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Self-Organization of Plutonomy in a Fair Competitive Society

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We investigate the origin of the emergence of the plutonomy, an extreme form of hierarchy, where the top 1% of households account for more wealth than the bottom 99%. For a model fair society where individuals participate in a competition with equal right, we show that the plutonomy can be self-organized when individuals divided into several groups compete with those in the same group for a certain period (season) and they are regrouped at the end of every season. In the fair society, the wealth flows steadily from lower groups to the highest group, which is the origin of the plutonomy. Using mean-field analysis, we show that the fraction of winners decreases in proportion to the inverse of the number of groups.

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1. Introduction

In the fall of 2011, many people occupied Wall Street in New York City with a slogan "We are the 99%". They blamed the traders in Wall Street for the emergence of the plutonomy, an extreme form of hierarchical society where the top 1% of households account for 33% of net worth, larger than the bottom 80% of households ([1, 2] and references therein). It is puzzling why the plutonomy has emerged in USA, "the Land of the Free and the Home of the Braves". This trend is observed worldwide. As in USA, wealth concentration occurs in the European advanced countries and Japan, where free competitions in the market are promoted. It is also enigmatic problem what mechanism causes the universal phenomenon of wealth concentration in a fair competitive society.

It has been shown that hierarchy can be self-organized in competitive societies where (1) an individual can fight with an opponent at a given frequency and the wealthier has a higher probability to win the fight, (2) the winner deprives the loser of its wealth, and (3) the wealth or debt of an individual relaxes to zero when it does not participate in fighting [3, 4]. The frequency of the fighting is controlled either by population of individuals [3] or by policies of the society [4]. When the frequency of fightings exceeds a critical value, the transfer of wealth occurs more frequently than the relaxation rate of the wealth, and winners who keep winning and losers who keep losing appear in the society, leading to a hierarchical society. In the hierarchical society, although winners, losers and middle class can be identified, the distribution of the wealth in the society is rather gradual and is completely different from that for the plutonomic society, where the number of the winners is a small fraction of the population and the wealth keeps accumulating in the winners. The key question is why the plutonomy, the extreme form of the hierarchical society, has emerged in recent times when the fair competition is the most important common value of the society.

In this paper, we introduce a model which incorporates groups and seasons in order to understand the effects of the fairness of society. Population are divided into several groups to compete among individuals at the same level for a season and they are regrouped at the end of every season. The fair society can be expressed for a society where the group size is small and a season is short. We show that the plutonomy is the inevitable consequence of a fair competitive society. The paper is organized as follows. In Sect. 2, we explain the model in detail. Results of Monte Carlo simulation are presented in Sect. 3. Section 4 is devoted to discussion.

2. Model

We begin with the competitive society introduced by [4]. In this society, each individual participates in fighting at a given Monte Carlo step with probability ρ. When it does, it chooses an opponent randomly from the rest of population and they fight each other. For the fight between individuals i and j, the winning probability, p_{ij}, of i against j is assumed to be

$$p_{ij} = \frac{1}{1+\mathrm{e}^{\eta(F_j - F_i)}}, \tag{1}$$

where F_i and F_j are the wealth of i and j, respectively, and η is a controlling positive parameter. The winning probability (1) guarantees that the wealthier always win when the difference in their wealths is extremely large and that the probability is one-half when their wealths are equal. We assume that two individuals participating in a fight must pay a fee c and that the winner gets a reward $2w$. That is, after the fight the wealth of the winner increases by $2w - c$ and that of the loser decreases by c. When $w = c$, then the winner gets and the loser loses wealth w after a fight. We take w as the unit of wealth in the following calculation. When all individuals take their turn for fighting, one Monte Carlo step is completed. We also assume that at each time step, the wealth (debt when it is negative) relaxes to zero following:

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$$F_i(t+1) = F_i(t) - \mu \tanh F_i(t), \quad (2)$$

where μ is a positive constant. Fujie and Odagaki [4] showed that when $\rho \geq \rho_c \equiv \frac{\mu}{2(2w-c)}$, a hierarchical society is self-organized. Here the order parameter of the self-organization is defined by:

$$\sigma^2 = \langle X_i^2 \rangle - \langle X_i \rangle^2, \quad (3)$$

where X_i is the fraction of fights that individual i won and $\langle \cdots \rangle$ is the average over all individuals. In the egalitarian society, $\sigma^2 = 0$ and in the hierarchical society $\sigma^2 > 0$.

