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# Nonlinear Waves in GaAs Semiconductor

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The nonlinear propagation of acoustic waves in electron-hole semiconductor plasmas is studied. For this purpose, the reductive perturbation method is employed to the basic equations obtaining the Gardner equation. The latter has been solved using an extended homogeneous balance method to obtain a set of analytical solutions including solitary wave solution. The effects of different physical parameters on the nonlinear structures are examined.

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#### 1. Introduction

Quantum plasma physics is a sub-branch of plasma physics started seriously during the last decade [1–4]. Its importance is raising due to its large variety of applications both in nanoscale studies and in astrophysics environments. Actually, the transport of electrons/holes in semiconductors [5, 6] and powerful laser beams up to densities significantly exceeding the liquid hydrogen density provides us with a good environment of quantum effects which can appear. The physics of quantum plasmas is also relevant in the context of quantum nanodiodes, nanophotonics and nanowires, nanoplasmonics [7], high-gain quantum free-electron lasers [8], microplasma systems, and small semiconductor devices, such as quantum wells [9] and piezomagnetic quantum dots [10].

Furthermore, quantum effects are also present in astrophysical environments, such as in cores of white dwarf stars and magnetars [11]. Nowadays, pair plasmas is an interesting field of study which means large ensembles of charged matter consisting of charged particle populations bearing opposite charge signs such as the electrons and holes, which are created in semiconductors at high densities. Simply, when a semiconductor is excited, electrons absorb the photon energy and transit from the valence to the conduction band via single and/or multi-photon absorption, depending on the photon energy and the bandgap energy. This inter-band transition of the electrons creates holes in the valence band, and this state may satisfy the plasma conditions. The created plasma possesses different quantum mechanical effects such as tunneling effect [1], exchange interactions and correlations [12], degenerate pressure [1].

Different nonlinear equations can be used to describe many complex phenomena in sciences, e.g. fluid mechanics, plasma physics, optical fibers, solid state physics, geophysics, etc. To obtain exact solutions of these nonlinear equations many techniques could be employed. For example, tanh-function method [13], extended tanh method [14], sine-cosine method [15], F-expansion method [16], general expansion method [17], G0/G method [18, 19], homogeneous balance (HB) [20]. HB method is direct and effective algebraic method to find the exact traveling wave solutions. Interestingly, the homogeneous balance method was extended to search for other kinds of exact solutions [21, 22] in addition to solitary solutions. Fan [23] described two new applications of the homogeneous balance method and explored for the Backlund transformation and similarity reduction of nonlinear partial differential equations. Our aim of the present work is to use the homogeneous

balance method to solve the evolution equation that describes the quantum semiconductor plasma system and obtain a class of solutions appropriate to characterize the possible nonlinear waves.

The paper is organized as follows: In Sect. 2 we present the governing equations for electron-hole semiconductor plasmas including different quantum effects. In Sect. 3 the reductive perturbation method is employed to derive the Gardner equation describing the system. In Sect. 4 we apply the extended homogeneous balance method to obtain the possible solutions of the Gardner equation. A discussion and the numerical results are also presented. Finally, the results are summarized in Sect. 5.

## 2. Governing equations

We consider a quantum hydrodynamics (QHD) model for two-species quantum plasmas, composed of electrons and holes. The one-dimensional QHD model consists of continuity and momentum equations for both electrons and holes are

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$$\frac{\partial n_{\rm e,h}}{\partial t} + \frac{\partial (n_{\rm e,h} u_{\rm e,h})}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_{\mathrm{e,h}}}{\partial t} + u_{\mathrm{e,h}} \frac{\partial u_{\mathrm{e,h}}}{\partial t} \pm m_{\mathrm{e,h}}^* \frac{\partial \phi}{\partial x} + \frac{1}{m_{\mathrm{e,h}}^* n_{\mathrm{e,h}}} \frac{\partial P_{\mathrm{e,h}}}{\partial x} \qquad (2)$$

$$+\frac{1}{m_{\rm e,h}^*}\frac{\partial V_{\rm xce,h}}{\partial x} - \frac{\hbar^2}{2m_{\rm e,h}^{*2}}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_{\rm e,h}}}\frac{\partial^2}{\partial x^2}\sqrt{n_{\rm e,h}}\right) = 0.$$

