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Quadrupole Moments and Deformation Parameters of the $^{166-180}$Hf, $^{180-186}$W and $^{152-168}$Sm Isotopes

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In this study, the quadrupole moments of the $^{166-180}$Hf, $^{180-186}$W and $^{152-168}$Sm isotopic chains have been calculated by using the Woods-Saxon mean field potential in the framework of the superfluid model of the atomic nucleus. Our calculation showed that the deformation parameter mainly used in literature overestimates the actual values of deformation parameters β_2 by 10% for the well-deformed rare-earth nuclei. Besides, the contribution of the hexadecapole deformation to all quadrupole moments in question is smaller than 0.01 barn. The results showed that the theoretical values of the quadrupole moments are in good agreement with the previous theoretical works which have been done only for Sm isotopes and the corresponding experimental data for nuclei in question. The best theoretical value for quadrupole moment has been reached for the ^{152}Sm isotope.

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1. Introduction

A comparison of the theoretical values of the quadrupole moments which are a significant quantity for deformed nuclei, with experimental data gives a chance to test the nuclear models. In fact, the quadrupole moment of a nucleus can be calculated by the microscopic [1] and phenomenological models [2]. The essential lack of the phenomenological models could not give sufficient information about the structure of the nuclear levels. On the other hand, the microscopic models which take into account the interaction between the nucleons in the nucleus is a more useful approximation to investigate the structure of the atomic nucleus. It is well known that the existence of the deformed nuclei have been recognized because of the fact that the experimental values of the quadrupole moments are several times higher than the values proposed by the single particle shell model [2].

Among the microscopic models, the most useful one, is certainly the BCS model which is based on the shell model [1]. A systematical calculation for the well deformed nuclei in the rare- earth region using the anisotropic Nilsson potential has been performed in [3, 4]. Recently, the discovery of the new deformed regions which are far from the stability region of the elements, has attracted attention on the neutron rich and the neutron deficient nuclei, i.e., exotic nuclei [5, 6]. Therefore, for the research on the structure, half-life and the other properties of these nuclei, correct determination of the mean field parameters is very important. However, neutron-rich nuclei in question have not been studied, satisfactorily. The most dependable calculations have been done for samarium isotopes in [7, 8]. Here, there are two aims. The first one is to present and to compare the results obtained by using the well-known superfluid model for $^{166-180}$Hf, $^{180-186}$W and $^{152-168}$Sm with the experimental data and the previous studies (available only for Sm isotopes). The second one is to determine the β_2 deformation parameters by comparing the theoretical values of the quadrupole moments with the experimental data.

2. Method

It is well known that the shape of the axially symmetric deformed nucleus is described by the deformation parameter (β_2) directly connected to the quadrupole moment (Q_0) which represents the homogeneous charge distribution [9, 10].

$$Q_0 = \frac{3ZR_0^2}{\sqrt{5\pi}}(\beta_2 + 0.36\beta_2^2 + \ldots), \quad (1)$$

where Z is the atomic number, $R_0 = 1.2A^{1/3}$ fm and $\beta_2 < 1$. The quadrupole deformed nuclei are labelled as prolate ($Q_0 > 0$) or oblate ($Q_0 < 0$). Generally in literature during the determination of the experimental value of β_2 deformation parameter in the first approximation is assumed as $\beta_2^2 \ll 1$ and by neglecting the β_2^2 term in the Eq. (1),

$$\beta_2^{(u)} = \frac{Q_0\sqrt{5\pi}}{3ZR_0^2} \quad (2)$$

is obtained [10].

