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The Ground State Nilsson Quantum Numbers of the Odd–Odd ^{138,140}Pr Nuclei

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In this work, we have determined the Nilsson quantum numbers of the ground state of the odd–odd ^{138,140}Pr nucleus for the first time. To achieve it, several low energy neutron–proton two quasiparticle levels have been calculated in ^{138,140}Pr nuclei using deformed Woods–Saxon potential basis. The Gamov–Teller $\beta^{(+)}$ decay transition matrix elements from these levels to the ground state of the neighbor ^{138,140}Ce nuclei and $\log ft$ values have been calculated. From the comparison of theoretical and experimental $\log ft$ values and neutron–proton K quantum numbers the ground state Nilsson quantum numbers of the odd–odd isotopes have been determined as $\{n[402]3/2-p[413]5/2\}_{1^+}$ for two isotopes.

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1. Introduction

During the last two decades, great success has been achieved in the measurement of nuclear excitations with low multipolarity. One of them is the observation of strong low-lying magnetic dipole excitations in deformed nuclei, which are frequently referred to as a scissors mode [1]. The study of these excitations gives valuable information about nuclear structure and nucleon-nucleon forces at low energy. Beta-decay studies are an important tool in the study of the nature and nuclear structure of the 1^+-states. They can provide information about the β-decay process itself as well as information on nuclear ground state mass differences, nuclear spin assignments and on the properties of the nuclear states involved. Nowadays β-decay properties of the scissors mode 1^+-states have not been yet investigated neither experimentally and nor theoretically in the even-even ^{138,140}Ce isotopes. These nuclei have high $Q_{\beta^{(+)}} = 4.437$ MeV between the ground states of ^{138}Pr and ^{138}Ce and $Q_{\beta^{(+)}} = 3.46$ MeV between ^{140}Pr and ^{140}Ce which made them attractive for beta decay investigation. Therefore, the isotopes ^{138,140}Ce are of special interest.

Only in a few studies the decay of ^{138}Pr was investigated [2, 3]. In these papers, the spin and parity of the ^{138}Pr, which are assigned to be 7^-, 8^- or 6^-, and isomeric transition in ^{138}Ce was discussed [2]. The half life of the ground state of ^{138}Pr (1^+) and the mass difference of the nuclei ^{138}Pr (1^+) and ^{138}Ce were found. The $\log ft$ values for beta-decay to the ground state of ^{138}Ce from ^{138}Pr (1^+) decay was calculated [3]. The $\log ft$ value from ^{140}Pr to the ground state of ^{140}Ce was determined to be 4.4. From the $\log ft$ values the spin and parity of the ground state of ^{140}Pr is assigned to be 1^+ by Hisatake et al. [4]. So far Nilsson quantum numbers of ground state of the odd–odd ^{138,140}Pr nuclei have not yet determined.

It is our intention, in this study, to investigate Nilsson quantum numbers of ground state of the odd–odd ^{138,140}Pr nuclei. For this, the allowed GT β-transitions from 1^+ ground state of the nuclei $^{138-140}$Pr to 0^+ ground state of the $^{138-140}$Ce are considered. Based on these investigations the aim of the future researches is to investigate beta transitions properties of the scissors mode 1^+ excitations in ^{138,140}Ce.

So determining appropriate neutron and proton Nilsson configurations ($Nn_z\Lambda\Sigma$) of the odd–odd ^{138,140}Pr is very important for the calculations of the $\log ft$ values of 1^+ excitations in ^{138,140}Ce.

2. Theory

According to superfluid model [5], the Hamiltonian describing the interaction of the nucleons is given by

$$H = \sum_{s\sigma} \left\{E_s\left(s\right) - \lambda_\tau\right\} a^+_{s\sigma} a_{s\sigma} - G_\tau \sum_{\tau s s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+}, \quad \tau = n, p, \tag{1}$$

where the E_s is the single-particle energies, G_τ is the pairing interaction constant and λ_τ is usually called "chemical potential". $a^+_{s\sigma}$ ($a_{s\sigma}$) are the creation (annihilation) operators (see Ref. [5]).

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In quasiparticle representation the GT β decay operators has the form [6]:

$$\beta^{+}_{\mathrm{GT}} = \sum_{np} \langle n|\sigma|p\rangle \left[\sqrt{2}\left(v_n u_p C_{np} - u_n v_p C^{+}_{np}\right) - \left(v_n v_p D^{+}_{np} + u_n u_p D_{np}\right)\right], \tag{2}$$

where the operators C_{np} and D_{np} are defined as:

$$C_{np} = \frac{1}{\sqrt{2}} \sum_{\rho} \alpha_{p\rho}\alpha_{n,-\rho},$$
$$D_{np} = \sum_{\rho} \rho \alpha^{+}_{n,-\rho}\alpha_{p,\rho}. \tag{3}$$

An analytic expression of matrix element for the ground state β transition ^{138,140}Pr $(1^+) \rightarrow ^{138,140}Ce(0^+)$ can be written in the following form by using Eqs. (2)–(4):

$$M^{\beta+}_{\mathrm{GT}} = \frac{2}{\sqrt{3}} \sigma_{n_1 p_1} U_{p_1} V_{n_1}. \tag{4}$$

Here the single particle matrix elements of the Pauli spin operator are denoted by $\sigma_{n_1 p_1}$.

