The Control of Entanglement in a Damped Jaynes–Cummings Model by Transient Effects

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(Received May 22, 2015; in final form August 14, 2015)

In this work, we study the entanglement dynamics of the damped Jaynes–Cummings model with the transient effects modeled as the linear sweep of the coupling coefficient between the atom and the field. We show that the decoherence of the entanglement can be controlled by the transient effects under the influence of damping. These effects can accelerate and can decelerate the decoherence of the entanglement during the time-evolution of the system, by adjusting the frequency of the Rabi oscillations.

DOI: 10.12693/APhysPolA.128.256

PACS: 03.65.Ud, 42.50.Ct, 03.65.Yz

1. Introduction

The Jaynes–Cummings model (JCM) [1] is a very convenient model to describe the interaction of a two-level atom with a cavity field under the rotating wave approximation. One advantage of this model is its suitability to include additional considerations into the interaction. One consideration is the inclusion of the transient effects into the interaction. Joshi and Lawande [2] modified the JCM coupling coefficient as the linear sweep case to include these effects and studied some dynamical properties. Linear sweep model was studied in detail elsewhere for the squeezing characteristics in a quantum harmonic oscillator [3, 4] and for the entanglement dynamics in the atom–field interaction [5–7].

In this paper, we consider another system to investigate the influence of transient effects on the entanglement properties of the atom–field system with the dissipation of the upper state. We show that the decoherence of the entanglement can be controlled by the transient effects under the influence of damping. These effects can accelerate and can decelerate the decoherence of the entanglement during the time-evolution of the system, by adjusting the frequency of the Rabi oscillations.

2. Theory

The Hamiltonian of the system with the resonance between the atom and the field is given by ($\hbar = 1$) [8–11]:

$$H = \frac{\omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \omega a^\dagger a + \lambda(a^\dagger |g\rangle\langle e| + a|e\rangle\langle g|) - \mathrm{i}\frac{\gamma}{2}|e\rangle\langle e|, \qquad (1)$$

where $|e\rangle, |g\rangle$ are the excited and the ground states of the atom and $a, a^\dagger$ are the field annihilation and creation operators. w is the atomic transition and the field frequency and λ is the coupling coefficient between the atom and the field. γ is the damping coefficient. When $\gamma = 0$, we have the JCM Hamiltonian [1].

In the presence of the transient effects, the coupling coefficient is modified as [2]:

$$\lambda(t) = \begin{cases} \lambda k t/T & \text{for } 0 \le t \le T, \\ 0 \text{ otherwise.} \end{cases}$$

In this linear sweep modification, the interaction strength achieves an adiabatic variation with small values of k and a sudden jump with large values of k, for a fixed value of T. In the real-world situations, these transient effects can be produced by a very slow shift or a sudden jump of the electric field interacting with the atom.

Then, we have the Hamiltonian

$$H = \frac{\omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \omega a^\dagger a + \frac{\lambda k t}{T}(a^\dagger |g\rangle\langle e| + a|e\rangle\langle g|) - \mathrm{i}\frac{\gamma}{2}|e\rangle\langle e|. \qquad (2)$$

The exact solution of the system is obtained by $|\psi(t)\rangle = \exp(-\mathrm{i}\int_0^t H(\acute{t})\mathrm{d}\acute{t}))|\psi(0)\rangle$ where $|\psi(0)\rangle$ is the initial state of the system and for simplicity we assume that the field is in the number state and the atom is in the excited state $|\psi(0)\rangle = |ne\rangle$. When the atom is in the excited state, some events such as spontaneous emission or the collision between the atoms will cause the damping. In this case, the excited state will partly decay to the other atomic states apart from the ground state. The state of the system will evolve in time into the state

$$|\psi(t)\rangle = A_n(t)|ne\rangle + B_n(t)|n+1g\rangle, \qquad (3)$$

where

$$A_n(t) = \exp\left(-\frac{t}{4}(\gamma + 2\mathrm{i}(w + 2nw))\right)$$

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$$\times\left(\cos\frac{t}{4}\sqrt{4x_n^2t^2-\gamma^2}-\gamma\frac{\sin\frac{t}{4}\sqrt{4x_n^2t^2-\gamma^2}}{\sqrt{4x_n^2t^2-\gamma^2}}\right), \quad (4)$$

$$B_n(t)=-2\mathrm{i}\exp\left(-\frac{t}{4}(\gamma+2\mathrm{i}(w+2nw))\right)$$

$$\times x_n t\frac{\sin\frac{t}{4}\sqrt{4x_n^2t^2-\gamma^2}}{\sqrt{4x_n^2t^2-\gamma^2}}, \quad (5)$$

where $x_n=\lambda k\sqrt{n+1}/T$.

