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Dependence of the Odd-Odd Nucleus ^{196}Au Level Density on the Parameters in Interacting Boson-Fermion-Fermion Model (IBFFM)

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The odd-odd nuclei are characterized by a level density which is high already in the low-energy region. This case displays a full complexity of the interwoven shell-model and collective degrees of freedom and thus provides an interesting testing ground for the pattern of nuclear level density. The level density of the odd-odd nucleus ^{196}Au is investigated in the interacting boson-fermion-fermion model which accounts for collectivity and complex interaction between quasiparticle and collective modes. In the present work, the IBFFM pattern of total and parametric dependent level densities is investigated and compared to the pattern found in previous investigations in the framework of combinatorial, thermodynamic and spectral distribution approaches.

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1. Introduction

The interacting boson model (IBM) of Arima and Iachello [1, 2] and its extensions, the interacting boson-fermion model (IBFM) [3] and the interacting boson-fermion-fermion model (IBFFM) [4], are of a particular interest for studies of the low-energy nuclear structure. This approach corresponds to a real physical system; it has a microscopic basis and successfully describes the low-lying nuclear phenomenology, accounting also for collective features.

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The general interest in the problem of nuclear level densities is based on several reasons. Nuclear level densities are important in nuclear reaction calculations, in particular in heavy-ion reactions [5], astrophysics applications [6] and in many applied problems in the areas of fission and fusion reactor design [7]. However, our first-hand knowledge of the density of levels is confined to a rather small region of excitation energy and angular momentum [8].

In the present work, the IBFFM pattern of total and parametric dependent level densities for the odd-odd nucleus ^{196}Au [9] is investigated and compared to the pattern found in previous investigations in the framework of combinatorial, thermodynamic, and spectral distribution approaches.

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2. IBFFM calculations

The calculation for ^{196}Au was performed in the IBFFM [1, 10] by coupling valence-shell proton and neutron quasi particles to the boson core of the IBM of Iachello and Arima [2]. The IBFFM Hamiltonian for an odd-odd nucleus reads

$$H_{\mathrm{IBFFM}} = H_{\mathrm{IBFM}}(\pi) + H_{\mathrm{IBFM}}(\nu) - H_{\mathrm{IBM}} + H_{\mathrm{RES}}(\pi\nu). \quad (1)$$

Here, $H_{\mathrm{IBFM}}(\pi)$ and $H_{\mathrm{IBFM}}(\nu)$ denote the IBFM Hamiltonian [5] for the neighboring odd-even and even-odd nuclei, ^{197}Au and ^{197}Hg, respectively. H_{IBM} denotes the IBM Hamiltonian [2] for the even-even core nucleus ^{198}Hg and $H_{\mathrm{RES}}(\pi\nu)$ denotes the residual proton–neutron interaction. The calculation was performed using the computer code "IBFFM" [11], which employs the TQM representation of IBM.

In the first step of the IBFFM calculation, the boson core was fitted to the low-lying levels in the even-even nucleus ^{198}Hg. In the second step of our calculation, we have adjusted the parameters in IBFM(π) of Eq. (1) to the low-energy spectrum of the odd-even nucleus ^{197}Au.

In the third step of our calculation, the parameters in IBFM(ν) of Eq. (1) were adjusted to the low-energy spectrum of the even-odd nucleus ^{197}Hg.

In the fourth step of our calculation, for the residual proton-neutron interaction, the dominant interactions are set to be spins $H_{\sigma\sigma}$ and tensors H_{T} interactions.

In the final fifth step of our calculation, we compute the full energy spectrum by diagonalising the Hamiltonian Eq. (1) in the IBFFM basis state

$$|(j_\pi j_\nu)j_{\pi\nu}, n_d \nu I; J\rangle. \quad (2)$$

Here, the proton quasiparticle angular momentum j_π and the neutron quasiparticle angular momentum j_ν are coupled to the angular momentum $j_{\pi\nu}$ and n_d d-bosons are coupled to the angular momentum I with additional seniority quantum number ν. The two-quasiparticle angular momentum $j_{\pi\nu}$ and the boson angular momentum I are coupled to the total angular momentum J. The basis states Eq. (2) contain also the $n_s = N - n_d$ s-bosons with the angular momentum equal to zero. The total number of IBFFM levels in the present calculation is 7582.

In the present work, we investigate the dependence of nuclear level density from the IBFFM parameters associated with this energy spectrum. The parameters for IBFFM calculation are taken from first, second, third and fourth steps from IBM, IBFM(π), IBFM(ν) and $H_{\mathrm{res}}(\pi\nu)$ respectively.

