Solution of Material Selection Problem Using Fuzzy Axiomatic Design and DEMATEL Methods

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Material selection is an important task for dental healthcare. A teeth can be filled with several filling materials such as a silver amalgam (mixed with silver, tin, zinc, and copper), a plastic, a porcelain, and composites. Choosing the best filling material is a complex process for a dentist because of human healthcare. There are many criteria for the selection process such as aesthetics, resistant, strength, rigid, long life, low cost, healthy. One of the novelty of this research that is the first research in the field of selecting dental material and another novelty that is used fuzzy axiomatic design and fuzzy DEMATEL methods together first time. First of all, the most effective criteria are determined in order to choose the best material. The weights of the criteria are determined using interaction among themselves. Interactions among criteria are found using DEMATEL method with taking into consideration of dentists' experiences on a teeth filling. Secondly, evaluating alternative materials is carried out using fuzzy axiomatic design method. The results illustrate that the proposed approach is suitable for selecting materials of a teeth filling. We believe that the approach can be applicable for other material selection problems.

1. Introduction

Dental filling is a restoration process for teeth damages due to a tooth decay. Dental health is directly related to healthcare, so selecting the best filling material is an important task. There are a few filling materials such as Ag–Sn (amalgam), Ni–Cr composites, Co–Cr composites, Au–Pt composites, composite resin (plastic–SiO_2). Primarily effective criteria should be determined in order to choose a suitable material. Long life (C1), healthy (C2), repairability (C3), strength (C4), heat resistance (C5), aesthetic (C6), formability (C7), corrosion resistance (C8), biocompatibility (C9), cost-effective (C10), easy access (C11), easy storage (C12), eco-friendly (C13), versatility (C14), rigid (C15) were considered for general filling material selecting problems. A novel approach is proposed to select a dental filling material in this study. We believe that our proposed model consists of two novelties in the literature. Our paper is first study deal with a dental filling material selection problem. The other one is that we used fuzzy axiomatic design and fuzzy DEMATEL methods together first time.

According to the literature review, 15 material selection criteria were determined to use for dental filling. Unlike other studies, the weights of these criteria were determined by considering interaction among themselves. When determining the weights of criteria, interactions of criteria were evaluated separately. The process of evaluating the criteria interactions was performed with specialist dentists in order to obtain useable criteria weights. The fuzzy decision making trial and evaluation laboratory (DEMATEL) method was preferred to determine criteria weights, because of considering criteria interactions and the criteria were fuzzy. After determining the weights of selection criteria, making a selection from the alternative materials is required. The evaluation process of the alternative materials should be made as fuzzy and based on the experience by the reason of fuzzy criteria. Based on this, fuzzy axiomatic design method was preferred to making a selection from the alternative materials. Triangular fuzzy expression design patterns were prepared with the same specialist dentists.

2. Method and application

2.1. Determining the criteria weights by fuzzy DEMATEL method

DEMATEL method is presented by Fontela and Gabus [1] which involves the complex causal relationships between criteria to create a structural model and a comprehensive method for analyzing. DEMATEL method divides the criteria into two groups particularly as a cause and an effect groups. Determining affecting or being affected criteria is an important step to solve complex problems. In our study, we calculated the importance of our decision criteria weights using the finite-difference (FD) method. The steps of the determining weights of criteria were the following:

Step 1: The triangular fuzzy numbers were determined corresponding to fuzzy expressions as in Table I.

Step 2: 88 different linguistic direct relation matrices were constituted by four dentists by considering the determined 15 criteria in our problem. One of them is shown in Table II. Then these matrices were converted to triangular fuzzy numeric form as Eq. (1) where $\tilde{z}_{ij} = (z_{ij,l} z_{ij,m} z_{ij,u})$.

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TABLE I

Triangular fuzzy numbers corresponding to fuzzy expressions.

