Special issue of the International Conference on Computational and Experimental Science and Engineering (ICCESEN 2014)

One-Loop Perturbative Corrections for Noncommutative $N = 1$ Supesymetric Gauge Theories in two Dimensions

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We calculate the contribution of the tadpole for gauge field of the noncommutative gauge supersymmetric theory in two dimensions in Minkowski space. We have found two types of divergences UV/IR similar to those found in the contribution of the ghost field loop.

DOI: 10.12693/APhysPolA.128.B-330

PACS: 14.80.Ly, 11.30.Pb, 14.80.Da, 12.60.Jv

1. Introduction

Noncommutative field theories possess specific properties of UV- and IR-divergencies which distinguish them from ordinary theories very essentially. It was shown for noncommutative scalar field models [1, 2] that the UV-divergencies lead to additional IR-singularities of effective action that was called the UV/IR-mixing. A study of extended supersymmetric model deserves a special attention. For example, maximally extended supersymmetric noncommutative $N = 4$ SYM is finite and has no UV/IR-mixing [3] what is convenient for investigating the low-energy behavior [4].

Our work deals mainly with the perturbative corrections to $N = 1$ noncommutative supersymmetric $U^*(1)$ gauge theory at one loop order with two dimensions in Minkowki space. We treat the deformation of the gauge theory $U^*(1)$ supersymmetric $N = 1$ in two dimensions. We quantify the theory to deduce the terms of gauge and ghosts fields of Faddeev Popov. Thereafter, we derive the Feynman rules of this theory, then we calculate the radiative corrections for the gauge field propagator of this theory at one loop order.

The present work gives the contribution of the tadpole for gauge field of the noncommutative gauge supersymmetric theory in two dimensions in Minkowski space.

2. Noncommutative Gauge Supersymmetric Theory in Two Dimensions

The noncommutative action of pure $U^*(1)$ gauge supersymmetric theory in

Minkowski space time with two dimensions is given by [5]:

$$S = \int \mathrm{d}x^2 \Big\{ -\frac{1}{4} F^*_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha}\left(\partial^\mu B_\mu\right)^2 - \frac{1}{2}\left(\partial_\mu M\right)^2$$
$$-\frac{\mathrm{i}}{2}\left(1 - \frac{1}{\alpha}\right)\bar{\lambda}\gamma^\mu \partial_\mu \lambda - \frac{\mathrm{i}}{2} g B^{\mu *} M * \overleftrightarrow{\partial}_\mu M$$
$$+\frac{1}{2} g \bar{\lambda}\gamma^{5*}\lambda^* M + \mathrm{i}\bar{c}\partial^\mu D_\mu c - \frac{1}{2} g \bar{\lambda}\gamma^{\mu *}\lambda^* B_\mu$$
$$+\frac{1}{4} g^2 \left(B^*_\mu M^* B^{\mu *} M - B^*_\mu B^{\mu *} M^* M\right) \Big\}, \qquad (1)$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \frac{\mathrm{i}g}{2}\left[B_\mu, B_\nu\right]$, $D_\mu M = \partial_\mu M + \frac{\mathrm{i}g}{2}\left[B_\nu, M\right]_*$.

As a result the propagator is the same as in the commutative counterpart but each vertex contain a supplementary phase factor which depends on deformation parameter [5].

2.1. The contribution of the tadpole for gauge field propagator

By using the Feynman rules, we obtain the contribution of the tadpole for gauge field propagator (Fig. 1):

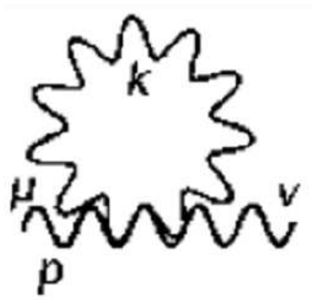

Fig. 1. The tadpole for gauge field propagator.

$$\pi_{\mu\theta} = \mathrm{i}g^2 g_{\mu\theta}\left(\alpha + 1\right)\int \frac{\mathrm{d}^2 k}{\left(2\pi\right)^2} \frac{\sin^2 \frac{1}{2}\left(k \wedge p\right)}{\left(k^2 + \mathrm{i}\varepsilon\right)}. \qquad (2)$$

This contribution is composed of planar and nonplanar diagrams:

$$\pi^{PL}_{\mu\theta} = \frac{1}{2}\mathrm{i}g^2 g_{\mu\theta}\left(\alpha + 1\right)\int \frac{\mathrm{d}^2 k}{\left(2\pi\right)^2} \frac{1}{\left(k^2 - m^2 + \mathrm{i}\varepsilon\right)}, \qquad (3)$$

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$$\pi_{\mu\theta}^{\mathrm{NPL}} = \frac{1}{2}\mathrm{i}g^2 g_{\mu\theta}(\alpha+1) \times \int \frac{\mathrm{d}^2k}{(2\pi)^2}\frac{1}{(k^2-m^2+\mathrm{i}\varepsilon)}\mathrm{e}^{\mathrm{i}k\rho}, \quad (4)$$

where $\rho^\mu = \theta^{\mu\nu}p_\nu$ and $\rho^2 = \rho_0^2 - \rho_1^2$.

Applying the Wick rotation, Schwinger formula and the Gauss integral with modified Bessel function:

$$\pi_{\mu\theta}^{(d)PL} = \frac{1}{4\pi}g^2 g_{\mu\theta}(\alpha+1) \times \left(\gamma + \ln\sqrt{\frac{m^2}{L^2}} + O\left(\frac{m^2}{L^2}\ln\frac{L}{m}\right)\right). \quad (5)$$

For the non-planar contribution, the situation is even worse in Minkowski space time due to its indefinite metric [2].

2.1.1. Space like case $\rho^2 < 0$

$$\pi_{\mu\theta}^{\mathrm{NPL}} = \frac{1}{4\pi}g^2 g_{\mu\theta}(\alpha+1) \times \left(\gamma + \ln\sqrt{\frac{m^2}{L_{1\mathrm{eff}}}} + O\left(\frac{m^2}{L_{1\mathrm{eff}}}\ln\frac{L_{1\mathrm{eff}}}{m}\right)\right), \quad (6)$$

where $L_{1\mathrm{eff}} = \frac{1}{4(\rho^1)^2 + \frac{1}{L^2}}$.

2.1.2. Time like $\rho^2 > 0$

$$\pi_{\mu\theta}^{\mathrm{NPL}} = \frac{1}{4\pi}g^2 g_{\mu\theta}(\alpha+1) \times \left(\gamma + \ln\sqrt{\frac{m^2}{L_{0\mathrm{eff}}}} + O\left(\frac{m^2}{L_{0\mathrm{eff}}}\ln\frac{L_{0\mathrm{eff}}}{m}\right)\right), \quad (7)$$

where $L_{0\mathrm{eff}} = \frac{1}{4(\mathrm{i}\rho^0)^2 + \frac{1}{L^2}}$.

The planar contributions give rise to ordinary UV singularity. The non-planar contributions have two types of divergences UV/IR.

3. Conclusion

We have found two types of divergences UV/IR similar to those found in the contribution of the Ghost field loop [5]. We have given the contribution of the tadpole for the gauge propagator at one loop order. This contribution is (UV-IR) divergent as the loop of the theory calculated in [5]. Finally, the persistence of the ultraviolet-infrared divergencies is interpreted as a further indication that UV-IR is essential property of Noncommutative field theories.

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