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Nonlinear Behavior of Composite Truncated Conical Shells Subjected to the Dynamic Loading

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In this paper, the non-linear (NL) behavior of composite truncated conical shells subjected to dynamic loading is studied. The basic equations are derived using the von Karman-Donnell-type of kinematic nonlinearity. These equations are reduced to a NL differential equation with the variable coefficient using the superposition principle and Galerkin method. The resulting equation is solved numerically using Runge-Kutta method and modified Budiansky-Roth criterion and the values of dimensionless NL critical time parameters are obtained. Finally, the effects of axial loading speed and orthotropy on the dimensionless NL critical time parameters of composite truncated conical shells are investigated.

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1. Introduction

As a common structure, the composite truncated conical shell has been widely applied in many fields such as mechanical, submarine and aerospace engineering technology, etc. Assessing their stability under different static and dynamic loadings and their sensitivity to variations in the model parameters is a challenging task, which helps better understanding the phenomenon and helps the design of efficient and reliable composite conical shell structures [1, 2]. It is well known, due to the difficulties emerging from the solution and theoretical analysis of NL stability problems of composite truncated conical shells, such as dynamic loads, has not been studied sufficiently [3–6]. The aim of the present paper is to study the NL stability of composite truncated conical shell subjected to a dynamic loading. The basic equations of orthotropic composite truncated conical shell are derived and then reduced to a NL differential equation with the variable coefficient using the Superposition and Galerkin methods. The resulting equation is solved numerically using Runge-Kutta method and modified Budiansky-Rooth criterion and the values of NL critical time parameters are obtained. Finally, the effects of the loading speed and orthotropy on the dimensionless NL critical time parameters of composite truncated conical shells are investigated.

2. Formulation of the problem

As shown in Fig. 1 a thin orthotropic truncated conical shell subjected to a time dependent axial load, $q(t)$, is considered. The axial load is expressed in the form: $q(t) = -(q_1 + q_0 t)$, where q_0 is the axial loading speed, q_1 is the static axial load and t is the time. The middle surface of the conical shell is defined as the reference surface. The structure is referred to a curvilinear coordinate system (S, θ, z), where S and θ axes lie along the generator and in the circumferential direction on the reference surface, respectively and z axis, being perpendicular to the plane of the first two axes, lies in the inwards normal direction of the cone. R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively, γ denotes the semi-vertex angle of the cone, L is the length, h is the thickness of the truncated cone and S_1 is the distance from the vertex to the small base. It is assumed that the local coordinate system, which determines the principal axes of material orthotropy, coincides with the global curvilinear coordinates.

The stress-strain relations for orthotropic truncated conical shells are given as follows [7]:

$$\begin{bmatrix} \sigma_S \\ \sigma_\theta \\ \sigma_{S\theta} \end{bmatrix} = \frac{1}{1-\nu_{S\theta}\nu_{\theta S}} \times \begin{bmatrix} E_S & \nu_{\theta S}E_S & 0 \\ \nu_{S\theta}E_\theta & E_\theta & 0 \\ 0 & 0 & 2G\left(1-\nu_{S\theta}\nu_{\theta S}\right) \end{bmatrix} \begin{bmatrix} \varepsilon_S \\ \varepsilon_\theta \\ \varepsilon_{S\theta} \end{bmatrix} \quad (1)$$

The relations between the forces and the Airy stress function are expressed as follows [1–3]:

$[(N_S, N_\theta N_{S\theta}), (M_S, M_\theta, M_{S\theta})] =$

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$$\int_{-h/2}^{h/2} (\sigma_S, \sigma_\theta, \sigma_{S\theta})[1, \varsigma] d\varsigma \tag{2}$$

The forces can be written in terms of a stress function Ψ as [4]:

$$(N_S, N_\theta, N_{S\theta}) = (\Psi,_{\phi\phi}/S^2 + \Psi,_S/S, \Psi,_{SS}, -\Psi,_{S\phi}/S + \Psi,_\phi/S^2). \tag{3}$$

