Acoustical Engineering 2015

Total Acoustic Power of Two Concentric Clamped Circular Plates Vibrating in a Fluid

W.P. Rdzanek*

University of Rzeszów, Department of Mechatronics and Control Science,
Faculty of Mathematics and Natural Sciences, Prof. St. Pigonia 1, 35-310 Rzeszów, Poland

This study focuses on the problem of sound radiation by two concentric clamped flat plates, circular and annular, into the half-space. The system of three coupled differential equations comprising two equations of motions of plates and the wave equation, is solved exactly. Vibrations of plates are axisymmetric and time-harmonic with a single excitation frequency. The initial phase difference of excitations can be nonzero. Attenuation due to fluid loading and material damping is included. Kirchoff-Love and Kelvin-Voigt theories are applied. The effect of initial phase difference of excitations on the acoustic power radiated is examined as well as errors resulting from neglecting the fluid loading.

DOI: 10.12693/APhysPolA.128.A-41

PACS: 43.20.Bi, 43.20.El, 43.20.Fn, 43.20.Rz

1. Introduction

Problems of sound radiation by systems of vibrating sources are important. The radiated sound field and energy quantities are of great interest in practical applications since a number of sources of that type appear in such devices as casings, partitions, architecture, transducer arrays, and active control systems. Majority of studies dealt with sound radiation of a single vibrating surface such as a piston, membrane, plate, or shell, where the fluid loading influences considerably responses of vibrating structures. Literature studies can be divided into three major groups dealing with sound radiation of a single surface source [1–30], a system of two or more surface sources [31–41], and systems of active control of noise and vibrations [42–54].

So far, the problem of sound radiation of two vibrating concentric plates, circular and annular, with fluid loading and material damping factors taken into account, was not solved using rigorous methods. Therefore, this problem is undertaken in the present paper.

2. Governing equations and their solutions

2.1. Helmholtz equation

Vibrating resilient concentric plates, circular and annular, are embedded into a flat rigid baffle (cf. Fig. 1). Vibrations are axisymmetric and time-harmonic. The initial phase difference of plates' excitations can be nonzero. The acoustic waves are radiated upwards. Attenuation of waves in the fluid is neglected. The fluid loading is taken into account as well as plates' internal damping. Normal components of vibration velocities of the plates are

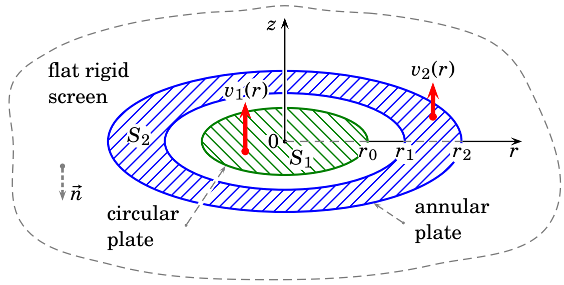

Fig. 1. Geometry of the problem.

$$v_\mu(r,t) = v_\mu(r)\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{1}$$

where index $\mu = 1$ applies to the circular plate of radii $r_{1,\mu} = 0$ and $r_{2,\mu} = r_0$; index $\mu = 2$ applies to the annular plate of radii $r_{1,\mu} = r_1$ and $r_{2,\mu} = r_2$, $0 < r_0 \le r_1 < r_2$; and ω is the angular frequency. The vibration velocity amplitude is

$$v_\mu(r) = -\mathrm{i}\omega W_\mu(r), \tag{2}$$

where W_μ is the transverse deflection amplitude. The acoustic pressure is

$$p(r,z,t) = p(r,z)\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{3}$$

whereas its amplitude is

$$p(r,z) = \sum_{\mu=1}^{2} p_\mu(r,z). \tag{4}$$

The wave processes in the fluid are governed by the Helmholtz equation

$$\left(\nabla^2 + k^2\right) p(r,z) = 0, \tag{5}$$

here $k = \omega/c$ is the acoustic wavenumber, c is the speed of sound, and the axisymmetric Laplace operator is