In order to model a fair competitive society, we divide the people in the society into several groups of the same number of individuals according to their wealths and assume that an individual fights with an opponent within the group it belongs to. We also introduce a season of competition so that all individuals are re-grouped after one season according to their wealth at the end of the season. Let the number of groups be M and the length of a season be S. We fix the total number, N, of individuals and the total number of the Monte Carlo steps, T. Therefore, N/M, the number of individuals in each group and T/S, the length of the Monte Carlo steps in each season, are controlling parameters of the model. Note that results shown here remain essentially unchanged even when T changes for a fixed value of T/S.

3. Results

Since the case of $M = 1$ and $T/S = 1$ corresponds to the original model [4], we investigate the structure of the hierarchical society by changing M and S. We are interested in the emergence of the plutonomy when the controlling parameters are changed for a given value of ρ, and thus we set $w = c = 1$, $\eta = 5$, $\mu = 0.5$, $N = 3000$ and $T = 10^6$. Note that μ and η could be absorbed to the scale of the time and of the wealth and thus the choice of these numbers are arbitrary. Unless we discuss time dependence of observables, we present data of various quantities at the end of 10^6 steps of the Monte Carlo simulation. Figure 1 shows ρ dependence of the order parameter

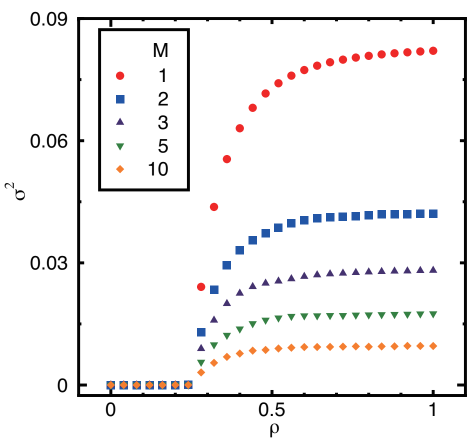

Fig. 1. ρ dependence of the order parameter for various values of M and a fixed value of $T/S = 10^2$.

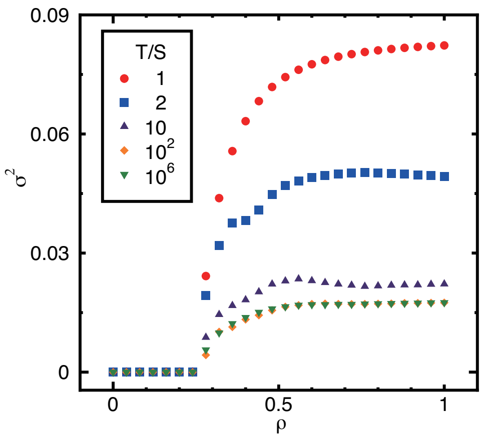

Fig. 2. The same as Fig. 1, but for various values of T/S and a fixed value of $M = 5$.

for various values of M and a fixed value of $T/S = 10^2$ and Fig. 2 shows ρ dependence of the order parameter for various values of T/S and a fixed value of $M = 5$. From Figs. 1 and 2, we observe that (i) the critical value for the emergence of hierarchy, $\rho_c = \frac{\mu}{2(2w-c)} = \frac{1}{4}$ for our choice of parameters [4], does not depend on M nor T/S and (ii) the hierarchical order measured by σ^2 gets smaller as M and T/S are increased. The society can be considered to be much fairer as the number of groups and/or the number of seasons are increased. Consequently, as the competition becomes fairer, the society becomes less inequal. We also observe in Fig. 2 that the order parameter σ^2 for $T/S > 1$ is not monotonically increasing function of ρ. Generally speaking, the order parameter increases as ρ is increased as seen for $T/S = 1$. For $T/S > 1$, winners in a lower group at the end of one season may fight with losers in a higher group in the next season. The formers may not keep winning and the latter may not keep losing depending on their wealth, which reduces the fluctuation in the winning probability.

Now, we investigate the structure of the hierarchical society in detail. Figure 3 shows (a) the wealth and (b) the winning average of all individuals plotted in the order of their ranking, respectively, for three different values of $M = 1, 5, 10$ and fixed values of $T/S = 10^2$ and $\rho = 1$. Figure 4 shows the same for three values $T/S = 1, 10, 10^6$ and fixed values $M = 5$ and $\rho = 1$. In the hierarchical society, we can easily identify three classes, the winners, the losers and the middle class [4] from the ranking dependence of the wealth or winning average.

In fact, we show in Fig. 5 how individuals move their ranking up or down at the end of some seasons. By observing the group of the wealthiest individuals, we can readily identify the winner whose ranking is unchanged. Note that the number of losers and winners are the same when $w = c$. As M or T/S is increased, the number of winners decreases and the number of individuals in the middle class increases, and thus the plutonomy is self-organized in the society.