Equations (1) and (2) are coupled by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0} (n_{\rm e} - n_{\rm h}). \tag{3}$$

Here  $n_j$ ,  $u_j$ ,  $V_{\text{xc}j}$ , and  $m_j^*$  denote the *j*-th species number density, velocity, exchange correlation potential, and effective mass, respectively, where j = e and h for electrons and holes. In Eq. (2), the positive sign is for holes and negative sign is for electrons. Furthermore, e is the magnitude charge of the electrons,  $\varepsilon_0$  is the vacuum dielectric constant,  $\phi$  is the electrostatic wave potential,  $P_e$  ( $P_h$ ) is the non-relativistic pressure of the degenerate electrons (holes).

The pressure of the non-relativistic degenerate plasma is assumed to obey the equation [3]:

$$P_{\rm e,h} = K_{\rm e,h} n_{\rm e,h}^{5l3},\tag{4}$$

where  $K_{\rm e,h} = \frac{3}{5} (\frac{\pi}{3})^{1/3} \frac{\pi \hbar^2}{m_{\rm e,h}^*}$ . The exchange-correlation potential  $V_{\rm xc}$  for the electrons and holes is given by [12]:

$$V_{\rm xce,h} = \frac{0.985}{\delta} n_{\rm e,h}^{1/3} \times \left( 1 + \frac{0.034}{a_{\rm Be,h}^* n_{\rm e,h}^{1/3}} \ln(1 + 18.37 a_{\rm Be,h}^* n_{\rm e,h}^{1/3}) \right), \qquad (5)$$

where  $a_{\text{Be,h}}^* = \delta \hbar^2 / m_{\text{e,h}}^* e^2$  is the effective Bohr radius and  $\delta$  is the effective dielectric constant of the material. It is seen that the second term on the right-hand side of Eq. (2) is the degenerate pressure due to the high number density of the electrons and holes. The third term on the right-hand side of Eq. (2) represents the quantum recoil force associated with the Bohm potential (quantum diffraction) due to the electrons/holes tunneling through a potential barrier. The last term in Eq. (2) represents the electron and hole exchange-correlation force between the identical particles when their wave functions overlap due to the high number density.

The following scaling can be used in order to normalize Eqs. (1)–(3) where  $V_{\rm Fe} = \left(\frac{k_{\rm B}T_{\rm Fe}}{m_e^*}\right)^{\frac{1}{2}}$  is the Fermi electron thermal speed,  $\omega_{\rm pe} = \left(\epsilon_0 \frac{m_e^*}{e^2 n_{\rm e0}}\right)^{-\frac{1}{2}}$  is the Fermi electron thermal speed,  $k_{\rm B}$  is the Boltzmann constant, and  $T_{\rm Fe}$  is the electron Fermi temperature in the non-relativistic and zero-temperature limits which is given by  $T_{\rm Fe} = \left(\frac{\hbar^2}{2k_{\rm B}m_e^*}\right) \left(3\pi^2 n_{\rm e0}\right)^{\frac{2}{3}}$ ,  $n_{\rm e0}$  ( $n_{\rm h0}$ ) denotes the unperturbed electrons (holes). The quasineutrality at equilibrium imposes the condition  $n_{\rm e0} = n_{\rm h0} = n_0$ . Using the above normalization, Eqs. (1)–(3) (dropping the overbar) can be cast into a dimensionless as

$$\frac{\partial n_{\rm e,h}}{\partial t} + \frac{\partial (n_{\rm e,h} u_{\rm e,h})}{\partial x} = 0, \tag{6}$$

$$\frac{\partial u_{\rm e,h}}{\partial t} + u_{\rm e,h} \frac{\partial u_{\rm e,h}}{\partial t} \pm M \frac{\partial \phi}{\partial x} + N n_{\rm e,h}^{-1/3} \frac{\partial n_{\rm e,h}}{\partial x}$$
(7)

$$+\frac{M}{k_{\rm B}T_{\rm Fe,h}}\frac{\partial V_{\rm xce,h}}{\partial x}-2H_{\rm e,h}^2\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_{\rm e,h}}}\frac{\partial^2}{\partial x^2}\sqrt{n_{\rm e,h}}\right)=0.$$

