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Effects of Material Non-Homogeneity and Two Parameter Elastic Foundation on Fundamental Frequency Parameters of Timoshenko Beams

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In the present study, effects of material non-homogeneity and two-parameter elastic foundation on the fundamental frequency parameters of the simply supported beams are examined. Material non-homogeneity is characterized taking into account the parabolic variations of Young's modulus and density along the thickness direction of the beam while the value of Poisson's ratio is assumed to remain constant. The foundation medium is assumed to be linear, homogeneous and isotropic, and it is modeled by the Pasternak model with two parameters for describing the reaction of the elastic foundation on the beam. At first, the equation of the motion including the effects of the material non-homogeneity and two-parameter elastic foundation is provided. Then, the solutions including fundamental frequency parameters versus various non-homogeneity, density and foundation parameters, and length to depth ratio adopting the Timoshenko beam theory as well as the Euler–Bernoulli beam theory are presented. To show the accuracy of the present results, a comparison is carried out and a good agreement is found.

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1. Introduction

The concept of beams on elastic foundations has been widely used in different fields of engineering, e.g., railroad tracks, highway pavements and pipelines. Since soil exhibits a very complex behavior, various foundation models have been developed for modeling it. Among these models, the Pasternak two-parameter elastic foundation (TPEF), which accounts for foundation shearing-stiffness neglected in Winkler one-parameter foundation model, is the commonly used one [1].

On the other hand, in the studies relating to the vibration analysis of beams resting on elastic foundation generally slender beams have been considered so that the Euler–Bernoulli beam theory (EBBT) is usually adopted [2, 3]. However, EBBT slightly overestimates the frequency parameters and the error increases with the increase of modes and thickness of the beam. Therefore, if the beam is moderately short and thick, the Timoshenko beam theory (TBT) gives more accurate results due to that it takes into account both the shear deformation and rotary inertia of the beam. Several researches have been conducted on the vibration of beams resting on TPEF adopting TBT [4–7].

All of the above mentioned studies are carried out for homogeneous (H) beams in sense that mechanical properties of the beam are taken to be constant throughout. However, plenty of materials exist in the nature, which are non-homogeneous (NH), either by working under high temperature environment or due to the physical composition and imperfections in the underlying materials. Because the use of these advanced materials in various fields of engineering has been increasing day by day, the vibration problems of NH beams resting on elastic foundation have also received the attention of numerous researchers [8–10].

From the review of available literature it is observed that the effects of material non-homogeneity (MNH) and TPEF on the values of fundamental frequency parameters (FFPs) of simply supported beam in which the MNH is characterized with the parabolic variation of the Young modulus and density along the thickness direction have not been dealt yet. In the present study an attempt is made to address this problem.

2. Formulation and solution of the problem

Consider an elastic beam of length L, height h, resting on TPEF as shown in Fig. 1a.

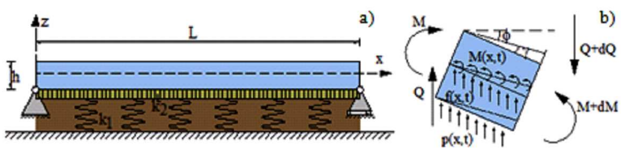

Fig. 1. (a) Coordinate and geometry of beam on TPEF, (b) beam element.

Pressure–displacement relation of the TPEF models are assumed to be [1]:

$$p(x,t) = k_1 w(x,t) - k_2 w''(x,t), \tag{1}$$

where $p(x,t)$ is the vertical foundation reaction, $w(x,t)$ is the function of transverse displacements of beam, k_1 is the modulus of sub-grade reaction, k_2 — shear

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foundation modulus. Note that, as $k_2 = 0$, the Pasternak foundation reduces to the Winkler foundation.

The equations including translational and rotational equilibrium conditions of the beam element are (Fig. 1b):

$$\frac{\partial Q}{\partial x} = \rho A \left(\partial^2 w/\partial t^2\right) + k_1 w - k_2 \left(\partial^2 w/\partial x^2\right), \qquad (2)$$

$$Q\,\mathrm{d}x - (\partial M/\partial x)\,\mathrm{d}x = \rho I \left(\partial^2 \Phi/\partial t^2\right) \mathrm{d}x, \qquad (3)$$

where Q, M, ρ, A, I, and Φ are shear force, bending moment, density, area, moment of inertia, and angle of rotation of beam element due to bending, respectively.