3. IBFFM parameters — dependent level density

3.1. Boson space

In the first step of the H_{IBFFM} calculation Eq.(1), in order to reduce the size of computations, the total number of boson in IBM Hamiltonian [2], which enters a maximum integer number of bosons $N = 6$, was reduced to $N = 5$, 4 and 3 respectively. It is assumed that this maximum number of bosons is equal to half of the total number of valence quasiparticles.)(

First, we will observe what happens to the level density if we change the value of the parameter N. Indeed, the first presupposition was that cutting the boson space causes the oscillatory behavior of the total density of states at high excitation energy and its deviations from the Gaussian distribution Eq. (1). It was expected that a reduction in the value of the parameter N under 5 will cause even greater oscillations and large deviations from the Gaussian distribution given by Eq. (3):

$$\rho_{\mathrm{G}}(E) = \frac{d}{\sqrt{2\pi}\sigma_M} \exp\left[\frac{(E-\varepsilon_M)^2}{2\sigma_M^2}\right]. \tag{3}$$

The IBFFM total level density as a function of the energy is represented in Fig. 1 for $N = 3, 4$ and in Fig. 2 for $N = 5, 6$. It can clearly be seen that they are qualitatively identical.

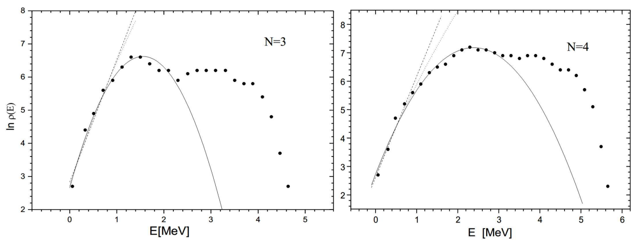

Fig. 1. Calculated total level density of ^{196}Au as a function of excitation energy for $N = 3$ (a) and $N = 4$ (b). A Gaussian was fitted to IBFFM below 3.2 MeV (respectively 5.2 MeV) of excitation energy (solid curve). The Bethe formula and the constant temperature Fermi model formula were fitted to IBFFM below 1 MeV of excitation energy (dashed and dot curves, respectively).

The low-energy section of the total IBFFM distribution again can be well fitted both by the Bethe equation (4) and by the constant temperature Fermi gas model equation, given respectively by Eq. (5):

$$\rho(E) = \frac{\exp(2\sqrt{a(E-E_1)})}{12\sqrt{2}\sigma a^{\frac{1}{4}}(E-E_1)^{5/4}}, \tag{4}$$

$$\rho(E) = \frac{1}{T}\exp\left(\frac{E-E_0}{T}\right), \tag{5}$$

while 55-60% of the total level density in the truncated IBFFM space can rather be well fitted by a Gaussian, distortions near the high-energy tail of the distribution (last 45–50%) have in each case the oscillatory behavior of the total level density. In Figs. 1 and 2, lines fitted with the above formulae are shown and the obtained parameter values are shown in Table I. Thus, the conclusion would be that dimensions of bosonic space do not affect the form of the total level density.fip(30)2Calculated total level density of ^{196}Au as a function of excitation energy for $N = 5$ (a) and $N = 6$ (b). A Gaussian was fitted to IBFFM below 7.5 MeV (respectively 8.2 MeV) of excitation energy (solid curve). The Bethe formula and the constant temperature Fermi model formula were fitted to IBFFM below 2 MeV of excitation energy (dashed and dot curves, respectively)

TABLE I

Values of the parameters d, σ_M and ε_M in the Gaussian fitted to the IBFFM total level and those of fit parameters a and E_1 in the Bethe formula and E_0, T in the constant temperature Fermi gas model for ^{196}Au. The values are given for different boson spaces.

	Gaussian distribution			Fermi gas model		Bethe formula	
N	d	σ_M [MeV]	ε_M [MeV]	E_0 [MeV]	T [MeV]	a [MeV^{-1}]	E_1 [MeV]
3	1779	0.92	2.31	–0.41	0.27	25.87	–0.38
4	4007	0.99	2.93	–0.75	0.35	18.27	–0.85
5	7582	1.41	4.11	–1.81	0.59	12.51	–1.54
6	12811	1.52	4.64	–0.79	0.38	18.38	–0.73