Fuzzy expressions	Triangular fuzzy numbers $(l,\ m,\ u)$
Very high influence (VH)	(0.75, 1.0, 1.0)
High influence (H)	(0.5. 0.75, 1.0)
Low influence (L)	(0.25, 0.5. 0.75)
Very low influence (VL)	(0, 0.25, 0.50)
No influence (NO)	(0, 0, 0.25)

TABLE II

An example of linguistic direct relation matrix for Ag–Sn.

RM_1E_1	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1	–	VH	VL	H	VL	VL	VL	H	L	VH	NO	NO	NO	VL	NO
C2	VL	–	VH	VH	H	VH	H	VL	H	H	NO	VL	NO	L	NO
C3	H	VH	–	H	H	H	VH	L	L	VL	NO	H	NO	L	NO
C4	VL	H	VL	–	L	VL	NO	NO	VL	VL	NO	H	NO	NO	NO
C5	VL	H	VL	VH	–	VH	L	NO	VH	H	NO	L	NO	L	NO
C6	H	VH	L	H	VL	–	L	NO	VL	VH	NO	VL	NO	VL	NO
C7	L	VH	H	H	VL	L	–	NO	L	H	NO	VH	NO	VL	NO
C8	VL	VL	H	L	VL	H	H	–	VL	VL	NO	VL	NO	VL	NO
C9	VL	H	VL	VL	H	H	L	NO	–	VH	NO	H	NO	NO	NO
C10	VL	VL	VL	L	L	VL	VL	VL	VH	–	NO	VH	NO	NO	NO
C11	L	H	H	VH	VH	H	VH	VL	VH	L	–	L	VH	VL	NO
C12	VL	H	L	L	L	VL	L	NO	L	H	NO	–	NO	VL	NO
C13	VH	H	VL	VH	VL	L	VL	VL	L	VH	VH	L	–	VL	VH
C14	H	H	VL	L	H	VL	VL	VL	L	L	NO	L	NO	–	NO
C15	VH	H	VL	VH	VL	VL	VL	VL	VL	VH	NO	VL	NO	VL	–

$$\tilde{Z} = \begin{bmatrix} 0 & \tilde{z}_{12} & \dots & \tilde{z}_{1n} \\ \tilde{z}_{21} & 0 & \dots & \tilde{z}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{n1} & \tilde{z}_{n2} & \dots & 0 \end{bmatrix}. \tag{1}$$

Step 3: Linguistic direct relation matrices were arranged in direct relation matrices to express a single matrix of different linguistic expert assessment. Afterwards by taking the arithmetic mean of 88 matrices they were reduced to a single evaluation matrix.

Step 4: The direct relation matrix was normalized as Eq. (2), where Eq. (3) and Eq. (4). Obtained single evaluation matrix was arranged to normalized direct relation matrix using Eqs. (3) and (4).

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{nn} \end{bmatrix}, \tag{2}$$

where

$$\tilde{x}_{ij} = \left(\frac{z_{ij,l}}{r_l}, \frac{z_{ij,m}}{r_m}, \frac{z_{ij,u}}{r_u} \right), \tag{3}$$

and

$$r_s = \max \left(\sum_{i=1, 1\le i \le n}^{n} z_{ij,s} \right) \quad \forall s = l, m, u. \tag{4}$$

Step 5: Total relation fuzzy matrix as seen in Eq. (5) was calculated. At first $\tilde{X}$ matrix is separated to sub-matrices as X_l, X_m, X_u. Let $t_{ij} = (t_{ij,l}, t_{ij,m}, t_{ij,u})$ where $t_{ij,s} = X_s(I - X_s)^{-1} \forall s = l, m, u$.

$$\tilde{T} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \dots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \dots & \tilde{t}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{n1} & \tilde{t}_{n2} & \dots & \tilde{t}_{nn} \end{bmatrix}. \tag{5}$$

Step 6: D_i and R_i vectors as Eq. (6) and Eq. (7) was calculated. (D_i is total row values and R_i is total column values of the $\tilde{T}$ matrix.

$$\tilde{D}_i = \sum_{j=1}^{n} \tilde{t}_{ij} \quad (i = 1, 2, \dots, n), \tag{6}$$

$$\tilde{R}_i = \sum_{i=1}^{n} \tilde{t}_{ij} (j = 1, 2, \dots, n). \tag{7}$$

Step 7: D_i and R_i vectors were defuzzified using Eq. (8).