The constitutive equations of flexural and shear stiffnesses are as follows, respectively:

$$M = -EI\left(\partial \Phi/\partial x\right); Q = \kappa G A \gamma, \qquad (4)$$

where E, κ, G, and γ are the Young modulus, shear correction factor, shear modulus and increase in the slope due to shear deformation.

The total slope of the beam is

$$\gamma = \partial w/\partial x - \Phi. \qquad (5)$$

Substitution of Eq. (5) into Eqs. (2) and (3) yield the following equations:

$$(\kappa G A + k_2)\left(\partial^2 w/\partial x^2\right) - \kappa G A\left(\partial \Phi/\partial x\right)$$
$$-\rho A\left(\partial^2 w/\partial t^2\right) - k_1 w = 0, \qquad (6)$$

$$EI\left(\partial^2 \Phi/\partial x^2\right) + \kappa G A\left(\partial \Phi/\partial x\right) - \kappa G A \Phi$$
$$-\rho I\left(\partial^2 \Phi/\partial t^2\right) = 0. \qquad (7)$$

Using Eqs. (6) and (7) after some mathematical operations the equation of motion for the free vibration of a H beam resting on TPEF is obtained as follows:

$$\left(EI + \frac{EIk_2}{\kappa G A}\right)\frac{\partial^4 w}{\partial x^4} - \left(\frac{EIk_1}{\kappa G A} + k_2\right)\frac{\partial^2 w}{\partial x^2}$$
$$-\left(\frac{EI\rho}{\kappa G} + I\rho + \frac{\rho I k_2}{\kappa G A}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2}$$
$$+\left(\rho A + \frac{\rho I k_1}{\kappa G A}\right)\frac{\partial^2 w}{\partial t^2} + \left(\frac{\rho^2 I}{\kappa G}\right)\frac{\partial^4 w}{\partial t^4} + k_1 w = 0. \qquad (8)$$

It is assumed that the MNH of the beam stems from the variation of Young's modulus and mass density along the thickness direction [9]:

$$E_1 = E[1 + \alpha_1 \lambda(\bar{z})], \quad \rho_1 = \rho[1 + \alpha_2 \lambda(\bar{z})], \qquad (9)$$

where $\bar{z} = z/h$, $\lambda(\bar{z})$ is the continuous function of MNH defining the variation of Young's modulus and density, α_1 and α_2 are the non-homogeneity and density parameters ($-0.5 \le \alpha_i \le 1, i = 1, 2$). Note that the value of Poisson's ratio is assumed to be constant.

The MNH functions of the beam are taken to be parabolic functions

$$\lambda(\bar{z}) = \bar{z}^2. \qquad (10)$$

Considering Eqs. (9) and (10) in Eq. (8), the governing equation for the free vibration of a NH Timoshenko beam resting on TPEF is obtained as follows:

$$\left(D_1 + \frac{D_1 k_2}{D_2 \kappa A}\right)\frac{\partial^4 w}{\partial x^4} - \left(\frac{D_1 k_1}{D_2 \kappa A} + k_2\right)\frac{\partial^2 w}{\partial x^2}$$
$$-\left(\frac{D_1 D_3}{D_2 \kappa} + I D_3 + \frac{D_3 I k_2}{D_2 \kappa A}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2}$$
$$+\left(D_3 A + \frac{D_3 I k_1}{D_2 \kappa A}\right)\frac{\partial^2 w}{\partial t^2}$$
$$+\left(\frac{D_3^2 I}{D_2 \kappa}\right)\frac{\partial^4 w}{\partial t^4} + k_1 w = 0, \qquad (11)$$

where D_1, D_2 and D_3 are flexural rigidity, shear modulus, and density of the NH beam, and the following definitions apply:

$$D_1 = Ebh^3 \int_{-1/2}^{1/2} \bar{z}^2 (1 + \alpha_1 \lambda(\bar{z}))\,\mathrm{d}\bar{z},$$

$$D_2 = \frac{E}{2(1+\nu)} \int_{-1/2}^{1/2} (1 + \alpha_1 \lambda(\bar{z}))\,\mathrm{d}\bar{z},$$

$$D_3 = \rho \int_{-1/2}^{1/2} (1 + \alpha_2 \lambda(\bar{z}))\,\mathrm{d}\bar{z}. \qquad (12)$$

The solution of Eq. (11) is sought by separation of variables. Assume that the displacement can be separated into spatial and temporal variables [11]:

$$w(x,t) = \xi(x)\eta(t), \qquad (13)$$

where ξ and η are dependent on position and time, respectively.