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}. \tag{6}$$

The acoustic pressures can be expressed as [1]

$$p_\mu(r,z) = k\varrho_0 c \int_0^\infty \tau \frac{\mathrm{e}^{\mathrm{i}z\sqrt{k^2-\tau^2}}}{\sqrt{k^2-\tau^2}} \bar{D}_\mu(\tau) J_0(\tau r)\,\mathrm{d}\tau, \tag{7}$$

*e-mail: wprdzank@ur.edu.pl

where $⨍$ denotes the principal value, the integration path is shown in Fig. 2, J is the Bessel function of the first kind, and the directivity pattern is

$$\bar{D}_\mu(\tau) = \int_{r_{1,\mu}}^{r_{2,\mu}} r v_\mu(r) J_0(\tau r)\,\mathrm{d}r. \tag{8}$$

The acoustic pressure amplitude satisfies the radiation condition [55] and the Neumann boundary condition

$$\left.\frac{\partial p(r,z)}{\partial z}\right|_{z=0} = \mathrm{i}\,k\varrho_0 c \begin{cases} v_\mu(r); & r_{1,\mu} \le r \le r_{2,\mu}, \\ 0; & \text{otherwise.} \end{cases} \tag{9}$$

The time-averaged acoustic power assumes the form of

$$\Pi = \sum_{\mu,\nu=1}^{2} \Pi_{\mu,\nu}, \tag{10}$$

where

$$\Pi_{\mu,\nu} = \pi \int_{r_{1,\mu}}^{r_{2,\mu}} r v_\mu^*(r) p_\nu(r,0)\,\mathrm{d}r = \pi k \varrho_0 c ⨍_0^\infty \frac{\tau \bar{D}_\mu^*(\tau)\bar{D}_\nu(\tau)\,\mathrm{d}\tau}{\sqrt{k^2-\tau^2}} \tag{11}$$

are the acoustic self-powers for $\mu = \nu$ and the acoustic mutual powers for $\mu \ne \nu$, whereas the symbol * denotes conjugate value.

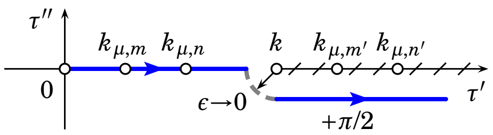

Fig. 2. Integration path in the plane of variable $\tau = \tau' + \mathrm{i}\tau''$.

2.2. Modes expansion method

The next step consists in applying modal expansions of physical quantities describing behavior of vibrating system presented in the previous subsection. The amplitude of transverse deflection of plates assumes the form

$$W_\mu(r) = \sum_{n=1}^{\infty} a_{\mu,n} W_{\mu,n}(r), \tag{12}$$

where $a_{\mu,n}$ are unknown coefficients,

$$W_{\mu,n}(r) = \beta_{\mu,n}\left[G_0(k_{\mu,n}r) + \alpha_{\mu,n} I_0(k_{\mu,n}r) - \delta_{2,\mu}\gamma_n K_0(k_{\mu,n}r)\right] \tag{13}$$

are eigenfunctions of axisymmetric modes $(0,n)$ (cf. [56]),

$$\beta_{\mu,n} = 2^{-1/2}\left[s^{2(\mu-1)} - \delta_{2,\mu}\right]^{1/2} \times \left[s^{2(\mu-1)} G_0^2(k_{\mu,n}r_{2,\mu}) - \delta_{2,\mu} G_0^2(k_{\mu,n}r_{1,\mu})\right]^{-1/2}, \tag{14a}$$

$$\xi_n = \frac{sS(k_{2,n}r_2) - S(k_{2,n}r_1)}{sT(k_{2,n}r_2) - T(k_{2,n}r_1)}, \tag{14b}$$

$$\begin{Bmatrix} S(u) \\ T(u) \end{Bmatrix} = \begin{Bmatrix} J_1(u) \\ Y_1(u) \end{Bmatrix} I_0(u) + \begin{Bmatrix} J_0(u) \\ Y_0(u) \end{Bmatrix} I_1(u), \tag{14c}$$