Equations (1) and (2) are coupled by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\rm e} - n_{\rm h}.\tag{8}$$

In Eqs. (6)–(8) M = 1; N = C for electrons and N = C0 for holes, where

$$C = (\pi/3)^{1/3} (\pi\hbar^2/m_{\rm e}^*) (n_{\rm e0}^{2/3}/k_{\rm B}T_{\rm Fe}),$$

and

$$C' = (\pi/3)^{1/3} (\pi \hbar^2/m_{\rm h}^*) (n_{\rm h0}^{2/3}/k_{\rm B}T_{\rm Fe}) M,$$
 with

$$M = m_{\rm e}^*/m_{\rm h}^*.$$

Here,  $H_{\rm e} = (\hbar \omega_{\rm pe}/2k_{\rm B}T_{\rm Fe})$  and  $H_{\rm h} = (\hbar \omega_{\rm ph}/2k_{\rm B}T_{\rm Fe})M$ , where  $\omega_{\rm ph} = (\varepsilon_0 m_{\rm h}^*/e^2 n_{\rm h0})^{-1/2}$  is the hole frequency.

In order to investigate the propagation of the nonlinear electrostatic waves in the quantum plasmas, we employ the reductive perturbation technique [1]. According to this method, the independent variables are stretched as  $\xi = \varepsilon^{1/2}(x - \lambda t)$  and  $\tau = \varepsilon^{3/2}t$  where  $\varepsilon$  is a small dimensionless expansion parameter; i.e  $0 < \varepsilon \ll 1$ , which characterizes the strength of the nonlinearity and  $\lambda$  is the acoustic wave phase speed normalized by  $V_{\rm Fe}$ . The dependent variables  $n_{\rm e}$ ,  $n_{\rm h}$ ,  $u_{\rm e}$ ,  $u_{\rm h}$  are expanded in terms of  $\varepsilon$  as

$$n_{\rm e,h} = 1 + \varepsilon n_{\rm e,h}^{(1)} + \varepsilon^2 n_{\rm e,h}^{(2)} + \dots$$
 (9)

$$u_{\rm e,h} = 1 + \varepsilon u_{\rm e,h}^{(1)} + \varepsilon^2 u_{\rm e,h}^{(2)} + \dots$$
 (10)

$$\phi = 1 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots \tag{11}$$

Substituting Eqs. (9)–(11) into Eqs. (6)–(8), then collecting terms of the same powers of  $\varepsilon$ , for the lowest-order, we get

$$u_{\rm e,h}^{(1)} = (\lambda - u_{\rm e,h}^{(0)}) n_{\rm e,h}^{(1)}, n_{\rm e}^{(1)} = -((\lambda - u_{\rm e}^{(0)})^2 - A - B - C)^{-1} \phi^{(1)}$$

and

$$n_{\rm h}^{(1)} = -((\lambda - u_{\rm h}^{(0)})^2 - A' - B' - C')^{-1}\phi^{(1)}$$

The Poisson equation gives the compatibility condition

$$((\lambda - u_{\rm e}^{(0)})^2 - A - B - C)^{-1} + M((\lambda - u_{\rm h}^{(0)})^2 - A' - B' - C')^{-1} = 0,$$

where

$$A = F/3k_{\rm B}T_{\rm Fe}, \ B = Gb/k_{\rm B}T_{\rm Fe}, \ A' = FM/3k_{\rm B}T_{\rm Fe},$$
$$B' = G'b'M/k_{\rm B}T_{\rm Fe}, \ F = 0.985\frac{e^2}{\delta}n_0^{1/3},$$

$$\begin{split} G &= 0.985 \times 0.034 (e^2/\delta a_{\rm Be}^*), \\ b &= \frac{1}{3} (18.37 a_{\rm Be}^* n_0^{1/3}/1 + 18.37 a_{\rm Be}^* n_0^{1/3}), \end{split}$$
 ad

$$\begin{split} G' &= 0.985 \times 0.034 (e^2/\delta a^*_{\rm Bh}), \\ b' &= \frac{1}{3} (18.37 a^*_{\rm Bh} n_0^{1/3}/1 + 18.37 a^*_{\rm Bh} n_0^{1/3}). \end{split}$$