$$\tilde{D}_i^{\text{def}} \text{ or } \tilde{R}_i^{\text{def}} = \begin{cases} u - \sqrt{\frac{(u-l)(u-m)}{2}}, & u-m > m-l, \\ \sqrt{\frac{(u-l)(u-m)}{2}} - 1, & u-m < m-l, \\ m, & \text{otherwise.} \end{cases} \tag{8}$$

Step 8: $\left(\tilde{D}_i^{\text{def}} - \tilde{R}_i^{\text{def}} \right)$ and $\left(\tilde{D}_i^{\text{def}} + \tilde{R}_i^{\text{def}} \right)$ were calculated. $\left(\tilde{D}_i^{\text{def}} + \tilde{R}_i^{\text{def}} \right)$ shows how important the criterion is (could be called prominence), $\left(\tilde{D}_i^{\text{def}} - \tilde{R}_i^{\text{def}} \right)$ shows which criterion is cause and which one is effect (could be called relation) [2]. Also, when the value of $\left(\tilde{D}_i^{\text{def}} - \tilde{R}_i^{\text{def}} \right)$ is negative the criterion belongs to the effect group, when is positive the criterion belongs to the cause group.

Step 9: The weights of the criteria were calculated using Eq. (9).

$$w_i = \left\{ \sqrt{\left(\tilde{D}_i^{\text{def}} + \tilde{R}_i^{\text{def}} \right)^2 + \left(\tilde{D}_i^{\text{def}} + \tilde{R}_i^{\text{def}} \right)^2} \right\}. \tag{9}$$

Step 10: The weights of the criteria were normalized using Eq. (10) [3]:

$$W_i = \frac{w_i}{\max_{1 \le i \le n}(w_i)} \quad \forall i = 1, 2, \dots, n. \tag{10}$$

As a results of these steps, normalized weights of material selection criteria were calculated and were shown in Table III. According to Table III, the first three important criteria were determined as healthy, strength and reparability.

2.2. Selecting the best material for dental filling by fuzzy axiomatic design method

Axiomatic design (AD) was developed to design a product, a system or a process by Suh [4]. The axiomatic

TABLE III

D_i, R_i, $\tilde{D}_i^{\text{def}}$, $\tilde{R}_i^{\text{def}}$, w_i and W_i values.

	D_i			R_i			$\tilde{D}_i^{\text{def}}$	$\tilde{R}_i^{\text{def}}$	w_i	W_i
	l	m	u	l	m	u				
Long life	1.083	1.446	2.466	1.005	1.590	2.682	1.626	1.725	3.353	0.735
Healthy	1.465	1.845	2.810	1.943	2.367	3.371	2.004	2.525	4.559	1
Reparability	1.352	1.690	2.699	1.133	1.710	2.781	1.875	1.842	3.717	0.815
Strength	0.775	1.095	2.177	1.893	2.296	3.259	1.306	2.448	3.924	0.861
Heat resistance	1.105	1.465	2.457	1.326	1.868	2.924	1.638	2.005	3.662	0.803
Aesthetic	0.891	1.290	2.282	1.223	1.786	2.778	1.451	1.9	3.381	0.742
Formability	0.984	1.364	2.423	0.905	1.470	2.540	1.55	1.605	3.155	0.692
Corrosion resistance	0.567	1.078	2.176	0.327	0.748	1.798	1.236	0.919	2.179	0.478
Biocompatibility	0.922	1.281	2.291	1.257	1.815	2.835	1.46	1.938	3.431	0.753
Cost-effective	0.589	1.059	2.057	1.738	2.188	3.151	1.201	2.326	3.703	0.812
Easy access	1.825	2.253	3.219	0.206	0.255	1.235	2.399	0.525	3.472	0.762
Easy storage	0.665	1.118	2.207	1.073	1.616	2.645	1.291	1.746	3.071	0.674
Eco-friendly	1.325	1.855	2.725	0.217	0.265	1.246	1.945	0.535	2.853	0.626
Versatility	0.787	1.252	2.371	0.313	0.874	1.937	1.429	1.008	2.474	0.543
Rigid	0.799	1.319	2.282	0.575	0.560	1.459	1.437	0.829	2.346	0.515

TABLE IV

Calculated total and weighted total information contents of the five alternative materials.