$$G_\nu(u) = J_\nu(u) - \delta_{2,\mu}\xi_n Y_\nu(u), \tag{14d}$$

for $\nu = 0, 1$; Y is Neumann function; I and K are modified Bessel and McDonald functions; $\delta_{2,\mu}$ is the Kronecker delta; $k_{\mu,n}$ is the structural wavenumber associated with the axisymmetric mode $(0,n)$; $k_{\mu,n}^4 = \omega_{\mu,n}^2 \varrho_\mu h_\mu / D_{0,\mu}$; $D_{0,\mu} = E_\mu h_\mu^3 / [12(1-\nu_\mu^2)]$ is the plates' bending stiffness; ϱ_μ and h_μ are density and thickness, respectively; E_μ and ν_μ are the Young modulus and the Poisson ratio, respectively; $\omega_{\mu,n} = 2\pi f_{\mu,n}$ is angular eigenfrequency in rad/s; and $f_{\mu,n}$ is the eigenfrequency in Hz. It was shown earlier [15] that coefficients $\alpha_{\mu,n}$ and γ_n are not necessary for analysis of sound radiation. The frequency equation is

$$\frac{s^{\mu-1} S(k_{\mu,n}r_{2,\mu}) - \delta_{2,\mu} S(k_{\mu,n}r_{1,\mu})}{\left[sT(k_{\mu,n}r_{2,\mu}) - T(k_{\mu,n}r_{1,\mu})\right]^{\mu-1}} = \delta_{2,\mu}\frac{sN(k_{\mu,n}r_{2,\mu}) - N(k_{\mu,n}r_{1,\mu})}{sR(k_{\mu,n}r_{2,\mu}) - R(k_{\mu,n}r_{1,\mu})}, \tag{15}$$

where

$$\begin{Bmatrix} N(u) \\ R(u) \end{Bmatrix} = \begin{Bmatrix} J_1(u) \\ Y_1(u) \end{Bmatrix} K_0(u) - \begin{Bmatrix} J_0(u) \\ Y_0(u) \end{Bmatrix} K_1(u). \tag{16}$$

Eigenfunctions satisfy the orthogonality condition

$$\frac{2\pi}{S_\mu}\int_{r_{1,\mu}}^{r_{2,\mu}} r W_{\mu,n}(r) W_{\mu,n'}(r)\,\mathrm{d}r = \delta_{nn'}, \tag{17}$$

where $S_\mu = \pi(r_{2,\mu}^2 - \delta_{2,\mu} r_{1,\mu}^2)$ is the plate surface area. Modal expansions of the vibration velocity, the directivity pattern, the acoustic pressure amplitude, and time-averaged acoustic self-powers and mutual powers can be expressed, respectively, as

$$v_\mu(r) = \sum_{n=1}^{\infty} \frac{\omega}{\omega_{\mu,n}} a_{\mu,n} v_{\mu,n}(r), \tag{18a}$$

$$\bar{D}_\mu(\tau) = \sum_{n=1}^{\infty} \frac{\omega}{\omega_{\mu,n}} a_{\mu,n} \bar{D}_{\mu,n}(\tau), \tag{18b}$$

$$p_\mu(r,z) = \sum_{n=1}^{\infty} \frac{\omega}{\omega_{\mu,n}} a_{\mu,n} p_{\mu,n}(r,z), \tag{18c}$$

$$\Pi_{\mu,\nu} = \frac{1}{2}\varrho_0 c\omega^2 \sqrt{S_\mu S_\nu} \sum_{n'=1}^{\infty}\sum_{n=1}^{\infty} a_{\mu,n'}^* a_{\nu,n} \zeta_{n',n}^{(\mu,\nu)}, \tag{18d}$$

where the modeshape and modal coefficients of the directivity, the acoustic pressure, and the acoustic self-impedance and mutual impedance are, respectively,

$$v_{\mu,n}(r) = -\mathrm{i}\omega_{\mu,n} W_{\mu,n}(r), \tag{19a}$$

$$\bar{D}_{\mu,n}(\tau) = \int_{r_{1,\mu}}^{r_{2,\mu}} r v_{\mu,n}(r) J_0(\tau r)\,\mathrm{d}r = \frac{-\mathrm{i}\omega_{\mu,n}\psi_{\mu,n}(\tau)}{k_{\mu,n}^4 - \tau^4}, \tag{19b}$$

