i=0}^{n-1} \frac{2n!}{2^{3i}(2n-2i)!i!}$$

$$\times \langle\psi_1| : (\Delta X_\delta)^{2n-2i} : |\psi_1\rangle + \frac{(2n-1)!!}{2^{2n}}. \tag{18}$$

Since $\langle\psi_1| : (\Delta X_\delta)^{2n-2i} : |\psi_1\rangle = 0$, therefore we have

$$\langle\psi_1| (\Delta X_\delta) |\psi_1\rangle = \sum_{i=0}^{n-1} \frac{2n!}{2^{3i}(2n-2i)!i!}$$
$$\times \langle\psi_1| : (\Delta X_\delta)^{2n-2i} : |\psi_1\rangle + \frac{(2n-1)!!}{2^{2n}}. \tag{19}$$

Now, tedious but straightforward calculations finally lead to

$$\langle\psi_1| : X_{\delta,}^{2n-2i} : |\psi_1\rangle = K^2|\alpha|^{2n-2i}$$
$$\times \left(\frac{\cos^{2n-2i}\delta + (-1)^{n-1}\sin^{2n-2i}\delta\,\mathrm{e}^{-2|\alpha|^2}}{2\cosh|\alpha|^2\,\mathrm{e}^{-2|\alpha|^2}} + \frac{r\cos((n-i)\delta-\phi)}{2^{2n-2i-1}} \right), \tag{20}$$

For simplicity we define squeezing factor,

$$S_{2n} = \frac{\langle\psi_1| (\Delta X_\delta)^{2n} |\psi_1\rangle - 2^{-2n}(2n-1)!!}{-2^{-2n}(2n-1)!!}. \tag{21}$$

This should be noted that the Hong–Mandel $2n$th-order squeezing occurs if $-1 \le S_{2n} < 0$. For $S_{2n} < 0$, we can define degree of squeezing by $D_{2n} = -S_{2n}$. Also we can call $100D_{2n}$ as percentage of squeezing. Using computer programming we get maximum the Hong–Mandel higher-order squeezing of X_δ in the superposed state $|\psi_1\rangle$ by minimizing S_{2n} with respect to the parameters $|\alpha|$, δ, r, and φ. The minimum value $(S_{2n})_{\min}$ of squeezing factor S_{2n}, and the values of $|\alpha|$, δ, r, and φ at which $(S_{2n})_{\min}$ occurs are reported in the following Table I.

TABLE I

Numerical values of $(S_{2n})_{\min}$ for even coherent state (left) and superposed states of even coherent state and vacuum state (right).

Order $2n$		$\delta = \pm\frac{\pi}{2}$		$\delta = \pm\frac{\pi}{2}$, $\varphi = 0$		
		$\lvert\alpha\rvert$	$(S_{2n})_{\min}$	$\lvert\alpha\rvert$	r	$(S_{2n})_{\min}$
second	2	0.80	–0.5569	1.60	1.07	–0.7404
fourth	4	0.65	–0.7296	1.30	1.10	–0.8895
sixth	6	0.56	–0.8084,	1.12	1.06	–0.9408
eighth	8	0.51	–0.8524	1.02	1.09	–0.9632

We note that the Hong–Mandel higher-order squeezing increases with increase of the order $2n$ of squeezing for even coherent state and also for superposition of even coherent state with vacuum state but higher-order squeezing exhibited by even coherent state enhances in its superposition with vacuum state. It can also be noted that $(S_{2n})_{\min}$ decreases with increase of the order $2n$ of squeezing and therefore we conclude any large amount of higher-order squeezing can be obtained by choosing suitably a large $2n$. We also note that we get more than 96% higher-order squeezing for orders greater than sixth in superposition of even coherent state with vacuum state. Variations of squeezing factor S_{2n} for ordinary, fourth-order, sixth-order, eighth-order, i.e., $2n = 2$, 4, 6 and 8 with the parameter $|\alpha|$ at $\delta = \frac{\pi}{2}$, $\varphi = 0$ near the minima are shown in Figs. 1–4, respectively.

