# Surface-Plasmon-Polaritons at the Interface of Graded Index Material and Semiconductor

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We have investigated the properties of structures incorporating graded index materials with parabolic permittivity profile. Surface-plasmon-polaritons at the interface of graded index material and semiconductor are studied by means of numerical simulations. We analyze the dependence of the dispersion characteristics on the graded index material profile as well as on the semiconductor concentration via the finite-difference time-domain simulations. Effects of the structure on dielectric and magnetic properties are taken into account by introducing the Drude model in the semiconductor dispersion.

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## 1. Introduction

Surface waves open the wide avenues for many physical phenomena forming a basis for a number of devices. Recently, subwavelength optical structures have been widely investigated and engineered to create unique anisotropic, dispersive, and nonlinear optical properties [1–7].

The wide avenues have been opened to apply the subwavelength structures in the integrated optics. Such structures can be used to optimize fiber-chip grating couplers [8], vertical-cavity surface-emitting lasers [9], and wavelength multiplexers [10], and to design lowloss waveguide crossings [11], high Q factor resonators [12], and ultra-broadband multimode interference couplers [13].

The potentiality to manipulate light at deep subwavelength scale is the blessing that has paid a significant attention to the surface plasmon polaritons during the past decades [14–16].

### 2. The state of the problem

In our proposal, an interface of parabolic graded index material (GRIN) waveguide and semiconductor for surface-plasmon-polaritons (SPPs) is synthesized numerically.

To design a parabolic GRIN waveguide for SPPs we will consider the interface of the GRIN medium and highly doped silicon. In order to synthesize structures with a graded index profile it suffices to vary the duty cycle of the sub-wavelength grating, as shown in Fig. 1.

It is interesting to notice that the case of heavy doped Si is considered, assuming that the doping level is  $N_1 = 5 \times 10^{19} \text{ cm}^{-3}$  [13]. An average effective mass  $m_1$  for electrons is  $0.26m_0$  with  $m_0$  being the free-electron mass, and  $\varepsilon_{\infty 1} = 11.68$ .



Fig. 1. Schematic of a semiconductor and graded index material interface realized by varying the duty cycle of the sub-wavelength grating [17].

## 3. Mathematical background

Using Maxwell's equations for nonmagnetic medium free of charges, one can derive a second order differential equation for a TM mode of the electromagnetic field in inhomogeneous and homogeneous media. Introducing a plane wave in the form

$$H = H(x) e^{i(\omega t - \beta z)}, \tag{1}$$

where  $\omega = 2\pi f$  with f being the frequency,  $\beta$  is the longitudinal propagation constant, z is direction of the wave propagation, x is direction transversal to the direction of the wave propagation.

Combining Maxwell's equations, a differential equation for the inhomogeneous part of the structure is obtained as

$$\frac{\mathrm{d}^2 H_{\mathrm{i}}}{\mathrm{d}x^2} - \frac{1}{\varepsilon_{\mathrm{i}}} \frac{\mathrm{d}\varepsilon_{\mathrm{i}}}{\mathrm{d}x} \frac{\mathrm{d}H_{\mathrm{i}}}{\mathrm{d}x} + \left(\omega^2 \mu_{\mathrm{i}}\varepsilon_{\mathrm{i}} - \beta^2\right) H_{\mathrm{i}} = 0, \quad (2)$$

where  $\varepsilon_{i} = \varepsilon(f, x)$  is spatially-varying frequencydependent dielectric permittivity. In the homogeneous part of the structure Eq. (2) reduces to the standard wave equation

$$\frac{\mathrm{d}^2 H_\mathrm{h}}{\mathrm{d}x^2} + \left[\omega^2 \mu_\mathrm{h} \varepsilon_\mathrm{h} - \beta^2\right] H_\mathrm{h} = 0. \tag{3}$$

Now we discuss the conditions satisfied by a wave propagating at a planar interface between two half-spaces one of which is inhomogeneous. In summary, the conditions for TM wave are as follows:

 $H_{\rm i} = H_{\rm h},$ 

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$$\frac{1}{\varepsilon_{i}}\frac{\partial H_{i}}{\partial z} = \frac{1}{\varepsilon_{h}}\frac{\partial H_{h}}{\partial z}.$$
(4)

The goal of obtaining the exact value of the longitudinal propagation constant can be achieved by solving Eq. (4). However, the main challenge that is arising here is the inhomogeneity of the magnetic field in the inhomogeneous medium, i.e.  $H_i = H_i(x)$ . The mentioned obstacle can be overcome by applying the appropriate numerical procedures of solving the wave equation in the inhomogeneous medium.

### 4. Numerical solutions

To obtain the insight into the propagation characteristics associated with the surface waves, numerical calculations have been performed using the analytical solutions presented above. This brings about interesting phenomena which is new and important for an actual design.

The method has been implemented in MATLAB code and simulations have been performed with the presence of the inhomogeneous layer. We consider an inhomogeneous medium shown in Fig. 1 for which the effective permittivity varies as the quadratic distribution given by

$$\varepsilon(x) = \varepsilon(x_0) \left[ 1 - 0.5\rho \left( x - x_0 \right)^2 \right]^2, \tag{5}$$

where  $x_0$  is the position of the optical axis and  $\rho$  is the gradient coefficient.

We numerically obtain the dispersion curves (the longitudinal propagation constant  $\beta$  versus propagation



Fig. 2. (a) Parabolic permittivity profile for the graded-index material. (b) Dispersion for the case of a different graded-index material profile.



Fig. 3. Dispersion for the case of a different concentration N of Si.

frequency  $\omega$ ) for the surface wave propagating at the boundary of two media. An average effective mass  $m_1$ for electrons is  $0.26m_0$  with  $m_0$  being the free-electron mass, and  $\varepsilon_{\infty 1} = 11.68$ .

The asymptotic frequencies of the surface waves can be tuned by changing the material design and its effective properties. The permittivities of the materials are displayed in Fig. 2a showing how they vary for different structure cases. The peak positions can be controlled through adjusting the structure of the materials under consideration. These properties are substantial in order to control the surface wave.

Figure 2b shows the longitudinal propagation constant  $\beta$  as a function of the propagation frequency, for different permittivity profiles (Fig. 2a). As can be seen from Fig. 2b, one can control the frequency range for surface waves by the permittivity profile of the inhomogeneous media. When the maximum value of the permittivity function is increased, the dispersion curve moves to a lower frequency range. The dependence of the frequency range for the surface wave existence on the permittivity profile provides additional degree of freedom to control surface waves.

To date it is well known that the presence of semiconductor in the structure can tune the properties of the investigated system in the easy way. Thus in Fig. 3 we present the dispersion curves of the surface wave dealing with the different concentrations of the Si.

#### 5. Conclusion

We have investigated properties of surface-plasmonpolaritons at the interface of graded index material and semiconductor. We have come up with an exact numerical wave solution when the permittivity profile varies according to a parabolic function.

Frequency range of surface wave existence can be engineered by varying GRIN material design. Consequently, the possibilities to tailor the surface waves are suggested. The former opens the wide avenues for different applications, i.e. GRIN lenses [18–20] used in free-space optics for focusing, collimation, and mode matching.

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