The entanglement properties of the system can be quantified by the Wootters concurrence [12]. It is defined as $C=\max(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4})$ where λ_is are the eigenvalues of the matrix $R=|\psi(t)\rangle\langle\psi(t)|(\sigma\otimes\sigma|\psi(t)\rangle\langle\psi(t)|^*\sigma\otimes\sigma)$, in the decreasing order. $C=0$ for the separable states and $C=1$ for the maximally entangled states.

3. Results and discussion

In the following figures, we analyze the influence of transient effects on the dynamics of both the entanglement and the transition probability of the excited state in the damped interaction of the atom-field. (In these, we assume that $\lambda=1$.)

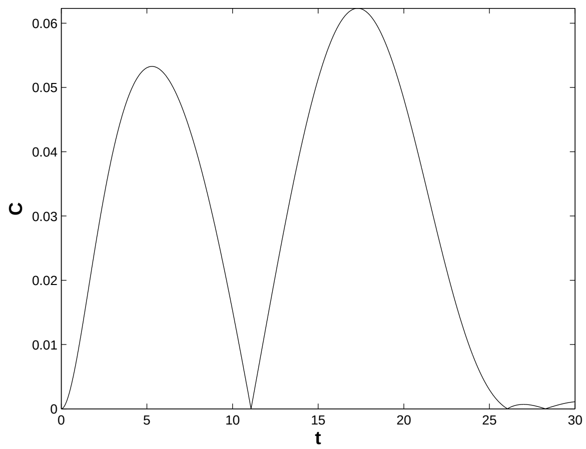

Fig. 1. Entanglement C as a function of time t. $n=3$, $\gamma=0.5$, $k=0.2$, and $T=30$.

Figure 1 shows the time-evolution of the entanglement in the case of the adiabatic variation interaction. It is clear that the entanglement has very small values and there is almost completely no decay in the considered time-interval, because the adiabatic variation induces a low frequency of the Rabi oscillations. So, it delays the growth of the entanglement after the interaction begins. In result, the damping weakens the entanglement very rapidly, almost completely without a decay in the oscillations. In this case, the entanglement in the system behaves as if it does not feel the decay effect of the damping. When the interaction strength is increased with the value of k for a fixed value of T (the interaction strength changes from the adiabatic variation to the sudden jump) as shown in Figs. 2 and 3, the entanglement achieves

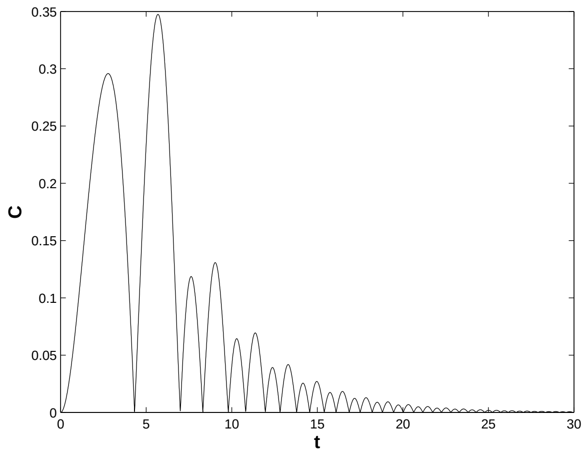

Fig. 2. As in Fig. 1, but for $k=2$.

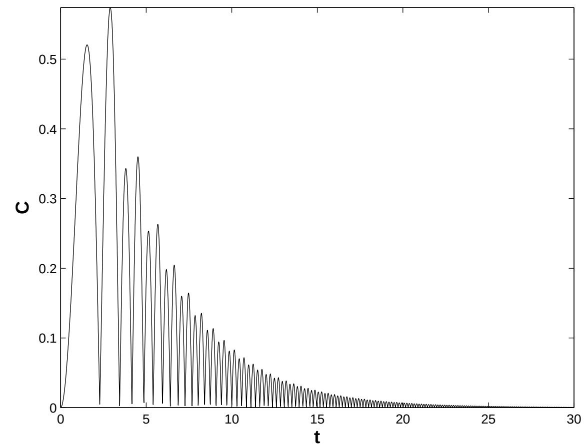

Fig. 3. As in Fig. 1, but for $k=8$.

higher values with an explicit decay in the oscillations. So, the entanglement in the system feels the damping effectively, because, in this case the Rabi oscillations have a higher frequency and accordingly the entanglement grows up and achieves higher values in a shorter time (as compared to the adiabatic variation case), after the interaction begins. But, the damping suppresses the amplitude of the entanglement oscillations in a finite time-interval.