It is customary to express the transition probability in terms of the product $ft_{1/2}$,

$$ft_{1/2} = \frac{6163.4}{|M_{\mathrm{if}}|^2}, \tag{5}$$

where $t_{1/2}$ is the half-life and f is a dimensionless quantity depending on the charge of the nucleus and the energy and multipolarity of the transition [7]. $|M_{\mathrm{if}}|^2$ is nuclear matrix element and it is defined as:

$$|M_{\mathrm{if}}|^2 = |M_{\mathrm{F}}|^2 + \left(\frac{g_{\mathrm{A}}}{g_{\mathrm{V}}}\right)^2 |M_{\mathrm{GT}}|^2, \tag{6}$$

where M_{F} and M_{GT} denote the Fermi and Gamov–Teller matrix elements. According to conservation of the momentum the Fermi matrix element does not contribute to nuclear matrix element in these transitions. Therefore Eq. (13) is in the form

$$|M_{\mathrm{if}}|^2 = \left(\frac{g_{\mathrm{A}}}{g_{\mathrm{V}}}\right)^2 |M_{\mathrm{GT}}|^2. \tag{7}$$

For the coupling constants appearing in Eq. (14), we take into account [7]:

$$\frac{g_{\mathrm{A}}}{g_{\mathrm{V}}} = -1.26. \tag{8}$$

Using (7) and (8) expressions Eq. (5) can be written

$$ft_{1/2} = \frac{6163.4}{(1.26)^2 |M_{\mathrm{GT}}|^2}. \tag{9}$$

3. Results and discussion

The single-particle energies are obtained from the deformed Woods–Saxon potential [8]. The basis contains all discrete and quasi-discrete levels in the energy region up to 4 MeV. The mean-field deformation parameters δ_2 are calculated according to [9] using deformation parameters β_2 defined from experimental quadrupole moments [10]. The pairing-interaction constants chosen according to Soloviev [5] are based on the single-particle levels corresponding to the nucleus in question. The calculated values of the pairing parameters Δ and λ and the mean field deformation parameters δ_2 for even–even ^{138,140}Ce are shown in Table I.

TABLE I

Pairing correlation parameters [MeV] and δ_2 values.

Nucleus	Δ_n	λ_n	Δ_p	λ_p	δ_2
^{138}Ce	0.81	–8.942	1.02	–6.275	0.086
^{140}Ce	1.19	–7.560	1.54	–7.110	0.087

3.1. Expected ground state configuration for ^{138}Pr

In order to achieve to our aim in this study, β-transitions from the selected levels of neutron–proton quasiparticle spectrum near the Fermi surface in the ^{138}Pr nucleus to ground state of neighbour ^{138}Ce nucleus have been investigated. The single-particle energies of neutrons and protons (E_n, E_p), matrix elements and $\varepsilon = \varepsilon_{n_1} + \varepsilon_{p_1}$ energies with two-quasiparticles of selected levels which mentioned above have been calculated in the framework of the superfluid model; $\log ft$ values are calculated for transitions from these levels (in Table II) to ground state of the ^{138}Ce nucleus. Neutron and proton core of Ce isotopes is used as vacuum for calculation in Pr isotopes. Obtained results are given in Table II.

TABLE II

Obtained results for the ground state transition ^{138}Pr $(1^+) \rightarrow ^{138}$Ce (0^+).

Neutron proton configurations $[Nn_z\Lambda\Sigma]$	E_n	E_p	Matrix element $\sigma_{n_1p_1}$	V_{n_1}	U_{p_1}	$\varepsilon_{n_1p_1}$ [MeV]	$\log ft$
[402]3/2–[413]5/2	-9.88	-6.18	-0.24	0.937	0.738	2.26	5.00
[400]1/2–[411]3/2	-10.08	-6.02	0.45	0.952	0.977	2.45	4.39
[400]1/2–[422]3/2	-10.08	-6.98	0.10	0.952	0.462	2.64	6.11
[402]3/2–[431]1/2	-9.88	-7.41	-0.05	0.937	0.357	2.76	6.98
[400]1/2–[431]3/2	-10.08	-7.41	0.01	0.952	0.357	2.92	8.49

As can be seen in Table II for the ground state of ^{138}Pr there are found five different two-quasiparticle states with spins and parities $I = 1^+$. Corresponding experimental $\log ft$ is 4.63 for this transition (see in Fig. 1) [3]. When comparing experimental data with the theoretical values, the $\log ft$ value of the lowest in energy level with the two-quasiparticle configuration of $\{n[402]3/2 - p[413]5/2\}$ is in good agreement with the experimental $\log ft$ value.

It is noted that the quantum number K was assigned as 3/2 for neutron and 5/2 for proton by Gromow and collaborators [3]. They considered to be $\{n(d_{3/2}) - p(d_{5/2})\}_{1^+}$ for the configuration of the ground state of ^{138}Pr which we found the same spin number theoretically. With this calculations it has been shown that quantum numbers of ground state of ^{138}Pr nucleus are $\{n\ [402]3/2 - p[413]5/2\}_{1^+}$.