The second-order in  $\varepsilon$  yields

$$n_{\rm e}^{(2)} = -\left[(\lambda - u_{\rm e}^{(0)})^2 - A - B - C\right]^{-3} \left[\frac{3}{2}(\lambda - u_{\rm e}^{(0)})^2 - \frac{1}{6}(C + 2A + 2B + 3bB)\right] (\phi^{(1)})^2 - \left[(\lambda - u_{\rm e}^{(0)})^2 - A - B - C\right]^{-1} \phi^{(2)}.$$

and

$$\begin{split} n_{\rm e}^{(2)} &= M^2 [(\lambda - u_{\rm e}^{(0)})^2 - A' - B' - C']^{-3} \left[ \frac{3}{2} (\lambda - u_{\rm e}^{(0)})^2 \right. \\ &\left. - \frac{1}{6} (C' + 2A' + 2B' + 3bB') \right] (\phi^{(1)})^2 - M [(\lambda - u_{\rm e}^{(0)})^2 \\ &\left. - A' - B' - C' \right]^{-1} \phi^{(2)}, \end{split}$$

while the Poisson equation gives where

$$\{[(\lambda - u_{\rm e}^{(0)})^2 - A - B - C]^{-1} + M[(\lambda - u_{\rm h}^{(0)})^2 - A' - B' - C']^{-1}\}\phi^{(2)} + \beta(\phi^{(1)})^2 = 0,$$
(12)

$$\beta = \frac{\frac{3}{2}(\lambda - u_{\rm e}^{(0)})^2 - \frac{1}{6}(C + 2A + 2B + 3bB)}{[(\lambda - u_{\rm e}^{(0)})^2 - A - B - C]^3} - M^2 \frac{\frac{3}{2}(\lambda - u_{\rm h}^{(0)})^2 - \frac{1}{6}(C' + 2A' + 2B' + 3bB')}{[(\lambda - u_{\rm h}^{(0)})^2 - A' - B' - C']^3}.$$
 (13)

The third-order in  $\varepsilon$  yields a system of equations in the third-order perturbed quantities. Solving this system with the aid of first-order and second-order results, we finally obtain the Gardner equation as

$$\frac{\partial u}{\partial \tau} + \alpha \beta u \frac{\partial u}{\partial \xi} + \alpha \gamma u^2 \frac{\partial u}{\partial \xi} + \frac{1}{2} \alpha \delta \frac{\partial^3 u}{\partial \xi^3} = 0.$$
(14)

where we replaced u by  $\varphi^{(1)}$  for simplicity. The coefficients  $\alpha$ ,  $\delta$  and  $\gamma$  are given as

$$\alpha = \left\{ \frac{2\left(\lambda - u_{e}^{(0)}\right)}{\left[\left(\lambda - u_{e}^{(0)}\right)^{2} - A - B - C\right]^{2}} + \frac{2\left(\lambda - u_{h}^{(0)}\right)M}{\left[\left(\lambda - u_{h}^{(0)}\right)^{2} - A' - B' - C'\right]^{2}} \right\}^{-1}, \quad (15)$$

$$\begin{split} \delta &= 1 - \frac{H_{\rm e}^2}{\left[ \left( \lambda - u_{\rm e}^{(0)} \right)^2 - A - B - C \right]^2} \\ &- \frac{H_{\rm h}^2 M}{\left[ \left( \lambda - u_{\rm h}^{(0)} \right)^2 - A' - B' - C' \right]^2}, \end{split} (16) \\ \gamma &= \frac{3 \left[ -3 \left( \lambda - u_{\rm e}^{(0)} \right)^2 + 2A/3 + 2B/3 + Bb + C/3 \right]}{2 \left[ \left( \lambda - u_{\rm e}^{(0)} \right)^2 - A - B - C \right]^5} \\ &+ \frac{3 \left[ \frac{C}{27} - 2 \left( \lambda - u_{\rm e}^{(0)} \right)^2 \right]}{2 \left[ \left( \lambda - u_{\rm e}^{(0)} \right)^2 - A - B - C \right]^4} \\ &+ \frac{3M^3 \left[ \frac{C}{27} - 2 \left( \lambda - u_{\rm h}^{(0)} \right)^2 \right]}{2 \left[ \left( \lambda - u_{\rm h}^{(0)} \right)^2 - A - B - C \right]^4} \\ &+ \frac{3 \left[ -3 \left( \lambda - u_{\rm h}^{(0)} \right)^2 + 2A'/3 + 2B'/3 + B'b' + C'/3 \right] M^3}{2 \left[ \left( \lambda - u_{\rm h}^{(0)} \right)^2 - A - B - C \right]^5} \end{split}$$