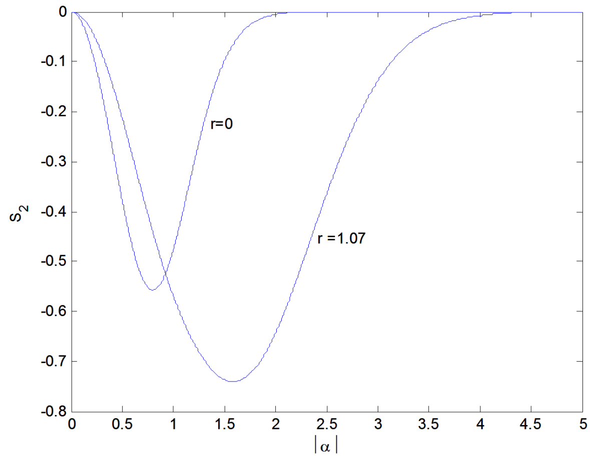

Fig. 1. Variation of S_2 with $|\alpha|$ for even coherent state ($r = 0$) and for superposed state of even coherent state and vacuum state at $\delta = \frac{\pi}{2}$, $\varphi = 0$.

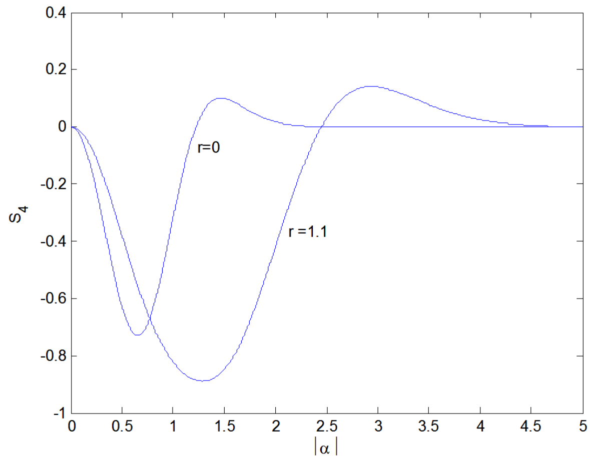

Fig. 2. As in Fig. 1, but for S_4.

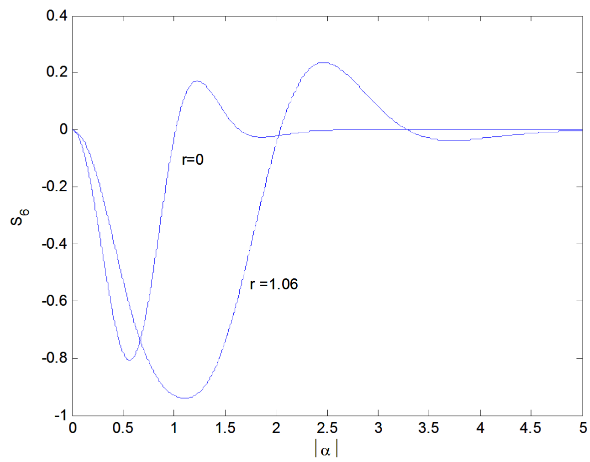

Fig. 3. As in Fig. 1, but for S_6.

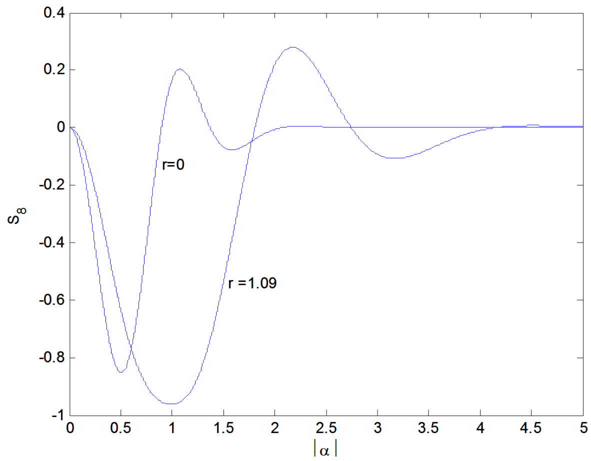

Fig. 4. As in Fig. 1, but for S_6.

3. Amplitude nth-power squeezing of $Y_\theta^{(n)}$ in the superposed state $|\psi\rangle$

Since $e^{-i\theta N} a e^{-i\theta N} = a e^{-i\theta}$ and $|\alpha\rangle = e^{-i\theta_\alpha N}|A\rangle$, where $N = a^+ a$, we have

$$\langle\psi|\,(\Delta Y_\theta^{(n)})^{2n}\,|\psi\rangle = \langle\psi_1|\,(\Delta Y_\delta^{(n)})^{2n}\,|\psi_1\rangle\,. \tag{22}$$