It is important to note that in the plutonomic society

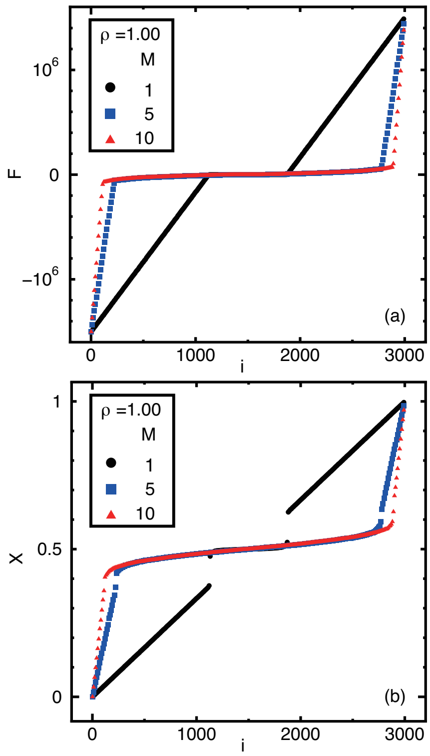

Fig. 3. (a) Wealths of all individuals and (b) winning averages of all individuals are plotted in the order of their ranking for $M = 1, 5, 10$ and fixed values of $T/S = 10^2$ and $\rho = 1$. For clear view, data for every 100 individuals is shown.

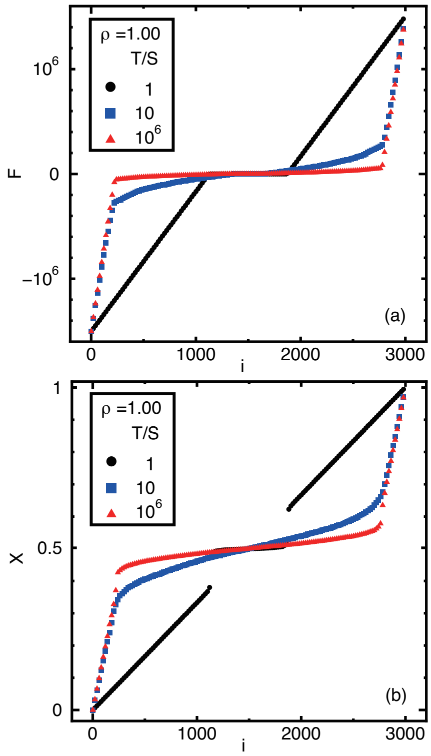

Fig. 4. The same as Fig. 3, but for $T/S = 1, 10, 10^6$ and fixed values of $M = 5$ and $\rho = 1$.

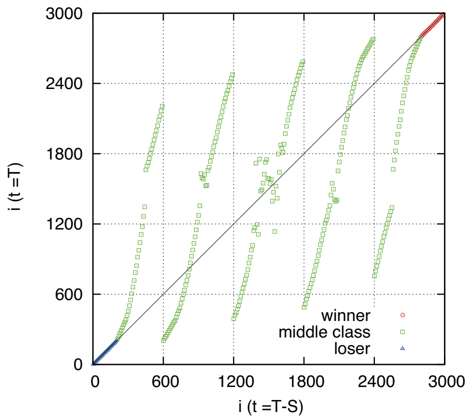

Fig. 5. Ranking change at the end of a season for $\rho = 1$ and $M = 5$. Most of individuals (green squares) change their ranking except for a few per cent of individuals among the top (red circles) and the bottom (blue triangles) groups. For clear view, data for every 100 individuals is shown.

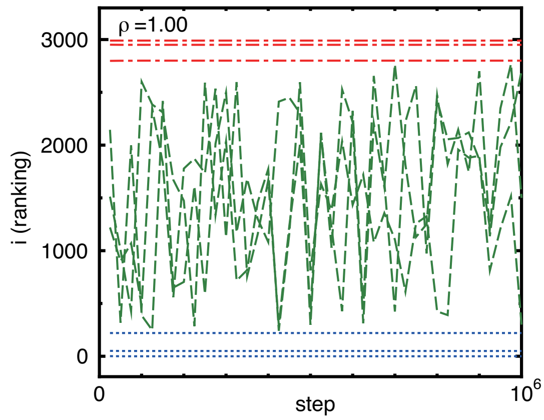

Fig. 6. Time dependence of ranking of representative three individuals in winners (top three dash-dot lines), losers (bottom three dotted lines) and the middle class (middle dashed lines).

the winners and losers are fixed. Figure 6 shows the time dependence of ranking of representative three individuals in winners (top three lines), losers (bottom three lines) and middle class (middle polygonal lines). While the ranking in the winners and losers are fixed, it always changes in time in the middle class.