#### 3. Solutions of Gardner equation

Consider the Gardner Eq. (14) in the form

$$\frac{\partial u}{\partial \tau} + \Gamma u \frac{\partial u}{\partial \xi} + \Lambda u^2 \frac{\partial u}{\partial \xi} + \Omega \frac{\partial^3 u}{\partial \xi^3} = 0, \qquad (18)$$

where  $\Gamma = \alpha\beta$ ,  $\Lambda = \alpha\gamma$ , and  $\Omega = \alpha\delta/2$ . We seek for special solution of Eq. (18), travelling wave solution, in the form

$$u(\xi,\tau) = u(\zeta), \quad \zeta = \zeta - \vartheta \tau,$$
 (19)

where  $\vartheta$  is a constant to be determined later. Using the transformation (19) in Eq. (18), the later reduces to a nonlinear ordinary differential equation. The next step is that the solution we are looking for is expressed in the form

$$u(\zeta) = \sum_{i=0}^{n} a_i \omega^i + \sum_{i=1}^{n} b_i (1+\omega)^{-i}, \qquad (20)$$

and

$$\omega' = k + M\omega + P\omega^2, \tag{21}$$

where  $a_i$  and  $b_i$  are constants, while k, M and P are parameters to be determined later,  $\omega = \omega(\zeta)$ , and  $\omega' = d\omega/d\zeta$ . Using the extended homogeneous balance method [23, 24], the solution of the Gardner equation of the type (14) will be

$$u_1(x,t) = \frac{i\sqrt{\Omega}\sqrt{\frac{\Gamma^2}{\Omega\Lambda}}\tan\left(\zeta\sqrt{\frac{\Gamma^2}{\Omega\Lambda}}/2\sqrt{6}\right)}{2\sqrt{\Lambda}} - \frac{\Gamma}{2\Lambda}, \quad (22)$$

and

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$$u_{2}(x,t) = \frac{i\sqrt{\Omega}\sqrt{\frac{\Gamma^{2}}{\Omega\Lambda}}\cot\left(\zeta\sqrt{\frac{\Gamma^{2}}{\Omega\Lambda}}/2\sqrt{6}\right)}{2\sqrt{\Lambda}} - \frac{\Gamma}{2\Lambda}, \quad (23)$$

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$$u_{3}(x,t) = -\frac{\Gamma}{2\Lambda}$$
$$-\frac{i\sqrt{\Omega}\sqrt{-\frac{\Gamma^{2}}{\Omega\Lambda}}\tanh\left(\zeta\sqrt{-\frac{\Gamma^{2}}{\Omega\Lambda}}/2\sqrt{6}\right)}{2\sqrt{\Lambda}},$$
(24)

$$u_{4}\left(x,t\right)=-\frac{\Gamma}{2\Lambda}$$

$$-\frac{i\sqrt{\Omega}\sqrt{\frac{\Gamma^2}{\Omega\Lambda}}\coth\left(\zeta\sqrt{-\frac{\Gamma^2}{\Omega\Lambda}}/2\sqrt{6}\right)}{2\sqrt{\Lambda}},$$
(25)

$$u_5(x,t) = \frac{\Gamma\zeta + 2\sqrt{6}i\sqrt{\Omega}\sqrt{\Lambda}}{2\zeta\Lambda},$$
(26)

$$u_6(x,t) = -\frac{\mathrm{i}\sqrt{6}\sqrt{\Omega}p_1\tanh\left(\zeta p_1\right)}{\sqrt{\Lambda}},\tag{27}$$

$$u_7(x,t) = -\frac{i\sqrt{6}\sqrt{\Omega}\coth\left(\zeta p_1\right)p_1}{\sqrt{\Lambda}},\tag{28}$$