$$p_{\mu,n}(r,z) = k\varrho_0 c ⨍_0^\infty \tau \frac{\mathrm{e}^{\mathrm{i}z\sqrt{k^2-\tau^2}}}{\sqrt{k^2-\tau^2}} \bar{D}_{\mu,n}(\tau) J_0(\tau r)\,\mathrm{d}\tau, \tag{19c}$$

$$\zeta_{n',n}^{(\mu,\nu)} = \zeta_{n,n'}^{(\nu,\mu)} =$$

$$\frac{2\pi k}{\sqrt{S_\mu S_\nu}}\int_0^\infty \frac{\tau\psi_{\mu,n'}(\tau)\psi_{\nu,n}(\tau)\,\mathrm{d}\tau}{(k_{\mu,n'}^4-\tau^4)(k_{\nu,n}^4-\tau^4)\sqrt{k^2-\tau^2}}, \tag{19d}$$

where the following symbol is introduced for brevity

$$\psi_{\mu,n}(\tau) = 2k_{\mu,n}^2 s^{-(\mu-1)} r_{2,\mu}\beta_{\mu,n}\Big\{ k_{\mu,n}\left[s^{\mu-1}\right.$$
$$\times G_1(k_{\mu,n}r_{2,\mu})J_0(\tau r_{2,\mu}) - \delta_{2,\mu}G_1(k_{\mu,n}r_{1,\mu})J_0(\tau r_{1,\mu})\left.\right]$$
$$-\tau\left[s^{\mu-1}G_0(k_{\mu,n}r_{2,\mu})J_1(\tau r_{2,\mu})\right.$$
$$\left.-\delta_{2,\mu}G_0(k_{\mu,n}r_{1,\mu})J_1(\tau r_{1,\mu})\right]\Big\}. \tag{20}$$

2.3. Forced vibrations

The total acoustic radiation power can be calculated on the basis of Eqs. (10) and (18d). For this purpose, it is necessary to calculate coefficients $a_{\mu,n}$ by solving the following system of equations of motion according to the Kelvin-Voigt theory:

$$D_{0,\mu}\nabla^4 W_\mu(r,t) + R_\mu \frac{\partial}{\partial t}\left[\nabla^4 W_\mu(r,t)\right]$$
$$+\varrho_\mu h_\mu \frac{\partial^2}{\partial t^2} W_\mu(r,t) = f_\mu(r,t) - p(r,0,t), \tag{21}$$

where R_μ is the dissipation factor, k_{D_μ} is the structural wavenumber, $k_{D_\mu}^4 = \omega^2\varrho_\mu h_\mu/D_\mu$, $D_\mu = D_{0,\mu}(1-\mathrm{i}\omega\eta_\mu)$ is the plate bending stiffness, $\eta_\mu = R_\mu/D_{0,\mu}$ is the plate material damping factor, $\bar{f}_\mu(r,t) = \bar{f}_\mu(r)\exp(-\mathrm{i}\omega t)$ is the surface excitation distribution, $p(r,0,t)$ is the acoustic pressure on surfaces of the sources and represents the fluid loading. It is assumed that only the upper surfaces of both plates are fluid-loaded. The other source of attenuation is dissipation represented by factor R_μ. The two equations of motion presented in Eq. (21) can be expressed as

$$\left(k_{D_\mu}^{-4}\nabla^4 - 1\right) W_\mu(r) + \frac{p(r,0)}{\omega^2\varrho_\mu h_\mu} = \frac{\bar{f}_\mu(r)}{\omega^2\varrho_\mu h_\mu}. \tag{22}$$