The relationship between deformation parameters ($\beta_2^{(u)}$) defined using approximation in Eq. (1) and the mean-field deformation parameter (δ) used in the numerical calculations of eigenvalue problem differ by less than 30% is given by

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$$\delta = \sqrt{\frac{45}{16\pi}}\beta_2^{(u)} - \frac{15}{8\pi}\beta_2^{(u)^2} + \frac{125}{32\pi}\beta_2^{(u)^4} - \dots . \quad (3)$$

As known, there is a simple relation between the quadrupole moments (Q_0) of the axial symmetric deformed nuclei and $B(E_2)$ values which represents the transition probability from the ground state to the lowest excited state with $K^\pi = 2^+$ given by

$$Q_0 = \sqrt{\frac{16\pi}{5}}\sqrt{\frac{B(E_2)}{e^2}}, \quad (4)$$

where e denotes the electric charge of the proton [2]. Here, $B(E_2)$ are basic experimental quantities that do not depend on nuclear models. However, for the well deformed nuclei, the values of the deformation parameter by using Eq. (2) do not agree with the values obtained by the experimental methods.

In this work using the Eq. (1) without using any approximation for the deformation parameter β_2, the formula that we obtained is given as,

$$\beta_2 = \frac{-1 + \sqrt{1 + 1.44\beta_2^{(u)}}}{0.72}. \quad (5)$$

Now the relation between the β_2 and mean-field deformation parameter δ slightly differs from Eq. (3) mainly used in literature and has a form

$$\delta = \sqrt{\frac{45}{16\pi}}\beta_2 - \frac{45}{32\pi}\beta_2^2 + \dots . \quad (6)$$

According to the superfluid model, the electric quadrupole moment of the nucleus is equal to the sum of the quadrupole moments of neutron and proton systems,

$$Q_0 = Q_0^n + Q_0^p, \quad (7)$$

where,

$$Q_0^n = 2\sum_\nu \langle\nu| r^2 Y_{20} |\rangle v_\nu^2;$$

$$Q_0^p = 2\sum_\nu \langle\mu| r^2 Y_{20} |\rangle v_\mu^2 \quad (8)$$

and $|\rangle$ ($|\rangle$) and ν (μ) denote the wave function of the neutron (proton) and quantum number set in the deformed mean field potential, respectively. In addition, the multiplying factor 2 before the sums is due to the z-component of the total angular momentum in the direction of symmetry axis that has twofold degeneracy. According to the superfluid model [1], the occupation probability of the particle in a level is given by

$$v_s^2 = \frac{1}{2}(1 - \frac{E_s - \lambda}{\varepsilon_s}), \quad (9)$$

where ε_s denotes the quasi-particle energy and E_s represents the mean field energy of any nucleon that has a quantum number set s. Besides, Δ and λ are the gap and the chemical potential parameters in the superfluid model, respectively. These parameters are calculated for the neutron and proton systems separately, using the fundamental formula of the superfluid model as follows,

$$\frac{2}{G} = \sum_s \frac{1}{\varepsilon_s}, \quad N = 2\sum_s v_s^2. \quad (10)$$

3. Numerical calculations and discussions

In this study, the single particle energies have been calculated using deformed Woods-Saxon potential [9, 10]. For the neutrons and the protons all energy levels occupied in the N= 2−7 shells from the bottom up to the 6 MeV in the nuclear potential well have been taken into account. For nucleus that has small deformation, Equation (2) does not influence the results anymore. However, for the well deformed nuclei, the values of the deformation parameter by using Eq. (2) do not agree with the values obtained by the experimental methods. Therefore, we calculated deformation parameter (β_2) by using Eq. (5) and compared these results with the results obtained from Eq. (2). In Fig. 1, the calculated values using Eq. (2) and Eq. (5) for Hf and W isotopic chains have been given as a function of mass number A.