Substituting Eq. 1 in Eq. 2, then considering the resulting expressions in the modified Donnell type NL stability and strain compatibility equations of composite truncated conical shells [5, 6], together with the relation 3, and then taking into account the independent variables $S = S_1 e^z$ and $\Phi = \Phi_1 e^{2z}$, the system of NL partial differential equations for w and Φ_1 can be written as:

$$L_{11}\Phi_1 + L_{12}w + L_{13}(\Phi_1, w) = 0;$$

$$L_{21}\Phi_1 - L_{22}w - L_{23}(w, w) = 0, \tag{4}$$

where $L_{ij}(i, j = 1, 2, 3)$ are differential operators and the following definitions apply:

$$\begin{aligned}
L_{11} &= (\tfrac{\partial^2}{\partial z^2} + 3\tfrac{\partial}{\partial z} + 2)S_1 e^{3z}\cot\gamma,\\
L_{12} &= -\Gamma_1\tfrac{\partial^4}{\partial\phi^4} - \Gamma_2\tfrac{\partial^4}{\partial z^2\partial\phi^2} + \Gamma_3\tfrac{\partial^3}{\partial z\partial\phi^2} - \Gamma_4\tfrac{\partial^2}{\partial\phi^2} - \Gamma_5\tfrac{\partial^4}{\partial z^4} + \Gamma_6\tfrac{\partial^3}{\partial z^3} - \Gamma_7\tfrac{\partial^2}{\partial z^2} + \Gamma_8\tfrac{\partial}{\partial z} - \rho h S_1^4 e^{4z} w,_{tt},\\
L_{13} &= e^{2z}(\tfrac{\partial^2}{\partial\phi^2} + \tfrac{\partial}{\partial z} + 2)(\tfrac{\partial^2}{\partial z^2} - \tfrac{\partial}{\partial z}) + e^{2z}(\tfrac{\partial^2}{\partial z^2} + 3\tfrac{\partial}{\partial z} + 2)(\tfrac{\partial^2}{\partial\phi^2} + \tfrac{\partial w}{\partial z}) - 2e^{2z}(\tfrac{\partial^2}{\partial z\partial\phi} + \tfrac{\partial}{\partial\phi})(\tfrac{\partial^2}{\partial z\partial\phi} - \tfrac{\partial}{\partial\phi}),\\
L_{21} &= \Delta_1 e^{2z}\tfrac{\partial^4}{\partial z^4} + \Delta_2 e^{2z}\tfrac{\partial^3}{\partial z^3} + \Delta_3 e^{2z}\tfrac{\partial^2}{\partial z^2} + \Delta_4 e^{2z}\tfrac{\partial}{\partial z} + \Delta_5 e^{2z}\tfrac{\partial^4}{\partial z^2\partial\phi^2} + \Delta_6 e^{2z}\tfrac{\partial^3}{\partial z\partial\phi^2} + \Delta_7 e^{2z}\tfrac{\partial^2}{\partial\phi^2} + \Delta_8 e^{2z}\tfrac{\partial^4}{\partial\phi^4},\\
L_{22} &= -S_1 e^z\cot\gamma(\tfrac{\partial^2}{\partial z^2} - \tfrac{\partial}{\partial z}), L_{23} = (\tfrac{\partial}{\partial\phi})^2 - 2\tfrac{\partial}{\partial\phi}\tfrac{\partial^2}{\partial z\partial\phi} + (\tfrac{\partial}{\partial z} - \tfrac{\partial^2}{\partial z^2})\tfrac{\partial^2}{\partial\phi^2} + (\tfrac{\partial^2}{\partial z\partial\phi})^2 + (\tfrac{\partial}{\partial z} - \tfrac{\partial^2}{\partial z^2})\tfrac{\partial}{\partial z}
\end{aligned} \tag{5}$$

where $\Gamma_i(i = 1, 2, ..., 8)$ and $\Delta_j(j = 1, 2..., 8)$ are parameters which are given in [5].

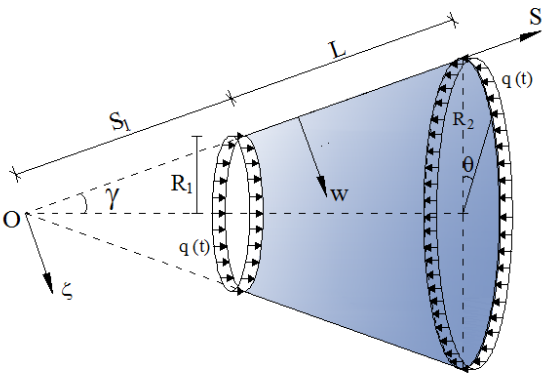

Fig. 1. Nomenclature and coordinate system of truncated conical shell subjected to a dynamic axial loading.