Next, Eqs. (4), (12), and (18c) are applied for $z=+0$, and the equations of free vibrations

$$\nabla^4 W_{\mu,n}(r) = k_{\mu,n}^4 W_{\mu,n}(r), \tag{23}$$

where $\nabla^4 = \nabla^2\nabla^2$ is the axisymmetric biharmonic operator, to obtain

$$\sum_{n'=1}^{\infty} a_{\mu,n'}\left(\frac{k_{\mu,n'}^4}{k_{D_\mu}^4} - 1\right) W_{\mu,n'}(r)$$
$$+\frac{1}{\omega^2\varrho_\mu h_\mu}\sum_{n'=1}^{\infty}\frac{\omega}{\omega_{\mu,n'}} a_{\mu,n'} p_{\mu,n'}(r,0)$$
$$+\frac{1}{\omega^2\varrho_\mu h_\mu}\sum_{n=1}^{\infty}\frac{\omega}{\omega_{\nu,n}} a_{\nu,n} p_{\nu,n}(r,0) = \frac{\bar{f}_\mu(r)}{\omega^2\varrho_\mu h_\mu}, \tag{24}$$

for $\nu = 2$ when $\mu = 1$ and $\nu = 1$ when $\mu = 2$. Multiplying both sides of the above expression by $\pi r v_{\mu,n'}^*(r)$, integrating over variable r from $r_{1,\mu}$ to $r_{2,\mu}$, and using the orthogonality of eigenfunction in Eq. (17), gives the system of equations

$$a_{\mu,n'}\left(\frac{k_{\mu,n'}^4}{k_{D_\mu}^4} - 1\right) - \mathrm{i}\,\varkappa_\mu \sum_{n=1}^{\infty} a_{\mu,n}\zeta_{n',n}^{(\mu,\mu)}$$
$$-\mathrm{i}\,\varkappa_\mu\sqrt{\frac{S_\nu}{S_\mu}}\sum_{n=1}^{\infty} a_{\nu,n}\zeta_{n',n}^{(\mu,\nu)} = \frac{\bar{f}_{\mu,n'}}{\omega^2\varrho_\mu h_\mu}, \tag{25}$$

where the modal excitation coefficient is

$$\bar{f}_{\mu,n'} = \frac{2\pi}{S_\mu}\int_{r_{1,\mu}}^{r_{2,\mu}} r\bar{f}_\mu(r) W_{\mu,n'}(r)\,\mathrm{d}r \tag{26}$$

and the dimensionless fluid loading coefficient is

$$\varkappa_\mu = \frac{1}{kh_\mu}\frac{\varrho_0}{\varrho_\mu}. \tag{27}$$

Values of coefficients $a_{\mu,n}$ are obtained by solving the above system of equations. The values can be then used to calculate the total acoustic power according to the formula

$$\Pi_{\mu,\nu} + \Pi_{\mu,\mu} = \tag{28}$$
$$\frac{-\mathrm{i}}{2\varkappa_\mu}\varrho_0 c\omega^2 S_\mu \sum_{n=1}^{\infty} a_{\mu,n}^*\left[a_{\mu,n}\left(\frac{k_{\mu,n}^4}{k_{D_\mu}^4} - 1\right) - \frac{\bar{f}_{\mu,n}}{\omega^2\varrho_\mu h_\mu}\right]$$

and the total acoustic power Π can be obtained by substituting it to Eq. (10).

The fluid loading can be neglected by assuming that

$$\varkappa_\mu \simeq 0. \tag{29}$$

In this case, Eq. (28) cannot be used and the coefficients can be obtained directly from

$$a_{\mu,n} = \frac{\bar{f}_{\mu,n}}{\omega^2\varrho_\mu h_\mu}\left(\frac{k_{\mu,n}^4}{k_{D_\mu}^4} - 1\right)^{-1}. \tag{30}$$