Weights	Multiplicative inverse of weights	Criteria\alternatives	Ag–Sn	Ni–Cr	Co–Cr	Au–Pt	Plastic–SiO$_2$
0.735	1.360	Long life	4.357	4.511	3.805	4.255	4.207
1.000	1.000	Healthy	4.255	4.781	4.781	4.781	4.357
0.815	1.227	Reparability	4.511	3.805	3.805	4.511	3.805
0.861	1.162	Strength	3.805	4.781	4.781	4.781	7.303
0.803	1.245	Heat resistance	3.805	3.805	4.511	3.805	4.781
0.742	1.348	Aesthetic	0.000	0.000	1.000	0.000	0.000
0.692	1.445	Formability	4.781	4.781	6.322	7.303	4.781
0.478	2.093	Corrosion resistance	4.357	4.207	3.805	6.322	4.511
0.753	1.329	Biocompatibility	4.781	4.781	3.805	3.805	4.511
0.812	1.231	Cost-effective	3.805	4.511	4.781	7.303	4.218
0.762	1.313	Easy access	4.781	7.303	4.781	7.303	3.805
0.674	1.485	Easy storage	3.805	3.805	3.805	4.207	4.781
0.626	1.598	Eco-friendly	3.805	4.781	4.781	7.303	4.357
0.543	1.843	Versatility	4.511	4.357	4.218	4.781	4.781
0.515	1.943	Rigid	0.585	1.000	2.322	1.000	0.000
Total information content			55.944	61.210	61.304	71.460	60.200
Weighted total information content			79.388	86.342	87.031	102.477	84.584

design method supports a designer or decision maker to provide a scientific basis for designs. One of the important notions in the axiomatic design is an information axiom. It asserts that a design is the best which has minimum information content. Information content is determined by the interaction between the design and system range. Information content of the common range is calculated using Eq. (11) [4]:

$$I = \log_2 \left(\frac{\text{System Range}}{\text{Common Range}} \right). \tag{11}$$

In our case, the problem data is not certain and it is fuzzy, therefore fuzzy axiomatic design (FAD) method can be used. FAD method allows defining the problem data as a linguistic type. Defined linguistic data are converted into real numbers using a designed triangular fuzzy expression graph. There is not any rule or approach to

determine a triangular fuzzy expression graph in the literature. Generally, the graphs are determined according to an expert opinion. We determined a graph with the experts for the digitization of the structure of linguistic variables' membership functions. The determined common triangular fuzzy expression design by four specialist dentists considering C1 (long life) is shown in Fig. 1.
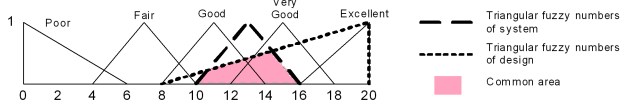

Fig. 1. Common triangular fuzzy expression design considering C1.

The five alternative filling materials were evaluated considering all criteria separately. Total information contents were calculated according to obtained evaluation data for each alternative material using Eq. (11). Calculated total and weighted total information contents of the five alternative materials are shown in Table IV. According to the results, material Ag–Sn should be preferred for dental filling.

3. Conclusion

Selecting the best material for dental filling is a difficult process that involves a high level experience. In this study a novel approach was used to select best dental filling material using fuzzy axiomatic design and fuzzy DEMATEL methods. According to dentists' experiences 15 different criteria were determined for 5 different dental filling materials. As a result of the study C2 (healthy) was the most important criterion and Ag–Sn is the best material for dental filling. We applied the novel approach on selection dental filling materials. We believe that the proposed approach can be applicable for other material selection problems as well.

References

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