ERRATUM

Scaling of the Thue–Morse Diffraction Measure *Acta Physica Polonica A* 125, 431 (2014), ERRATUM

M. Baake[a], U. Grimm[b] and J. Nilsson[a]

[a]Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany

[b]Department of Mathematics and Statistics, The Open University, Walton Hall, Milton Keynes MK7 6AA, UK

In the proofreading stage we overlooked a number of typographical errors. (On-line version is correct, this erratum applies to printed version only.)

original DOI: 10.12693/APhysPolA.125.431

actual DOI: 10.12693/APhysPolA.129.1263

PACS/topics: 99.10.Cd

Under point 2 of Proposition 1, the last line must read $\lim_{N\to\infty} \frac{1}{N} \operatorname{card}\bigl(\{x_n \mid n \le N\} \cap [a,b)\bigr) = b-a$.

Under points 3 and 4 of the same proposition, it must read $\frac{1}{N}\sum_{n=1}^{N}$ rather than $\frac{1}{n}\sum_{n=1}^{n}$. The same correction applies to the last equation in the right column of page 431.

Furthermore, the first formula under Case B (on page 432) must begin as

$$\beta(k) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \frac{\log\bigl(1-\cos(2^{n+1}\pi k)\bigr)}{\log(2)}.$$

More importantly, our argument for Case B on the basis of uniform distribution modulo 1 of the sequence $\bigl(2^n x\bigr)_{n\in\mathbb{N}}$ for almost all $x \in \mathbb{R}$ is incomplete, because the function f defined by $f(x) = \log\bigl(1-\cos(2\pi x)\bigr)$ is only Riemann integrable on $[0,1]$ in the generalised sense (meaning that it is an improper integral), which is insufficient here. However, f is properly integrable on $[0,1]$ in the Lebesgue sense and has an obvious 1-periodic extension to $\mathbb{R}$.

Consequently, rather than employing uniform distribution, one can argue with the dynamical system defined by the map T of the unit interval $[0,1]$ into itself, given by $x \mapsto 2x \bmod 1$. It is well-known that T leaves Lebesgue measure invariant and is ergodic relative to it, so that the Birkhoff sums satisfy

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n(k)) = \int_0^1 f(x)\,\mathrm{d}x$$

for Lebesgue almost every $k \in \mathbb{R}$ by an application of the ergodic theorem.

This still gives the result of Case B for almost all wave numbers $k \in \mathbb{R}$, though it might be that they differ from the wave numbers with uniform distribution of $\bigl(2^n x\bigr)_{n\in\mathbb{N}}$ on a null set. This has no further consequence on the analysis presented in paper.

Acknowledgments

We thank Gerhard Keller, Marc Keßeböhmer and Tanja Schindler for helpful comments and discussions.