Assuming that the excitations of the two plates are uniform,

$$\bar{f}_\mu(r) = \begin{cases}\bar{f}_{0,\mu}; & r_{1,\mu} \le r \le r_{2,\mu},\\ 0; & \text{otherwise},\end{cases} \tag{31}$$

the modal excitation coefficient assumes the form

$$\bar{f}_{\mu,n} = \tag{32}$$
$$\frac{4s^{\mu-1}\bar{f}_{0,\mu}\beta_{\mu,n}\left[s^{\mu-1}G_1(k_{\mu,n}r_{2,\mu}) - \delta_{2,\mu}G_1(k_{\mu,n}r_{1,\mu})\right]}{k_{\mu,n}r_{2,\mu}\left[s^{2(\mu-1)} - \delta_{2,\mu}\right]},$$

where the initial phase difference of excitations of both plates is

$$\varphi = \arg(\bar{f}_{0,1}) - \arg(\bar{f}_{0,2}). \tag{33}$$

3. Numerical analysis

Values of physical quantities (see Table I) have been selected to assure nearly overlapping of the second, third and forth eigenfrequencies of the annular plate and the corresponding eigenfrequencies of the circular plate (see Table II) to highlight some strong interactions via fluid. The influence of phase shift between excitations of both plates on the total acoustic power radiated is examined. Three representative values of the shift have been selected, i.e. $\varphi = 0$ (both sources vibrating in phase), π (in anti-phase), and $\pi/2$ (out-of-phase). It is illustrated in Figs. 3a and b, where some noticeable differences in the

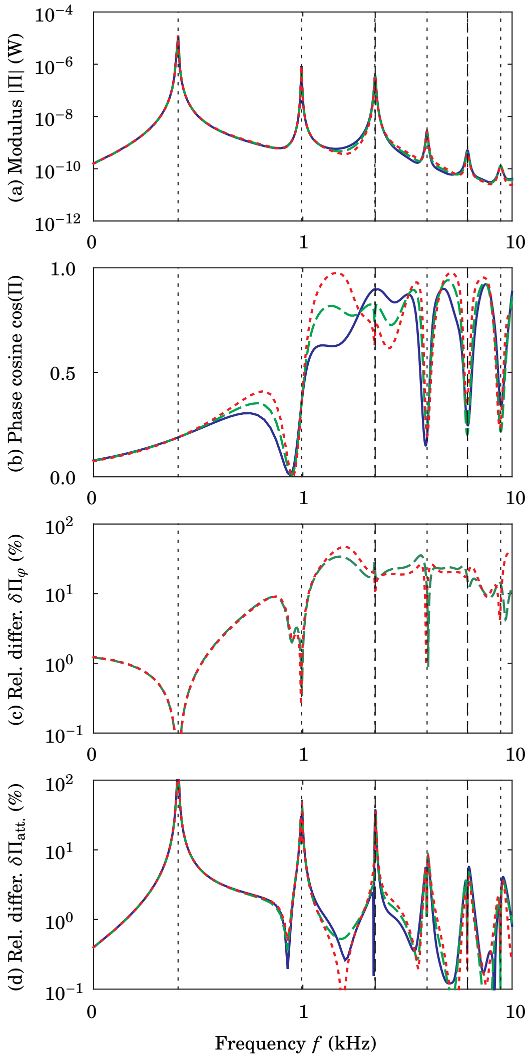

Fig. 3. Plots representing the acoustic power Π: (a) modulus; (b) phase cosine; (c) relative percentage difference due to changes in φ; (d) relative percentage difference due to fluid loading neglecting. Key of lines for φ equal to: 0 — solid, $\pi/2$ — dashed, and π — dotted.

acoustic power can be observed around the overlapping eigenfrequencies. For different frequencies, the effect is considerably weaker. The relative percentage difference is further introduced

$$\delta\Pi = \frac{|\Pi_0 - \Pi_\varphi|}{\frac{1}{2}|\Pi_0 + \Pi_\varphi|} 100\% \tag{34}$$

for the purpose of quantitative analysis (see Fig. 3c), where Π_0 is the reference acoustic power for φ equal to zero and Π_φ for different values of φ. This measure does not exceed 10% for frequencies smaller than the second eigenfrequency of circular plate and assumes values of several dozen percent for higher frequencies. Figure. 3d shows errors when fluid loading is neglected. It can be seen that they assume considerable values when the vibration frequency is close to eigenfrequencies of both plates.

TABLE I

Values of parameters selected for numerical calculations.