Besides, the following expressions are satisfied for the boundary conditions of the simply supported beam:

$$\xi(0) = \xi^{II}(0) = \xi(L) = \xi^{II}(L) = 0. \qquad (14)$$

Substituting Eqs. (13) and (14) in Eq. (11), respectively, and after some mathematical operations lead to the following equation:

$$\left(D_1 + \frac{D_1 k_2}{D_2 \kappa A}\right)\left(\frac{n\pi}{L}\right)^4 + \left(\frac{D_1 k_1}{D_2 \kappa A} + k_2\right)\left(\frac{n\pi}{L}\right)^2$$
$$-\omega^2 \left[\left(\frac{D_1 D_3}{\kappa D_2} + I D_3 + \frac{D_3 I k_2}{D_2 \kappa A}\right)\left(\frac{n\pi}{L}\right)^2\right.$$
$$\left. + \left(D_3 A + \frac{D_3 I k_1}{D_2 \kappa A}\right)\right]$$
$$+\omega^4 \left(\frac{D_3^2 I}{\kappa D_2}\right) + k_1 = 0. \qquad (15)$$

Consequently, Eq. (15) is a quadratic equation in ω^2 and yields two values of ω^2, in which the smaller one corresponds to the free vibration of the NH Timoshenko beam resting on TPEF. Note that the equation for the free vibration of the NH Euler–Bernoulli beam resting on TPEF can be obtained setting the terms including $D_3 I$ equal to zero and letting $D_2 \to \infty$ in Eq. (11) and reapplying the above given solution procedure.

Comparison of FFPs of H beam resting on TPEF. TABLE I

	Ω							
	$K_P = 0$				$K_P = 1$			
	Refs. [4, 5]		Present study		Refs. [4, 5]		Present study	
K_W	EBBT	TBT	EBBT	TBT	EBBT	TBT	EBBT	TBT
0	9.8696	9.2740	9.8696	9.2740	13.9577	13.4473	13.9577	13.4473
10	10.3638	9.7848	10.3639	9.7848	14.3115	13.8045	14.3115	13.8045
10^2	14.0502	13.5408	14.0502	13.5407	17.1703	16.6781	17.1703	16.6781
10^3	33.1272	32.5378	33.1272	32.5378	34.5661	33.9613	34.5661	33.9613

FFPs of beams versus non-homogeneity and density parameters. TABLE II

		Ω								
		$K_W = 0, K_P = 0$			$K_W = 1000, K_P = 0$			$K_W = 1000, K_P = 1$		
α_1	α_2	–0.5	0	1	–0.5	0	1	–0.5	0	1
–0.5	EBBT	9.70	9.49	9.12	33.73	33.02	31.72	35.20	34.46	33.11
	TBT	9.14	8.95	8.60	33.14	32.44	31.17	34.60	33.87	32.54
0	EBBT	10.08	9.87	9.48	33.84	33.13	31.83	35.31	34.57	33.21
	TBT	9.49	9.29	8.93	33.24	32.54	31.26	34.69	33.96	32.63
1	EBBT	10.81	10.58	10.17	34.06	33.35	32.04	35.52	34.78	33.41
	TBT	10.15	9.94	9.55	33.44	32.74	31.45	34.89	34.15	32.81

3. Numerical results and discussion

In this section illustrative studies are given to examine the present problem. All numerical results are expressed according the following non-dimensional parameters:

$$\Omega = \omega L^2 \sqrt{\rho A / EI}, \quad K_W = k_1 L^4 / EI,$$
$$K_P = k_2 L^2 / \left(\pi^2 EI\right). \tag{16}$$

3.1. Comparative study

The FFPs of homogeneous beam resting on the Pasternak type TPEF versus elastic foundation parameters are compared with those of Refs. [4, 5] in Table I for $n = 1$, $\kappa = 5/6$, $L/h = 5$, $\alpha_1 = \alpha_2 = 0$, $\nu = 0.3$. The obtained results validate the accuracy of present formulations.