If we look at the IBFFM spin-dependent level density for $N = 5$, Fig. 3, we see that again we have an identical behavior as that for $N = 6$. Spin-dependent level density again can be described by modified spin-dependent formula given by Eq. (6):

$$Y_H(J) = D - \frac{1}{\sqrt{3}}\left(\frac{2}{\sigma}\right)^3 \mathrm{sh}\left(\frac{\sqrt{3}\sigma}{2\eta^3}(j+0.5)^2\right) \tag{6}$$

and in the low-spin limit (up to approximately half of the maximum spin), the modified formula is reduced to the form of the Bethe formula given by Eq. (7):

$$Y_H^B = D - \frac{1}{2\sigma^2}\left(J + \frac{1}{2}\right)^2. \tag{7}$$

Based on this, we can conclude that the dimensions of bosonic space has no impact on the form of spin-dependent level density. The parameter values obtained by fitting the spin dependent level density with Eq. (6) and Eq. (7) are given in Table II.

If we compare the parameter values in Table II with those in [8] and [13], we see that we now have somewhat lower value for the spin cut-off parameter σ and slightly higher values for the spin correction parameter η. In the present calculation, the average ratio of η and σ is 1.82

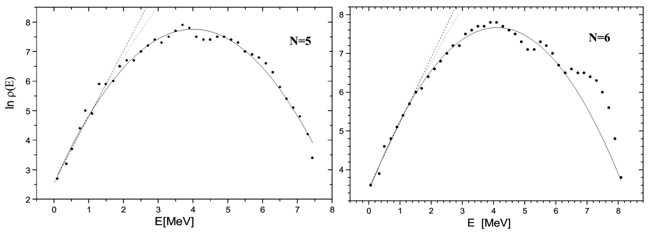

Fig. 3. Calculated spin distributions for ^{196}Au, for $N = 5$ and $N = 6$. The calculated values $y_H = \ln(N(J)/(J+0.5))$ are presented for the energy intervals 0–1 MeV (closed squares), 1–2 MeV (closed diamonds), 2–3 MeV (closed triangles) and 0–5 MeV (closed inversed triangles). $N(J)$ denotes the number of levels of spin J obtained by IBFFM calculation in the corresponding energy intervals. Dashed lines present the fits of spin-dependent Bethe formula to the state spins $J = 4{-}8$, and solid lines the fits of modified spin-dependent formula.

for $N = 4$, 1.76 for $N = 6$ and 1.74 for $N = 5$ which are somewhat lower than the ratios ≈2–3 obtained in previous combinatorial calculations for ^{114}Cd and ^{244}Am [14], and ≈1.8 for ^{132}Pr [8] calculated in IBFFM. Let us note that the reduction of the maximum number of bosons under 5 will certainly not bring any improvement in the calculation, as well as increasing the maximum number of bosons also does not bring improvement in the calculation of the total level densities of energy states.

3.2. Fermion space

In previous calculations, cuttings in the boson space were necessarily introduced, and now we will remove two proton quasiparticles. These are $\pi_{d_{3/2}}$ and $\pi_{s_{1/2}}$ states with energies of 0.47 MeV and 0.82 MeV, respectively, and appropriate probabilities 0.70 and 0.85. Other values of the interaction parameters are as in the previous calculations.

TABLE II

IBFFM values for parameters σ, η, D in the modified spin distribution formula and the values of σ, D in the Bethe formula for ^{196}Au. The values are given for four energy intervals in the low-lying section of the spectrum for $N = 4$ and $N = 6$.

Bin energy [MeV]	Bethe formula ($N = 4$)		Modified Bethe formula ($N = 4$)			Bethe formula ($N = 6$)		Modified Bethe formula ($N = 6$)		
	D	σ	D	σ	η	D	σ	D	σ	η
0-1	3.36	4.13	2.16	4.06	6.91	2.09	5.05	2.63	4.68	7.57
1-2	3.97	4.46	3.24	5.12	9.907	3.00	5.19	3.08	5.38	9.27
2-3	4.14	5.15	3.79	5.32	10.08	3.75	5.56	3.78	5.51	10.61
0-5	4.41	6.08	4.80	5.72	10.18	5.19	5.76	5.13	5.92	10.62

The total IBFFM level density is shown in Fig. 4, where from Fig. 4a, we can see that the total IBFFM level densities monotonically increase at low excitation energy (up to 3 MeV). In this part, total IBFFM level densities can be described again by the Bethe expression for the total density of states and the Fermi gas model. At energies greater than 3 MeV, the total level density of states is nearly constant up to 4.5 MeV and then monotonically decreases to 6.6 MeV. A careful comparison for Fig. 4a with Figs. 2 and 3, we can see that lowering fermion space caused a decrease in the number of states as expected. The low side is identical to the shape, and then we have a different behavior.