Plate geometry	
circular plate, outer radius	r_0
(= annular plate, inner radius)	$r_1 = 0.10$ m
annular plate, outer radius	$r_2 = 0.15$ m
thickness (both plates)	$h_\mu = 10^{-3}$ m
Material parameters (both plates)	
density	$\varrho_\mu = 7850.0$ kg/m^3
Young modulus	$E_\mu = 210.0 \times 10^9$ N/m^2
Poisson ratio	$\nu_\mu = 0.3$
damping factor	$\eta_\mu = 10^{-6}$ s/rad
excitation	$\bar{f}_{0,\mu} = 0.5$ N/m^2
Fluid (air)	
temperature	$T_0 = 293.15$ K
density	$\varrho_0 = 1.2930$ kg/m^3
speed of sound	$c = 340$ m/s

TABLE II

Initial eigenfrequencies $f_{\mu,n}$ selected for numerical calculations.

n	1	2	3	4	5
$f_{1,n}$ kHz	0.253422	0.986594	2.21039	3.92404	6.12743
$f_{2,n}$ kHz	2.217860	6.116600	11.9936	19.8280	–
n	6	7	8	9	10
$f_{1,n}$ kHz	8.82053	12.0033	15.6758	19.8379	–
$f_{2,n}$ kHz	–	–	–	–	–

4. Conclusions

Modal analysis was used to include the fluid loading into the problem of sound radiation considered herein. Modal acoustic impedance coefficients were calculated using Eq. (19d), where approximate formulas presented in [13–16, 18] can also be used. Strong fluid loading was observed for frequencies around eigenfrequencies of both plates. This effect is even stronger when the eigenfrequencies nearly overlap, where errors reach values of dozens and hundreds percent if fluid loading is neglected. However, the errors do not exceed 1% for different frequencies.

Acknowledgments

The research presented in this paper was partially supported under the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge Project at the University of Rzeszów.

References

[1] L.V. King, *Can. J. Res.* **11**, 135 (1934).
[2] C.J. Bouwkamp, *Physica* **16**, 1 (1950).
[3] R.L. Pritchard, *J. Acoust. Soc. Am.* **32**, 730 (1960).

[4] C.L. Scandrett, J.L. Day, S.R. Baker, *J. Acoust. Soc. Am.* **109**, 2715 (2001).

[5] N. Hashimoto, *Applied Acoustics* **62**, 429 (2001).

[6] J.P. Arenas, *Int. J. Occup. Saf. Ergo.* **15**, 401 (2009).

[7] J.P. Arenas, M.J. Crocker, *Int. J. Acoust. Vib.* **7**, 217 (2002).

[8] J.P. Arenas, *J. Comput. Acoust.* **16**, 321 (2008).

[9] M. González-Montenegro, R. Jordan, A. Lenzi, J.P. Arenas, *Arch. Acoust.* **39**, 249 (2014).

[10] K. Kolber, A. Snakowska, M. Kozupa, *Arch. Acoust.* **39**, 511 (2014).

[11] J.P. Arenas, *Lat. Am. Appl. Res.* **39**, 345 (2009).

[12] M. Inalpolat, M. Caliskan, R. Singh, *Noise Control Eng.* **58**, 145 (2010).

[13] H. Levine, F.G. Leppington, *J. Sound Vib.* **121**, 269 (1988).

[14] W.J. Rdzanek, *Arch. Acoust.* **17**, 439 (1992).

[15] W.P. Rdzanek, Z. Engel, *Appl. Acoust.* **60**, 29 (2000).

[16] W.P. Rdzanek, *J. Sound Vib.* **249**, 307 (2002).

[17] T. Mellow, *J. Acoust. Soc. Am.* **120**, 90 (2006).

[18] T. Mellow, L. Kärkkäinen, *J. Acoust. Soc. Am.* **121**, 3527 (2007).

[19] T. Mellow, L. Kärkkäinen, *J. Acoust. Soc. Am.* **123**, 2598 (2008).

[20] S.D. Vishwakarma, A.K. Pandey, J.M. Parpia, D.R. Southworth, H.G. Craighead, R. Pratap, *J. Microelectromech. S.* **23**, 334 (2014).