$$u_8(x,t) = -\frac{\mathrm{i}\sqrt{\frac{3}{2}}\sqrt{\Omega}\left(\sinh\left(\zeta\right) + \sqrt{r^2 - 1}\right)}{\sqrt{\Lambda}(r + \cosh\left(\zeta\right))},\qquad(29)$$

$$_{9}\left(x,t\right)=\tag{30}$$

$$\frac{\mathrm{i}\sqrt{\frac{3}{2}}\sqrt{\Omega}\left(\cosh\left(\frac{\zeta}{2}\right) + 2M\sinh\left(\frac{\zeta}{2}\right)\right)}{\sinh\left(\frac{\zeta}{2}\right)\sqrt{\Lambda}},$$
$$\mathrm{i}\sqrt{6}\sqrt{\Omega}\coth\left(\zeta\right)$$

$$u_{10}(x,t) = -\frac{\mathrm{i}\sqrt{6}\sqrt{\Omega}\coth\left(\zeta\right)}{\sqrt{\Lambda}},\tag{31}$$

$$u_{11}(x,t) = \frac{-\sqrt{6}\sqrt{-(k-P)^2 \Omega P^2} - \sqrt{\Lambda} a_0 \left[P - p_1 \left(P + \tanh\left(\zeta p_1\right)\right)\right]^2}}{P\sqrt{\Lambda} \left[p_1 \left(P + \tanh\left(\zeta p_1\right)\right) - P\right]},$$
(32)

$$u_{12}(x,t) = \frac{-\sqrt{6}\sqrt{-(k-P)^2} \Omega P^2 - \sqrt{\Lambda} a_0 \left[P - (P + \coth(\zeta p_1)) p_1\right]^2}}{P\sqrt{\Lambda} \left[(P + \coth(\zeta p_1)) p_1 - P\right]},$$
(33)

$$u_{13}(x,t) = \frac{-8\sqrt{6}P^2\sqrt{-(k-P)^2 \Omega} (r + \cosh(\zeta))^2 - \sqrt{\Lambda}(2r^2 + \cosh(2\zeta) + 4\sqrt{r^2 - 1}\sinh(\zeta) - 3)a_0}{4P\sqrt{\Lambda}(r + \cosh(\zeta))(\sinh(\zeta) + \sqrt{r^2 - 1})},$$
(34)

$$u_{14}(x,t) = (35)$$

$$\frac{4\sqrt{6}\sqrt{-(k-P)^2 \Omega}P^2 - \sqrt{\Lambda}\left(4P + \coth\left(\frac{\zeta}{2}\right)\right)^2 a_0}{2P\sqrt{\Lambda}\left(4P + \coth\left(\zeta\right) + \operatorname{csch}\left(\zeta\right)\right)},$$

$$u_{15}\left(x,t\right) = \tag{36}$$

$$\frac{-\sqrt{6}\sqrt{-(k-P)^2}\,\Omega\tanh\left(\zeta\right)P^2-\sqrt{\Lambda}\coth\left(\zeta\right)a_0}{P\sqrt{\Lambda}}.$$

## 4. Results and discussion

We have considered a collisionless, unmagnetized plasma, whose constituents are electrons and holes. To investigate the nonlinear dynamics of the acoustic waves, the reductive perturbation technique is employed to obtain the Gardner equation. The latter is solved using an extended homogeneous balance method which gives different classes of solutions. These solutions include many types, like rational, periodical, shock solutions, etc. For example, the solutions (22) and (23) are examples exhibiting the sinusoidal-type periodical solutions, which develop a singularity at a finite point, i.e. for

any fixed  $\tau = \tau_0$  there exists a value of  $\zeta_0$  at which these solutions blow up; see Fig. 1a.



Fig. 1. (a) Three-dimensional profile of the periodic solution [given by Eq. (22)] for fixed values of  $\hat{\alpha} = 0.6$ ,  $\beta = 0.1$ , and q = 0.7. (b) Three-dimensional profile of the explosive/blowup pulse [given by Eq. (28)] for the same parameters as in (a).