3.2. Numerical examples

Example 1. Table II shows the FFPs of beams with and without elastic foundations versus non-homogeneity and density parameters α_1 and α_2 for $L/h = 5$, $\kappa = 5/6$, $\upsilon = 0.25$. It is seen that values of FFPs increase with the increase of α_1, while they decrease with the increase of α_2 in all cases. Besides, the effect of MNH on the values of FFPs varies not only according to α_1 and α_2, but also presence and type of elastic foundations and adopted beam theory.

Example 2. Table III shows FFPs of beams with and without elastic foundations versus foundation parameters, K_W and K_P for $\alpha_1 = 1$, $\alpha_2 = -0.5$, $L/h = 5$, $\kappa = 5/6$, $\upsilon = 0.25$. It is found that values of FFPs increase with consideration of the foundation parameters. Moreover, the effect of MNH on the values FFPs decreases with the increase of K_W and K_P, and becomes least efficient for the Pasternak foundation.

TABLE III

FFPs of beams versus foundation parameters.

		Ω			
		H Case		NH case	
K_W	K_P	EBBT	TBT	EBBT	TBT
0	0	9.87	9.29	10.81	10.15
10		10.36	9.80	11.28	10.64
10^2	0	14.05	13.55	14.87	14.30
10^3		33.13	32.54	34.06	33.44
10		12.50	11.98	13.35	12.75
10^2	0.5	15.69	15.20	16.49	15.94
10^3		33.86	33.26	34.80	34.17
10		14.31	13.81	15.13	14.56
10^2	1	17.17	16.69	17.97	17.42
10^3		34.57	33.96	35.52	34.89

Example 3. Table IV shows FFPs of beams with and without elastic foundations for $\alpha_1 = -0.5$, $\alpha_2 = 1$, $\kappa = 5/6$, $\nu = 0.25$ versus the length to depth ratio, L/h. It is observed that values of FFPs for TBT increase with the increase in the ratio L/h. Besides, the effect of variation of ratio, L/h, on the values of FFPs is more pronounced in foundationless case. Moreover, the effect of MNH on the values of FFPs increases with the increase in the ratio, L/h, in foundationless case, while it changes irregularly with consideration of elastic foundations.

TABLE IV

FFPs of beams versus the length to depth ratio.

		Ω								
		$K_{\mathrm{W}}=0, K_{\mathrm{P}}=0$			$K_{\mathrm{W}}=1000, K_{\mathrm{P}}=0$			$K_{\mathrm{W}}=1000, K_{\mathrm{P}}=1$		
Case	L/h	2	5	10	2	5	10	2	5	10
H	EBBT	9.87	9.87	9.87	33.13	33.13	33.13	34.57	34.57	34.57
	TBT	7.46	9.29	9.71	29.14	32.54	32.96	30.00	33.96	34.40
NH	EBBT	9.12	9.12	9.12	31.72	31.72	31.72	33.11	33.11	33.11
	TBT	6.93	8.60	8.98	27.66	31.17	31.57	28.43	32.54	32.95

4. Conclusions

In this study, effects of MNH and TPEF on the FFPs of the simply supported beams are examined using TBT as well as EBBT. Solutions including FFPs for various combinations of non-homogeneity, density and foundation parameters, and length to depth ratios are reported.

Briefly, the obtained results can be summarized as follows:

1. The values of FFPs increase with consideration of the elastic foundations;
2. The variation of non-homogeneity and density parameters have adverse effects on the values of FFPs;
3. The presence of elastic foundation decreases the effects of MNH and length to depth ratio on the values of FFPs;
4. MNH is more influential on the values of FFPs in the Winkler foundation and EBBT cases in comparison with the Pasternak foundation and TBT cases.

The presented results prove the importance of the effects of MNH and TPEF on FFPs of beams.

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