[21] J.S. Anderson, M. Bratos-Anderson, *Acta Acust. United Ac.* **91**, 61 (2005).

[22] M.S. Kozień, *Acta Phys. Pol. A* **121**, A132 (2012).

[23] R. Zhou, M.J. Crocker, *J. Vib. Acoust.* **132**, 0310121 (2010).

[24] K. Szemela, *J. Comput. Acoust.* **21**, 1350016 (2013).

[25] K. Szemela, *Acta Acust. United Ac.* **100**, 604 (2014).

[26] D. Kuo, Y.C. Shiah, J.H. Huang, *J. Vib. Acoust.* **133**, 031015 (2011).

[27] S. Oberst, J.C.S. Lai, S. Marburg, *J. Sound Vib.* **332**, 2284 (2013).

[28] Hyeongill Lee, *P. I. Mech. Eng. C-J. Mec.* **227**, 919 (2013).

[29] D. Karkosiński, *Arch. Elect. Eng.* **56**, 265 (2007).

[30] H. Lee, R. Singh, *J. Sound Vib.* **282**, 313 (2005).

[31] W. Thompson Jr., *J. Sound Vib.* **17**, 221 (1971).

[32] P. Witkowski, *Arch. Acoust.* **22**, 463 (1997).

[33] J. Lee, I. Seo, S.-M. Han, *Sensor. Actuat. A-Phys.* **90**, 1 (2001).

[34] H. Lee, J. Tak, W. Moon, G. Lim, *J. Acoust. Soc. Am.* **115**, 666 (2004).

[35] K. Sha, J. Yang, W.-S. Gan, *J. Sound Vib.* **282**, 179 (2005).

[36] P.A. Nishamol, J. Mathew, D.D. Ebenezer, *J. Sound Vib.* **323**, 989 (2009).

[37] H. Lee, D. Kang, W. Moon, *J. Acoust. Soc. Am.* **125**, 1879 (2009).

[38] Z. Kulka, *Arch. Acoust.* **36**, 419 (2011).

[39] K. Szemela, W.P. Rdzanek, W.J. Rdzanek, *Arch. Acoust.* **37**, 463 (2012).

[40] K.K. Park, B.T. Khuri-Yakub, *J. Acoust. Soc. Am.* **132**, 2292 (2012).

[41] W.P. Rdzanek, in: *Proc. Forum Acusticum*, Kraków 2014, p. 190.

[42] R. Trojanowski, J. Wiciak, *Acta Phys. Pol. A* **121**, A148 (2012).

[43] J. Wiciak, R. Trojanowski, *Acta Phys. Pol. A* **125**, A179 (2014).

[44] L. Leniowska, *Arch. Acoust.* **33**, 531 (2008).

[45] M. Pawełczyk, *Arch. Acoust.* **33**, 509 (2008).

[46] W.M. Zawieska, *Int. J. Occup. Saf. Ergo.* **13**, 381 (2007).

[47] A. Brański, S. Szela, *Arch. Acoust.* **33**, 521 (2008).

[48] A. Brański, S. Szela, *Acta Phys. Pol. A* **119**, 942 (2011).

[49] S.M. Hasheminejad, M. Maleki, *Smart Mater. Struct.* **18**, 055013 (2009).

[50] B. Gheshlaghi, S.M. Hasheminejad, *Thin Solid Films* **537**, 212 (2013).

[51] S.M. Hasheminejad, H. Keshavarzpour, *J. Sound Vib.* **332**, 4798 (2013).

[52] M.S. Kozień, J. Wiciak, *Acta Phys. Pol. A* **118**, 1186 (2010).

[53] S. Wrona, M. Pawełczyk, *Arch. Acoust.* **38**, 529 (2013).

[54] K. Mazur, M. Pawełczyk, *Arch. Acoust.* **38**, 537 (2013).

[55] A. Sommerfeld, *Partial Differential Equations in Physics*, Academic Press, New York 1964.

[56] A.W. Leissa, *Vibration of Plates*, NASA, Washington 1969.