3. Solution of the problem

Assuming that the conical shell is simply supported and displacement function satisfies the boundary condition of $w = 0$ at $z = 0$ and $z = z_0$. The solution of the second equation of (4) is sought in the following form [4]:

$$w = f_1(t) e^z \sin(\beta_1 z)\sin(\beta_2\phi) + f_2(t)\sin^2 e^z(\beta_1 z) \tag{6}$$

where f_1 and f_2 are unknown time dependent functions of the displacement w, $\beta_1 = m\pi/z_0$, $\beta_2 = n/\sin\gamma$, $z_0 = \ln(1 + L/S_1)$, m nand n are number of half-waves along a generatrix and full-waves in the circumferential direction, respectively. Substituting Eq. 6 into the second equation of the system 4 and solving the resulting differential equation with superposition method, a particular solution is obtained as follows:

$$\begin{aligned}
\Phi_1 &= (\Lambda_1 f_2^2 + \Lambda_2 f_1^2 + \Lambda_3 f_2)\cos(2\beta_1 z) + (\Lambda_4 f_2^2 + \Lambda_5 f_1^2 + \Lambda_6 f_2)\sin(2\beta_1 z) + \Lambda_7 f_1^2\cos(2\beta_1 z)\cos(2\beta_2\phi)\\
&+\Lambda_8 f_1^2\sin(2\beta_1 z)\cos(2\beta_2\phi) + (\Lambda_9 f_1 f_2 + \Lambda_{10} f_1)\cos(\beta_1 z)\sin(\beta_2\phi) + (\Lambda_{11} f_1 f_2 + \Lambda_{12} f_1)\sin(\beta_1 z)\sin(\beta_2\phi)\\
&+\Lambda_{13} f_1 f_2\cos(3\beta_1 z)\sin(\beta_2\phi) + \Lambda_{14} f_1 f_2\sin(3\beta_1 z)\sin(\beta_2\phi) + \Lambda_{15} f_1 f_2\cos(4\beta_1 z) + \Lambda_{16} f_1 f_2\sin(4\beta_1 z)\\
&+\Lambda_{17} f_1^2\cos(2\beta_2\phi) - \left[1 + e^{2z} + \cos(\beta_2\phi)\right](q_1 + q_0 t)S_1^2/2,
\end{aligned} \tag{7}$$

where the coefficients $\Lambda_i(i = 1, 2, ..., 16)$ are given in [6]. Taking into account Eq. 6 and Eq. 7 in Eq. 4, then integrating them using Galerkin method (see, ref. [4]), we obtain a system of two NL differential equations, which after elimination of f_2, leads to the following equation in the dimensionless form,

$$\eta,_{t_1 t_1} + Q[(1 - t_1)\eta - (Q_1 + Q_2 + Q_5)a\eta^3 - Q_4\eta^3 - Q_3 a^2\eta^5] = 0, \tag{8}$$

if the following non-dimensional parameters are introduced:

$$\eta = f_1/h; t_1 = \frac{q_0 t}{h E_S q_{1\mathrm{Lin}}^{\mathrm{stcr}}}; Q = \frac{(q_{1\mathrm{Lin}}^{\mathrm{stcr}})^3}{\lambda_8};$$

$$\lambda_8 = \frac{\lambda_7 q_0^2}{E_S^2 h^2}; Q_i = \frac{\lambda_i}{q_{1\mathrm{Lin}}^{\mathrm{stcr}}}; (i = 1, 2, ..., 7),$$

$$a = \frac{\Lambda_5 + \Lambda_7 - \Lambda_2\lambda_5}{(\lambda_2 + \lambda_3 + \lambda_6)\Lambda_2 - \lambda_1\Lambda_1 - \Lambda_3}, \tag{9}$$

where coefficients $\lambda_i, (i = 1, 2, ..., 7)$ are given in [4, 5]. The Eq. 8 is solved numerically by using Runge-Kutta method with the following initial conditions: $t_1 = 0, \eta = \eta_0 = 0.001, \eta,_{t_1} = 0$. Using modified Budiansky-Roth criterion, the values of dimensionless NL and L critical time parameters t_{1cr}^{NL} and t_{1cr}^{L}, and corresponding dynamical wave numbers (m, n) are found for different loading speeds, material properties and truncated shell characteristics.