Noting that the winners appear in the wealthiest group, we can estimate the number of winners when $T/S = 10^6$ on the basis of the mean field analysis [4]. According to the mean field analysis, the fraction of winners in a class, x_W, is given by

$$x_W = 1 - \frac{2\rho c + \mu}{4\rho w}. \tag{4}$$

For our choice of parameters, $x_W = 3/8$, and thus the number of winners will be $\frac{3}{8} \times \frac{1}{5} = 7.5\%$ since the wealthiest group consists of $1/5$ of all individuals. This estimation agrees quite well with the number read in Fig. 4a.

We show in Fig. 7 M dependence of the fractions of

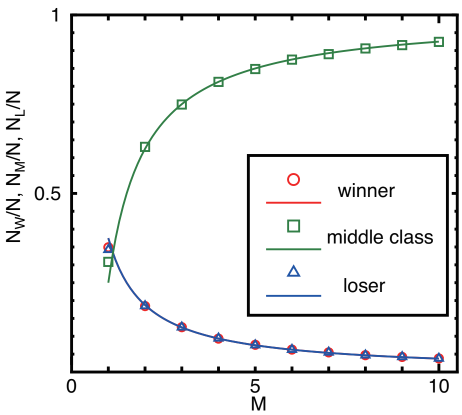

Fig. 7. M dependence of the fractions of winners, losers and middle class for $T/S = 10^2$ and $\rho = 1$. Solid curves are the estimation obtained by the mean field analysis.

winners, losers and middle class for $T/S = 10^2$ and $\rho = 1$. The M dependence of the fraction of the winners and the losers can be represented quite well by the mean field value $3/8M$ and the fraction of the middle class is given by $1-3/4M$ as shown in Fig. 7. Decrement of the winners as M is increased indicates that inequality is enhanced. In fact, as M is increased, the Gini coefficient [5] increases from 0.50 for $M = 1$ to 0.84 for $M = 5$.

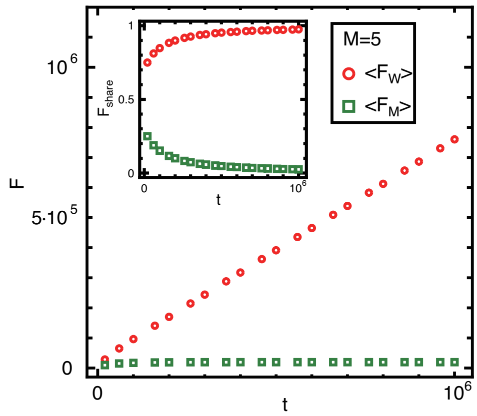

Fig. 8. Time dependence of the average wealth of individuals in the winners and in the middle class. The inset shows the share of the wealth of the winners and the middle class among the total positive wealth.

We can understand why the plutonomy is self-organized in our model society. For each season, hierarchy in each group is formed at the end of the season if ρ is larger than the critical value. In the next season, winners in lower groups must challenge to the winners in higher groups and they will lose in the fighting except for the winners in the wealthiest group who keep winning all the time and become wealthier and wealthier as shown in Fig. 5. In fact, we show the flow of wealth in Fig. 8, where the average wealth of individuals in the winners and in the middle class are plotted as functions of time. While the average wealth in the middle class stays unchanged (= zero), the average wealth in the winners keeps increasing. Note that the accumulation of wealth into the top 1% has been observed in the U.S. which is one of the important characteristics of the plutonomy in the U.S. [6].

4. Conclusion

We have studied the self-organization of hierarchy in a model society where people classified into several groups according to their wealth participate in fightings for a season and they are re-grouped at the beginning of the next season. As one can see in Fig. 5 for the case of $\rho = 1$, a moderate hierarchy in each group at the beginning of a season is enhanced at the end of the season. Winners in a group must fight with people in the upper group in the next season, ending as losers at the end of the season. Only exceptions are the winners in the strongest group who will keep winning. The wealth flows from the poorers to the wealthiest individuals continually, and the plutonomy is self-organized.

In the real society, one can easily see that people usually compete with others in the same level and when they become rich, they will try to fight with wealthier people. Thus, the present model must contain the essential mechanism of the emergence of the plutonomy in the United States of America or in the fair and equal-right competitive society.

In the present model, we set $w = c$ so that the cost c and the reward $2w - c$ of a fight are equal for all groups (so-called zero sum rule). In order to test the robustness of the results, we investigated the case where the cost and reward depend on the groups, keeping the zero-sum condition and found no essential differences in the results.

In passing, a similar emergence of plutonomy has been observed in the challenging society where individuals performing challenging-random-walk in random-order tend to fight in effect among the same group of their wealth [7].

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