The revealed dynamical behavior of the entanglement production above is directly related to the transition probability of the atomic excited state to decay to other states apart from the ground state due to the damping. We observe the similar dynamical results for the transition probability as like for the entanglement with the same parameters, as shown in Figs. 4–6. In the adiabatic variation case, the transition probability takes small values almost completely without a decay. As we reach to the sudden jump in the interaction, the transition probability takes higher values with an apparent decay in a finite time-interval.

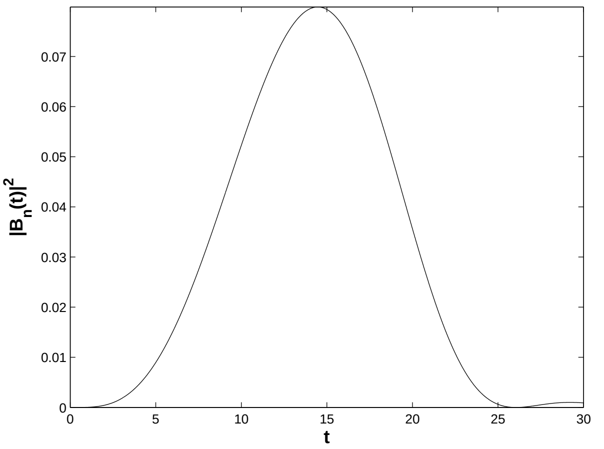

Fig. 4. The transition probability $|B_n(t)|^2$ as a function of time t. $n = 3$, $\gamma = 0.5$, $k = 0.2$ and $T = 30$.

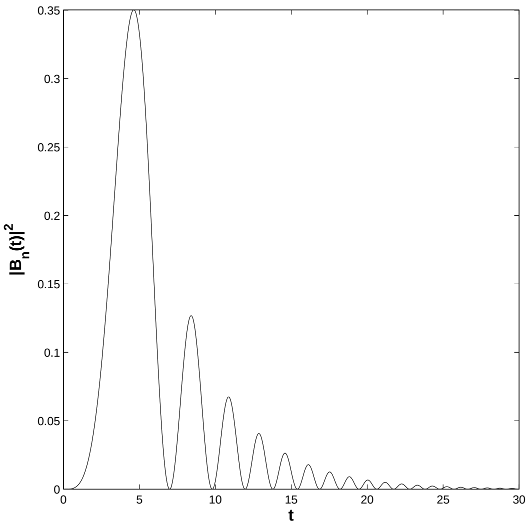

Fig. 5. As in Fig. 4, but for $k = 2$.

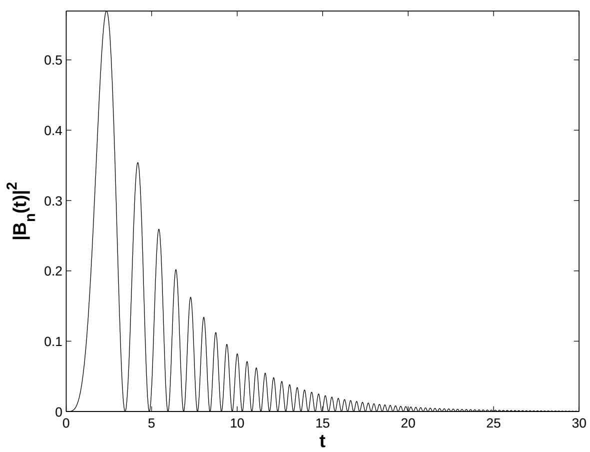

Fig. 5. As in Fig. 4, but for $k = 8$.

Due to the linear time-dependence of the interaction between the atom and the field, the frequency of the Rabi oscillations in the entanglement and in the transition probability is governed by $e^{\pm i \lambda t^2}$ instead of $e^{\pm i \lambda t}$. In this case, the frequency of these Rabi oscillations grows up and then speeds up, as time passes. What controls this dynamical behavior is the parameters of the transient effects (k and T). For a constant value of T, this dynamical behavior will evolve slowly for a small value of k (adiabatic variation) and will evolve fast for a high value of k (sudden jump). So, under the influence of damping, a sudden jump interaction produces more powerful entanglement and enhances the transition probability with an apparent decay, by adjusting the frequency of the Rabi oscillations. Therefore, the decoherence of both the entanglement and the transition probability can be controlled by the transient effects under the influence of damping.

4. Summary

We study the entanglement dynamics of the damped Jaynes–Cummings model with the transient effects modeled as the linear sweep of the coupling coefficient between the atom and the field. We show that the decoherence of the entanglement can be controlled by the transient effects under the influence of damping. These effects can accelerate and can decelerate the decoherence of the entanglement during the time-evolution of the system, by adjusting the frequency of the Rabi oscillations.

Acknowledgments

I am grateful to the referee for the valuable contributions to improving and interpreting the results of this work.

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