4. Numerical results and discussion

Numerical results are presented in this subsection for stability of glass/epoxy conical shells subjected to the time dependent axial load. In Table, the values of L and NL dimensionless critical time parameters for different form of material properties and loading speeds are presented. Here, the data are chosen as $R_1/h = 100; L/R_1 = 1; \gamma = 30°$. The values of dimensionless linear critical static axial load and corresponding wave numbers are found as follows: $q_{1\text{Lin}}^{\text{stcr}}(m,n) = 1.74\times10^{-3}(3,6)$, $0.993\times10^{-3}(2,8)$, $0.763\times10^{-3}(1,9)$, $0.596\times10^{-3}(1,9)$ for the left part of Table and $q_{1\text{Lin}}^{\text{stcr}}(m,n) = 1.851\times10^{-3}(2,9)$ for the right part of Table. An examination of Table shows that the values of t_{1cr}^{L} and t_{1cr}^{NL}, and corresponding wave numbers orthotropic shells increase with increasing q_0 and E_S/E_θ. The influence of non-linearity on the values of dimensionless critical time parameters of orthotropic shells is 22.19%; 27.39%; 18.35% ; 14.82%, for $q_0 = 1\times10^9; 2\times10^9; 3\times10^9; 7\times10^9$ (N/m s), respectively. The influence of non-linearity on the values of dimensionless critical time parameters is considerable and changes irregularly, as E_S/E_θ increases. For example, the influence of non-linearity on the values of t_{1cr}^{NL} is 23.94%; 17.47%; 19.96%; 18.89%, for $E_S/E_\theta = 5; 15; 25; 40$, respectively.

Variations of t_{1cr}^{L} and t_{1cr}^{NL} versus the E_S/E_θ and axial loading sped, q_0. TABLE

$q_0 = 1\times10^9$ (N/m×s); $E_S = 2\times10^{11}$ Pa; $\nu_{S\theta} = 0.3$; $\rho = 8000$ kg/m^3			$E_S = 53.7791$ GPa; $E_\theta = 17.9264$ GPa; $G = 8.9632$ GPa; $\nu_{S\theta} = 0.25$; $\rho_0 = 2004$ kg/m^3 [6]		
E_S/E_θ	$t_{1cr}^{L}(\eta,m,n)$	$t_{1cr}^{NL}(\eta,m,n)$	q_0(N m/s)	$t_{1cr}^{L}(\eta,m,n)$	$t_{1cr}^{NL}(\eta,m,n)$
5	2.345(0.440, 3, 21)	1.894(0.440, 4, 21)	1×10^9	3.844(0.269, 3, 21)	2.991(0.269, 4, 21)
15	3.080(0.237, 3, 29)	2.542(0.237, 4, 29)	3×10^9	6.797(0.230, 4, 27)	4.935(0.230, 5, 27)
25	4.048(0.376, 3, 33)	3.240(0.376, 4, 33)	5×10^9	8.243(0.791, 5, 33)	6.730(0.791, 6, 33)
40	4.118(0.222, 4, 48)	3.340(0.222, 5, 48)	7×10^9	8.954(0.857, 6, 38)	7.627(0.857, 7, 38)

5. Conclusions

The non-linear behavior of composite truncated conical shells subjected to a dynamic loading is investigated. The basic equations are derived using the von Karman–Donnell- type of kinematic non-linearity and solved numerically using the superposition principle, Galerkin and Runge-Kutta methods and modified Budiansky-Roth criterion. The numerical results support the following conclusions: The values of t_{1cr}^{L} and t_{1cr}^{NL}, and corresponding wave numbers orthotropic shells increase with increasing q_0 and E_S/E_θ. The influence of non-linearity on the values of dimensionless critical time parameters is considerable and changes irregularly, as E_S/E_θ increases.

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