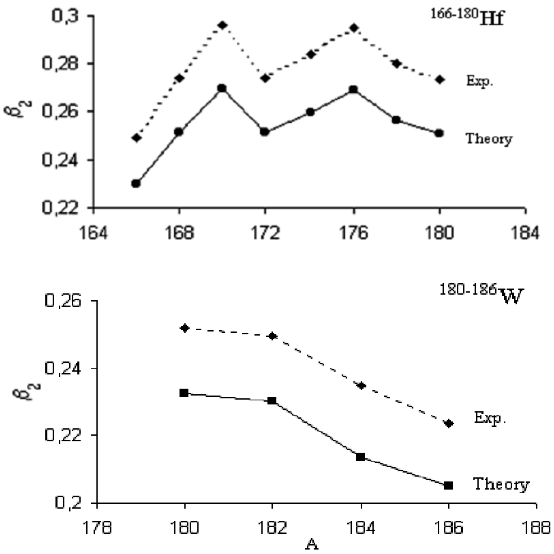

Fig. 1. The quadrupole deformation parameters for the $^{166-180}$Hf and $^{180-186}$W isotopes. Dashed and solid line denote the values obtained using Eq. (2) and Eq. (5), respectively.

We observed that Eq. (2) which is obtained by neglegting β_2^2 term gives a 10% bigger than the actual values. These results prove the significance of Eq. (5) for the experimental determination of the deformation parameter. Such a picture is peculiar for the investigated Sm isotopes.

Studies on the deformed nuclei via Coulomb excitation experiments showed that there exists also hexadecapole deformation in these nuclei [11, 12 and references therein]. The parameter β_4, which represents hexadecapole deformation, is about ten times smaller than the values of the parameter β_2 for any nucleus. It should be stated that for the nuclei at the beginning of the $150 \leq A \leq 190$ region, the parameter β_4 is a positive quantity but β_4 decrease with increasing A. Finally, for the last nuclei at the end of this region, it takes negative values. [1, p. 264].

In Table I, using the experimental data [7], the calculated values of the parameter β_4 have been listed only for Hf isotopes as an example. Such pictures are also peculiar for W and Sm isotopes. In this table, the quadrupole moments for $\beta_4 = 0$ have been given for comparison. As can be seen, taking the hexadecapole deformation into account is not much effective on the results. The contribution of the hexadecapole deformation to the total deformation is not over 1% for all nuclei under study. The parameters β_2^{th} and δ^{th} in Table I have been obtained theoretically by fitting the quadrupole moments to the experimental data [13].

TABLE I

The calculated values of the quadrupole moments for the $^{166-180}$Hf isotopes by taking into account β_4.

Nucleus	β_2^{th}	δ^{th}	Q_{th} [barn] $\beta_4 = -0.02$	Q_{th} [barn] $\beta_4 = 0$
$^{166}_{72}$Hf	0.0957	0.0857	5.868	5.89
$^{168}_{72}$Hf	0.2479	0.2349	6.542	6.561
$^{170}_{72}$Hf	0.3603	0.3554	7.032	7.072
$^{172}_{72}$Hf	0.2445	0.2317	6.622	6.641
$^{174}_{72}$Hf	0.3062	0.2968	6.92	6.95
$^{176}_{72}$Hf	0.3777	0.3754	7.235	7.28
$^{178}_{72}$Hf	0.295	0.2852	6.936	6.961
$^{180}_{72}$Hf	0.2631	0.2516	6.813	6.836

Using the Eq. (5) for Hf and W isotopic chains, theoretical values of quadrupole moments as a function of the mass number A, have been shown in Fig. 2. In these figures the experimental values were taken from [13] for comparison.

As can be seen from Table I and Fig. 2, the theoretical results agree with the experimental ones. The reason of the disagreement in ^{166}Hf and ^{186}W is that our assumption for the calculations of the quadrupole moment is not valid for these nuclei. These results show that the lowest excited 2^+ state is not related with the rotation of the nucleus in the nuclei near the closed shells.

The results of quadrupole moments for $^{152-168}$Sm isotopes have been presented in Table II. Besides, the predictions in [7, 8], and the experimental values [13] (they are available only for $^{152-154}$Sm isotopes) have been presented in Table II.