Here $|\psi\rangle = e^{-i\theta_\alpha N}|\psi_1\rangle$, $|\psi_1\rangle = K[|A,+\rangle + r e^{-i\phi}|0\rangle]$, $|A,+\rangle = K_+[|A\rangle + |-A\rangle]$, and $\delta = \theta_\alpha - \theta$. For simplicity we define squeezing factor for amplitude nth power squeezing,

$$(S_a)_n = \frac{\langle\psi_1|\,(\Delta Y_\delta^{(n)})^n\,|\psi_1\rangle - \frac{1}{4}\left|\langle\psi_1|\,[Y_\delta^{(n)}, Y_{\delta+\pi/2}^{(n)}]\,|\psi_1\rangle\right|}{\frac{1}{4}\left|\langle\psi_1|\,[Y_\delta^{(n)}, Y_{\delta+\pi/2}^{(n)}]\,|\psi_1\rangle\right|}. \tag{23}$$

This should be noted that amplitude nth-power squeezing occurs if $-1 \le (S_a)_n < 0$. For $(S_a)_n < 0$, we can define degree of amplitude nth-power squeezing by $(D_a)_n = -(S_a)_n$. Also we can call $100(D_a)_n$ as percentage of amplitude nth-power squeezing. Now we have for even n,

$$\langle\psi_1|\,a^n\,|\psi_1\rangle = K^2|\alpha|^n\left[2rK_+ e^{-|\alpha|^2/2} e^{-i\phi}2 + r^2\right], \tag{24}$$

$$\langle\psi_1|\,a^{\dagger n} a^n\,|\psi_1\rangle = K^2 r^2 |\alpha|^{2n}, \tag{25}$$

and for odd n,

$$\langle\psi_1|\,a^n\,|\psi_1\rangle = 0, \tag{26}$$

$$\langle\psi_1|\,a^{\dagger n} a^n\,|\psi_1\rangle = K^2 r^2 |\alpha|^{2n}\tanh|\alpha|^2. \tag{27}$$

Using Eqs. (23)–(27) and computer programming we get maximum amplitude nth-power squeezing of Y_δ in the superposed state $|\psi_1\rangle$ by minimizing $(S_a)_n$ with respect to the parameters $|\alpha|$, δ, r and φ. The minimum value $(S_a)_{n,\min}$ of amplitude nth-power squeezing factor $(S_a)_n$, and the values of $|\alpha|$, δ, r and φ at which $(S_a)_{n,\min}$ occurs are reported in the following Table II.

We conclude that amplitude nth-power squeezing in even coherent state and superposition state of even coherent state with vacuum state occurs only for odd n. We also note that amplitude odd-power squeezing increases with increasing the order n of squeezing for even coherent state and also for superposition of even coherent state with vacuum state. This should also be noted that amplitude odd-power squeezing exhibited by even coherent state enhances in its superposition with vacuum state. Variations of squeezing factor $(S_a)_n$ for first, second, third, and fourth-power, i.e., $n = 1$, 2, 3 and 4 with the parameter $|\alpha|$ at $\delta = \pi/2$, $\varphi = 0$ with $r = 0$ and $r = 1$ are shown in Figs. 5–8, respectively.

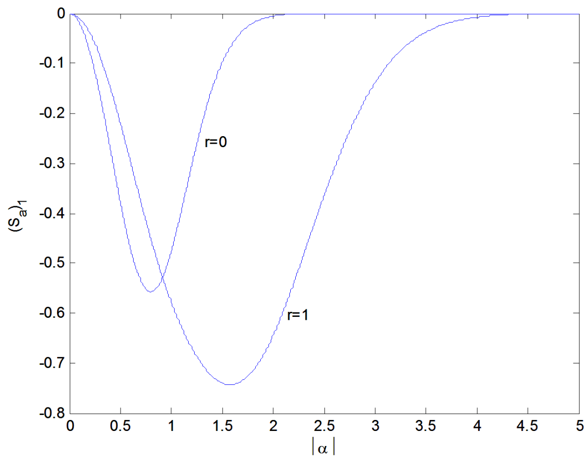

Fig. 5. Variation of $(S_a)_1$ with $|\alpha|$ for even coherent state ($r = 0$) and for superposed state of even coherent state and vacuum state at $\delta = \frac{\pi}{2}$, $\varphi = 0$.

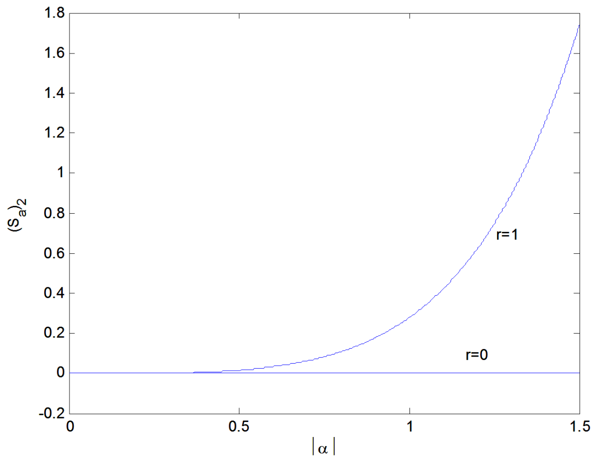

Fig. 6. As in Fig. 5, but for $(S_a)_2$.