Fitting the expressions Eq. (4) and Eq. (5) on IBFFM total level densities up to 6.6 MeV has given the following values of the parameters:
Bethe: $a = 11.96$ MeV^{-1}, $E_1 = -1.75$ MeV,
Fermi: $T = 0.608$ MeV, $E_0 = -1.95$ MeV.

If we compare the value of a and E_1 for the case as shown in Fig. 2 and Fig. 4a, we can see that the results corresponding to Fig. 2 are better than those to Fig. 4a.

Figure 4b illustrates spin dependent level densities with cut-off fermion space. If we compare Fig. 4b and Fig. 3 we see that the spin distributions not only have similar behavior but are almost identical.

Let us now see in Table III the parameter values σ, η and D which are obtained by fitting expressions Eq. (6) and Eq. (7) with the results obtained from IBFFM.

TABLE III

IBFFM values for parameters σ, η, D in the modified spin distribution formula and the values of σ, D in the Bethe formula for ^{196}Au. The values are given for four energy intervals in the low-lying section of the spectrum with cut-off fermion space.

Bin energy [MeV]	Bethe formula		Modified Bethe formula		
	D	σ	D	σ	η
0-1	1.78	4.76	1.69	5.27	7.79
1-2	3.03	4.96	2.93	5.47	9.60
2-3	3.75	5.23	3.69	5.58	10.18
0-5	5.16	5.42	4.98	5.81	11.40

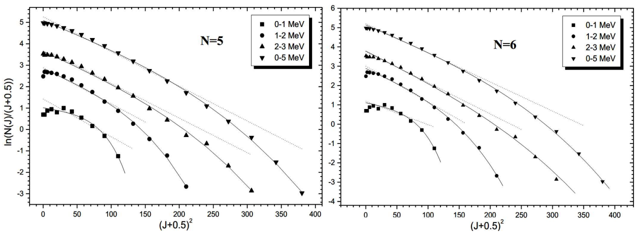

Fig. 4. (a) Total level density of ^{196}Au as a function of excitation energy. A Gaussian was fitted to IBFFM (solid curve). The Bethe formula and the constant temperature formula were fitted to IBFFM (dashed and dot curves, respectively) and (b) spin dependent level densities with cut-off fermion space. Fits with the Bethe formula to the state spins (dashed lines) and modified spin-dependent formula (solid lines) are shown.

We see that the values for the spin "cutoff" parameter σ are somewhat higher compared to those given in Table II. If we recall once again the value of the spin parameters for rigid body $\sigma_{\rm rig} = 5.1$ for $E \approx 2$ MeV, we see that the σ value is higher than that of rigid body. From the total level density of states, it can be seen that the complete fermion space at low energies better describes the level density, and thus the full spectrum. If this is taken into account, it can be said that full fermion space gives a more accurate value for the parameter σ at low energies.

4. Conclusion

The calculation of nuclear level densities in the IBFFM for ^{196}Au reveals several interesting features. The total level density in the truncated IBFFM space can be rather well fitted by a Gaussian, except for distortions near the high-energy tail of the distribution. The low-energy section of the total IBFFM distribution can be well fitted both by the Bethe formula and by the constant temperature Fermi gas model.

The IBFFM spin distributions exhibit pronounced deviation from spin-dependent Bethe formula, the high-spin reduction (for spins $J \geq J_{\rm max\,/2)}$ with monotonically increasing reduction towards the maximum possible yrast spin (yrast spin is a state of a nucleus with a minimum of energy for a given spin). The high-spin reduction is similar as in the combinatorial calculations and can be well fitted by the modified spin-dependent level density formula which was introduced to fit the results of combinatorial calculations. The value of the spin cut off parameter σ calculated in IBFFM is slightly higher than the rigid body value. The values of the new spin truncation parameter η calculated in IBFFM for different number of bosons are by an average factor 1.77 higher than the values of σ, and the introduction of η causes only a minor change of the value of the parameter σ with respect to fits with the Bethe formula. We can conclude that the dimensions of bosonic space has no impact on the form of spin-dependent level density.

Also or therefore, the reduction of the maximum number of bosons under 5 will certainly not bring any improvement in the calculation.

From the total level density of states, it can be seen that the complete fermion space at low energies does better describe the level density, and the full. If this is taken into account, it can be said that full fermion space gives a more accurate value for the parameter σ at low energies. It can be concluded that for a better description of the level density of states in low-lying part of the spectrum, it is preferable to not decrease the fermion space. However, removing the proton states has not settled the oscillations in the medium and high-energy excitations. Therefore, fermion space does not affect the oscillatory behavior of the total level density of states in these areas.

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