The decay schemes of ^{138}Pr are shown in Fig. 1. The half-life of ^{138}Pr is 1.5 ± 0.15 min and Q is energy difference between ground state of ^{138}Pr and ^{138}Ce nuclei and value is 4.460 ± 40 MeV [3].

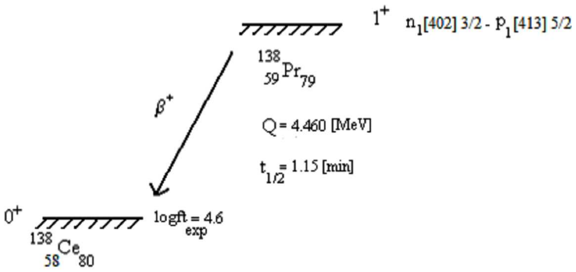

Fig. 1. Decay scheme of beta transitions from ground state of the odd–odd ^{138}Pr nucleus to ground state of even–even ^{138}Ce nucleus.

3.2. *Expected ground state configuration for ^{140}Pr*

For the ground state of ^{140}Pr there are found five different two-quasiparticle states whose spins and parities are $I = 1^+$ using the same method. Obtained results are given in Table III.

TABLE III

Obtained results for the ground state transitions ^{140}Pr $(1^+) \rightarrow ^{140}$Ce (0^+).

Neutron proton configurations $[Nn_z\Lambda\Sigma]$	E_n	E_p	Matrix element $\sigma_{n_1p_1}$	V_{n_1}	U_{p_1}	$\varepsilon_{n_1p_1}$ [MeV]	$\log ft$
[402]3/2–[413]5/2	-9.81	-6.83	0.24	0.970	0.766	4.11	4.93
[400]1/2–[411]3/2	-10.01	-6.68	0.20	0.974	0.796	4.33	5.00
[400]1/2–[422]3/2	-10.01	-7.64	-0.10	0.974	0.578	4.36	5.89
[402]3/2–[431]1/2	-9.81	-8.08	-0.05	0.970	0.482	4.36	6.68
[400]1/2–[431]3/2	-10.01	-8.08	0.009	0.974	0.482	4.55	8.19

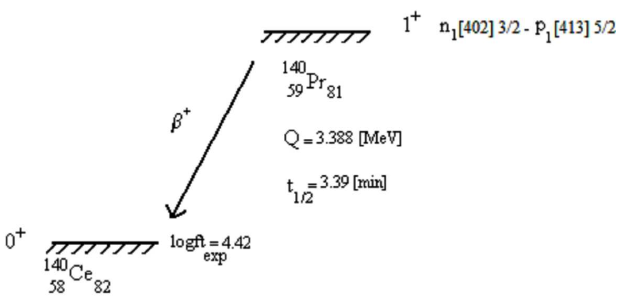

Fig. 2. Decay scheme of beta transitions from ground state of the odd–odd ^{140}Pr nucleus to ground state of even–even ^{140}Ce nucleus.

The decay schemes of ^{140}Pr are shown in Fig. 2. The half-life of ^{140}Pr is 3.39 min and Q is energy difference between ground state of ^{140}Pr and ^{140}Ce nuclei and value is 3.388 MeV [2]. For the ground state of ^{140}Pr five different two-quasiparticle states with spins and parities $I = 1^+$ is found. The theoretically calculated $\log ft$ values have been compared with experimental results. As can be seen in Table III the experimental $\log ft = 4.42$ value [2] is in agreement with the theoretical one for the lowest energy level whose the Nilsson quantum numbers are $\{n[402]3/2 - p[413]5/2\}_{1^+}$. The quantum number K of this configuration for the neutron and proton, as was pointed out by Hisatake [4], are identified as 3/2 and 5/2, respectively. The determination of the Nilsson quantum numbers of ground state of the odd-odd ^{138,140}Pr nuclei will serve as a basis for calculating various characteristics such as beta decay [11], magnetic moment [12–16] and magnetic dipole transitions [17–23] of even and odd mass nuclei.

4. Conclusion

Our main interest in this study was to determine the Nilsson quantum numbers of ground state of the odd–odd ^{138,140}Pr nuclei using superfluid model. Logft values were successfully calculated for the β-transitions from the selected levels of neutron–proton quasiparticle spectrum near the Fermi surface in the odd–odd nuclei to ground state of neighbour even–even nuclei. The calculation results show that the quantum number K is in very good agreement with the earlier studies and the Nilsson quantum numbers of ground state of the odd–odd ^{138,140}Pr nuclei were characterized as $\{n[402]3/2-p[413]5/2\}_{1^+}$. It is clearly time to explore beta transitions properties of the scissors mode 1^+ excitations in ^{138,140}Ce. For this our primary aim will be to investigate the transition probability of the decay from the ground state of the parent nucleus to 1^+ states in the daughter Ce nucleus, and we intend to do so soon.

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