TABLE I

Numerical values of $(S_a)_{n,\min}$ for even coherent state (left) and superposed states of even coherent state and vacuum state (right).

Order $2n$		$\delta = \pm\frac{\pi}{2}$		$\delta = \pm\frac{\pi}{2}, \varphi = 0, r = 1$	
		$\lvert\alpha\rvert$	$(S_a)_{n,\min}$	$\lvert\alpha\rvert$	$(S_a)_{n,\min}$
first	1	0.80	–0.5569	1.60	–0.7404
second	2		no squeezing		
third	3	0.56	–0.8084	1.12	–0.9408
fourth	4		no squeezing		

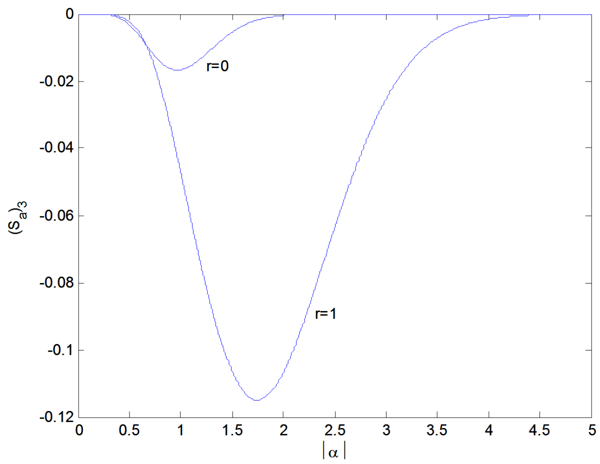

Fig. 7. As in Fig. 5, but for $(S_a)_3$.

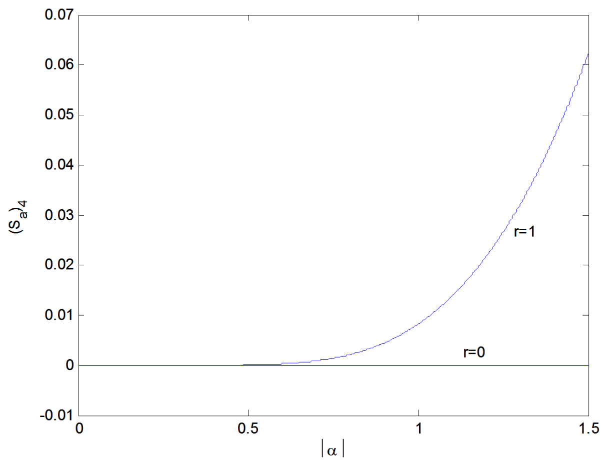

Fig. 8. As in Fig. 5, but for $(S_a)_4$.

4. Conclusions

In the present paper we investigated the Hong–Mandel $2n$ th-order squeezing of the operator $X_\theta \equiv X_1 \cos\theta + X_2 \sin\theta$ and amplitude n th-power squeezing of the operator $Y_\theta^{(n)} \equiv Y_1^{(n)} \cos\theta + Y_2^{(n)} \sin\theta$ in superposed state, $|\psi\rangle = K[|\alpha,+\rangle + r\,\mathrm{e}^{-\mathrm{i}\varphi}\,|0\rangle]$, of vacuum state and even coherent state defined by $|\alpha,+\rangle = K_+\,[|\alpha\rangle + |-\alpha\rangle]$. Here operators $X_{1,2}$ are defined by $X_1 + \mathrm{i}X_2 = a$, operators $Y_{1,2}^{(n)}$ are defined by $Y_1^{(n)} + Y_2^{(n)} = a^n$, a is the annihilation operator, α, θ, r and φ are arbitrary and the only restriction on these is the normalization condition of the superposed state $|\psi\rangle$. We conclude that the Hong–Mandel $2n$ th-order squeezing and amplitude odd-power squeezing exhibited by even coherent state enhance in its superposition with vacuum state. It has also been concluded that any large amount of the Hong–Mandel $2n$ th-order squeezing and amplitude n th-power squeezing can be obtained by choosing suitably a large n. We also note that we get more than 96% the Hong–Mandel $2n$ th-order squeezing and more than 94% amplitude n th-order squeezing for n greater than 3. Variations of these higher-orders squeezing with different parameters near its maxima have also been discussed.

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