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Jack 3/5 State from Two-Body Interaction

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We investigate the Read–Rezayi parafermion state of correlated electrons at the fractional Landau level filling $\nu = 3/5$. It is a Jack polynomial generated by contact four-body repulsion. We show by exact diagonalization that it is also emerges from a suitable short-range two-body interaction. We find that it closely matches Coulomb ground state in the second Landau level of non-relativistic fermions, and thus possibly describes the $\nu = 13/5$ (and, by conjugation, $\nu = 12/5$) fractional quantum Hall effect in GaAs.

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1. Introduction

Fractional quantum Hall effect (FQHE) [1] reveals spontaneous emergence of a series of correlated, non-degenerate, gapped quantum fluid ground states in a system of essentially two-dimensional (2D) electrons with the Coulomb interaction filling particular fractions ν of an isolated, massively degerate Landau level (LL) formed in a high magnetic field. Since orbitals of the lowest LL are essentially monomials (in the complex 2D coordinate x), the many-electron wave-functions defining universality classes for different FQH states are seeked among complex antisymmetric polynomials. (This also applies to FQHE in higher LLs, via mapping onto the lowest LL.)

2. Theory

Virtually all known FQH states are explained by the composite fermion (CF) theory [2] which postulates binding by electrons an even number ($2p$) of vortices of the many-body wave function. Most invoke one kind of essentially free CFs filling an integral number n of effective CF LLs (ΛLs); this is the Jain series of fractions: $\nu = n/(2pn \pm 1)$, equating FQHE with integral QHE of CFs. Others depend on CF–CF interaction and their description involves extensions such as CF pairing/condensation [3] or additional CF degrees of freedom ("partitions") [4]. Among the latter is the "parafermion" $\nu = k/(k+2)$ series of states [5] generated as unique zero-energy ground states of contact $(k+1)$-body repulsions, exemplifying the Jack polynomials [6].

The Jack polynomials J^{α}_{λ} are indexed by integer partitions λ and real numbers α. They are eigenstates of the following Laplace–Beltrami Hamiltonian defined in the space of symmetric polynomials

$$H_{\mathrm{LB}}(\alpha) = \alpha \sum_i (x_i \partial_i)(x_i \partial_i) + \sum_{i<j} (x_i + x_j)(x_i - x_j)^{-1}(x_i \partial_i - x_j \partial_j). \quad (1)$$

An explicit recursion construction exists for symmetric Jack polynomials [7]; fermionic Jacks have an additional antisymmetrizing Vandermonde factor.

The Jacks relevant for FQHE must yield uniform wave functions; in the Haldane spherical [8] geometry this means no total angular momentum $L = 0$. This sets the value of α, while partition λ determines the filling factor ν [9].

3. Results

Let us consider a particular Jack: Read–Rezayi (RR) $k = 3$ parafermion state [5], expected to form in the first excited ($n = 1$) LL of massive spinless electrons and thus a plausible candidate for the $\nu = 13/5$ FQHE in GaAs.

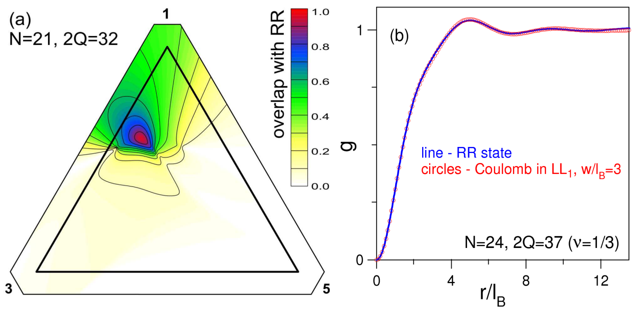

Fig. 1. (a) Contour map of overlaps of the RR state with ground states of two-body pseudopotentials (V_1, V_3, V_5), calculated on a sphere for $N = 21$ particles. Corners of the inner triangle correspond to one positive pseudopotential V_m (m as indicated) and all others vanishing; the inside to $V_{m=1,3,5} > 0$ and $V_{m>5} = 0$ [10]. (b) Pair correlation functions $g(r)$ of RR state and Coulomb ground state in the $n = 1$ LL (layer width $w/l_B = 3$; l_B is the magnetic length), calculated for $N = 24$.

First, we checked if it can be accurately generated by a suitable short-range two-body interaction. We diagonalized two-body interactions with arbitrary three leading pseudopotentials $V_1 + V_3 + V_5 = 1$ (V_m defined as dependence of pair energy V on relative angular momentum m [8]). In Fig. 1a we show the map of overlaps of the lowest $L = 0$ eigenstate of (V_1, V_3, V_5) with the RR state, obtained for a fairly large system of $N = 21$ particles on a sphere (results for smaller systems are similar). Indeed, for $V_1 : V_3 : V_5 \approx 6 : 3 : 1$ the overlap reaches 97%, which demonstrates that a suitable two-body interaction can simulate a higher-order interaction.

Second, from similar maps we have also determined optimum model interactions (V_1, V_3, V_5) which simulate Coulomb ground states in different LLs. We found that only the $n = 1$ LL in GaAs (especially in wider quasi-2D layers) is simulated by nearly the same model as the RR state.

Finally, we have directly compared the RR and Coulomb states; the overlaps are listed in Table I; the pair correlation functions are drawn in Fig. 1b. Indeed, the $\nu = 13/5$ FQHE in GaAs appears a manifestation of the RR state, described by a Jack wave function. By particle–hole conjugation, the (also observed) $\nu = 12/5$ FQHE state is the corresponding "anti-Jack".

TABLE I

Overlaps of the RR state with different Coulomb $L = 0$ ground states on a sphere. Columns: electron number N, magnetic flux $2Q$, dimension of the diagonalized N-body subspace with $L_z = 0$, and the overlaps with Coulomb states in the $n = 0$ and 1 LLs in GaAs (LL_n) and in the $n = 1$ and 2 LLs in graphene (G-LL_n). Layer width is zero except for $\mathrm{LL}_1^{\mathrm{wide}}$ corresponding to 3 magnetic lengths.

N	$2Q$	Dim	LL_0	LL_1	$\mathrm{LL}_1^{\mathrm{wide}}$	G-LL_1	G-LL_2
18	27	2×10^5	0.5399	0.9369	0.8995	0.5458	0.3584
21	32	5×10^6	0.5689	0.8990	0.9316	0.5714	0.1332
24	37	1×10^8	0.3442	0.8100	0.8792	0.3468	0.1408

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