The solution (26) represents the rational-type solutions, which may be helpful to explain the creation of very high energy in the plasma system. Because the rational solution is a discrete joint union of manifolds, particle systems describe the motion of a pole of the evolution equation.

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Note that these excitations never reach zero, except in a very specific combination of parameter values. Our prediction for a potential excitation blowup indicates that an instability in the system may occur due to the effect of nonlinearity. In simple terms, the balance between dispersion and nonlinearity may be disturbed by variations of plasma quantities (e.g. temperature, pressure, density, etc.). This might locally destroy localized excitation stability leading to an amplitude increase to very high values; since this represents an increase in the electric potential, it might lead to an acceleration of the moving particles, it is important to notice that Eqs. (28) are different forms of explosive/blowup solutions as depicted in Fig. 1b.

Another different nonlinear wave that could be of interest is represented by solution (24), which represents the shock waves. Equation (24) can be written as

$$u\left(\zeta\right) = \frac{1}{2}\phi_{\rm m}\left(1 - \tanh\left(\frac{2\zeta}{W}\right)\right),\tag{37}$$

where  $\phi_{\rm m}$  and W are the amplitude and width of the shocks, respectively, and are given by

$$\phi_{\rm m} = -\frac{\Gamma}{\Lambda} \text{ and } W = 2 \frac{\sqrt{-\frac{12}{\Omega\Lambda}}}{|\phi_{\rm m}|}.$$
 (38)

It is clear from Eq. (37) that to have shock waves  $\Lambda$  should acquire negative value, i.e.  $\Lambda < 0$ . The GaAs semiconductor will be used as an example to investigate numerically the coefficient  $\Lambda$ .



Fig. 2. The contour plot of the coefficient  $\Lambda$  with  $u_{\rm e}^{(0)}$ and  $u_{\rm h}^{(0)}$  for GaAs semiconductor parameters  $n_0 = 4.7 \times 10^{22} {\rm m}^{-3}$ ,  $m_{\rm e}^* = 0.067 m_{\rm e}$ ,  $m_{\rm h}^* = 0.5 m_{\rm e}$ .

The numerical analyses in Fig. 2 define the possible regions of negativity that is represented by yellow zone, while for white zone  $\Lambda$  is greater than zero. Therefore, in our numerical analysis of the shock wave profile we will be limited within the yellow region.

Now, we shall study the variation of the shock wave profile against the streaming velocities of electrons and holes as depicted in Fig. 3a and b.

Figure 2a shows the shock or kink wave profile for constant electron streaming  $u_{\rm e}^{(0)} = 0.4$  and different values of



Fig. 3. (a) The shock wave profile for  $u_{\rm h}^{(0)} = 0.4$  and different values of hole streaming  $u_{\rm h}^{(0)} = 0.1$  (solid curve), 0.15 (dotted curve), and 0.2 (dashed curve). (b) The shock wave profile for  $u_{\rm h}^{(0)} = 0.1$  and different values of electron streaming  $u_{\rm e}^{(0)} = 0.2$  (solid curve), 0.25 (dotted curve), and 0.3 (dashed curve).

hole streaming of 0.1 (solid curve), 0.15 (dotted curve), and 0.2 (dashed curve). Streaming holes leads to increase the shock wave amplitude. Shock or kink wave profile for constant hole streaming  $u_{\rm h}^{(0)} = 0.1$  and different values of electron streaming of 0.2 (solid curve), 0.25 (dotted curve), and 0.3 (dashed curve) is depicted in Fig. 2b. Electron streaming leads to increase the shock wave amplitude, however the streaming holes have more influence to increase the wave amplitude. Therefore, the increase of hole streaming would lead to make the shock amplitude taller and accelerate the particles due to generation of high potential shock wave.

### 5. Conclusion

In this paper, we have studied the nonlinear propagation of acoustic waves in electron-hole semiconductor plasmas. We have derived the Gardner equation describing the system. Using an extended homogeneous balance method we obtain class of solutions of the Gardner equation. These solutions include different rational solutions and shock wave solution. We have used the present model to investigate the behavior of the nonlinear structures in GaAs semiconductor plasma. The effects of different physical parameters on the nonlinear structures are examined, which indicates that the nonlinear pulses suffer amplitude and width modifications due to change of the electrons and holes streaming parameters.

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