As can be seen from Table II, for the ^{152}Sm, the present work gives better agreement with the corresponding experimental data [13] than the other theoretical calculations [7, 8]. However, for ^{154}Sm, the results of [8], is the best one for experiment. In [8], the authors used FBCS method and stated that except for the first three nuclei their results systematically were lower than those of [7]. For this reason, they claimed that discrepancy might be attributed to the particle number fluctuation in the BCS theory which had not been taken into account in [8]. According to them, the well known shortcoming of the BCS theory could be emphasized in the neutron rich nuclei. However, although any projection method that conserves particle number was not used in the present work, as seen in Table II, except ^{152}Sm isotope the values that we obtain for quadrupole moments is lower than those of [7] and except the ^{160,162,164}Sm isotopes are lower than those of [8]. As is well known, the mathematical approximation used in the description of pairing correlations lead to the non conservation of particle number. To counteract this effect, the particle number is conserved only in average in the BCS model introducing Lagrange parameters called 'chemical potentials'.

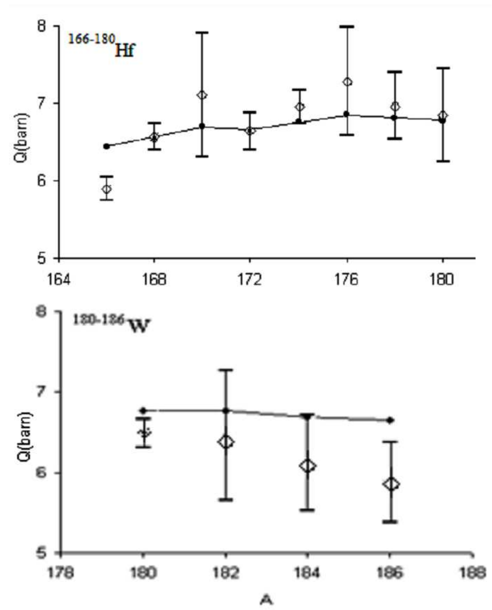

Fig. 2. Variation of the experimental and the theoretical values of the electric quadrupole moments versus atomic mass number A for the $^{166-180}$Hf and $^{180-186}$W isotopes. Where • and ◇ denote the theoretical and the experimental values, respectively. The experimental values are given in the experimental error limits.

TABLE II

Quadrupole moments (in barn) of the even-even $^{152-168}$Sm nuclei. The predictions of the present work are compared with the other theoretical studies [7, 8] and the experimental data [13] available only for $^{152-154}$Sm isotopes.

Nucleus	β_2	δ_2	Q (this work)	Q [7]	Q [8]	$Q_{\text{exp.}}$ [13]
$^{152}_{62}$Sm	0.243	0.203	5.91	5.69	5.96	5.88
$^{154}_{62}$Sm	0.270	0.222	6.11	6.22	6.56	6.62
$^{156}_{62}$Sm	0.279	0.229	6.22	6.42	6.55	–
$^{158}_{62}$Sm	0.279	0.229	6.25	6.62	6.40	–
$^{160}_{62}$Sm	0.290	0.236	6.33	6.81	5.96	–
$^{162}_{62}Sm$	0.300	0.243	6.44	6.96	6.09	–
$^{164}_{62}$Sm	0.302	0.245	6.46	7.18	6.02	–
$^{166}_{62}$Sm	0.294	0.239	6.37	7.30	6.49	–
$^{168}_{62}$Sm	0.284	0.232	6.33	7.23	6.61	–

4. Conclusion

We conclude that in the framework of the microscopic model, together with permanent deformed nuclei, also for the nuclei in the extremities of the regions in question quadrupole moment can be calculated successfully. The fitted values of the deformation parameter are in good agreement with the experimental values that obtained from the electric quadrupole transitions.

We also conclude that it seems a weak argument that the present discrepancy among the predictions of the methods might be attributed to the particle number fluctuations in the BCS theory. It is possible to obtain acceptable results for the electric quadrupole moments of the natural and also the exotic nuclei using the BCS theory.

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