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Inverse Edelstein Effect: an Heuristic Derivation

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We provide a heuristic derivation of the "Inverse Edelstein Effect" (IEE), in which a non-equilibrium spin accumulation in the plane of a two-dimensional (interfacial) electron gas drives an electric current perpendicular to its own direction. The drift-diffusion equations that govern the effect are derived and applied to the interpretation of recent experiments. A brief analysis based on the Kubo formula shows that the result is valid also outside the diffusive regime, i.e. when spin and momentum relaxation become comparable.

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1. Introduction

A recent experiment [1], in which by means of spin pumping a non equilibrium spin polarization is created at the interface of a silver-bismuth hybrid system, has made possible to efficiently convert spin into charge. This phenomenon could be accounted for by the Rashba spin-orbit coupling (SOC) existing at the interface. According to this interpretation, the mechanism at play is the IEE [2], also known in semiconducting systems as spin-galvanic effect [3], for which we have given a microscopic theory elsewhere [4]. Here we provide an alternative heuristic derivation, which elucidates various physical aspects characterizing this effect.

We consider a two-dimensional electron gas (2DEG) with Rashba SOC and impurity scattering. The Hamiltonian reads

$$H = \frac{p^2}{2m} + \alpha(p_y\sigma^x - p_x\sigma^y) + V(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{p} = (p_x, p_y)$ is the momentum operator, $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ the vector of Pauli matrices with α the SOC strength. Finally, $V(\boldsymbol{x})$ is a random function with variance $\langle V(\boldsymbol{x})V(\boldsymbol{x}')\rangle = (2\pi N_0\tau)^{-1}\delta(x - x')$, mimicking the breaking of translational invariance due to the imperfections of the host lattice. In the above $N_0 = m/2\pi$ is the density of states in the absence of both SOC and impurity scattering with m the effective electron mass (we are using units such that $\hbar = 1$). The parameter τ has the physical meaning of an inverse scattering rate due to the interaction with the impurities. When $\alpha = 0$, Hamiltonian (1) is the standard minimal model for electrical conduction in metallic diffusive systems characterized by a Drude conductivity $\sigma_D = 2e^2N_0D$, with $D = v_F^2\tau/2$ being the diffusion coefficient expressed in terms of τ and the Fermi velocity v_F.

At fixed momentum $\boldsymbol{p}$, the electron spin operator $\hat{\boldsymbol{S}} = \boldsymbol{\sigma}/2$ obeys the precession equation

$$\frac{\mathrm{d}S^a}{\mathrm{d}t} = 2\alpha p\, \varepsilon_{abc}\, (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{z}})_b S^c, \tag{2}$$

ε_{abc} being the Ricci tensor. Due to impurity scattering, momentum relaxes over a time τ, which is short compared to the typical times controlling the spin dynamics. By integrating Eq.(2) over the time τ, one gets for the average value of the S^z component

$$S^z(\tau, \boldsymbol{p}) = 2\alpha p\tau\hat{p}_y S^y(0, \boldsymbol{p}), \tag{3}$$

$S^y(0, \boldsymbol{p})$ being the initial polarization along the y direction of electrons with momentum $\boldsymbol{p}$. As a consequence there will be a spin current $J_y^z(\boldsymbol{p})$ associated to the electrons with momentum $\boldsymbol{p}$

$$J_y^z(\tau, \boldsymbol{p}) = \frac{p_y}{m} S^z(\tau, \boldsymbol{p}) = 2\alpha m \left(\frac{p}{m}\right)^2 \hat{p}_y^2 \tau S^y(0, \boldsymbol{p}). \tag{4}$$

The total contribution to the spin current is obtained by summing over all momenta and the result depends on the initial polarization. By assuming that the initial polarization is due to different chemical potentials $\mu_\pm$ for electrons having their spin parallel (antiparallel) to the positive y axis, we write

$$S^y(0, \boldsymbol{p}) = \frac{1}{2}\left[f(\varepsilon_{\boldsymbol{p}} - \mu_+) - f(\varepsilon_{\boldsymbol{p}} - \mu_-)\right] \approx -\frac{1}{2}\partial_{\varepsilon_{\boldsymbol{p}}} f(\varepsilon_{\boldsymbol{p}} - (\mu_+ + \mu_-)/2)(\mu_+ - \mu_-). \tag{5}$$

By defining the total spin density S^y and spin current J_y^z as

$$S^y = \int \frac{\mathrm{d}^2p}{(2\pi)^2} S^y(0, \boldsymbol{p}), \tag{6}$$

$$J_y^z = \int \frac{\mathrm{d}^2p}{(2\pi)^2} J_y^z(\tau, \boldsymbol{p}) \tag{7}$$

we obtain

$$J_y^z = D2\alpha m S^y. \tag{8}$$

In obtaining Eq.(8) we have taken out of the integral in (7) the factor $(p/m)^2 \approx v_F^2$ thanks to the factor $\partial_{\varepsilon_{\boldsymbol{p}}} f(\varepsilon_{\boldsymbol{p}})$ which differs from zero only at the Fermi surface. By repeating the reasoning for an initial polarization along the x direction, we may write the general result

$$J_i^a = \varepsilon_{abc} D A_i^b S^c, \tag{9}$$

where the tensor A_i^a has as non vanishing components

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only $A^x_y = 2m\alpha = -A^y_x$. Eq.(9) may be interpreted as a diffusive current in spin space. To see this, let us rewrite Hamiltonian (1) as

$$H = \frac{1}{2m}(\boldsymbol{p} + \boldsymbol{A})^2, \quad \boldsymbol{A} = \sum_a \frac{1}{2}\boldsymbol{A}^a\sigma^a,$$
$$(\boldsymbol{A}^a)_i = A^a_i, \tag{10}$$

where we have rewritten the SOC in terms of a spin-dependent vector potential with spin components $\boldsymbol{A}^a$ and neglected the constant term $\boldsymbol{A}^2/2m = m\alpha^2$. By requiring the local gauge invariance of the Hamiltonian (9) under the $SU(2)$ group of transformations, one is directly led to define the covariant derivative

$$(\nabla_i O)^a = \partial_i O - \varepsilon_{abc}A^b_i O^c, \tag{11}$$

when acting on an observable O^a, ∂_i being the standard space derivative.† Eq.(9) can be rewritten then allowing also for the standard diffusion due to the space derivative of the spin density

$$J^a_i = -D(\nabla_i S)^a. \tag{12}$$

The introduction of the $SU(2)$ vector potential can be further motivated by showing how, as a consequence, one obtains the well known mechanism of D'yakonov-Perel' spin relaxation (DPSR). By requiring the continuity equation for the spin density in terms of covariant derivatives one has

$$\partial_t S^a = -(\nabla_i J_i)^a, \tag{13}$$
$$\partial_t S^a = -\partial_i J^a_i - D\varepsilon_{abc}A^b_i\partial_i S^c$$
$$-(\varepsilon_{abc})^2(A^b_i)^2 D S^a, \tag{14}$$

where in going from the first to the second line we used Eqs.(11-12). One then obtains the well known DPSR $1/\tau_{\rm DP} = (2m\alpha)^2 D$ for $S^{x,y}$ and $2/\tau_{\rm DP}$ for S^z. It is worthwhile to recall that the DPSR arises as a consequence of the collisions with the impurities. During the time τ between two collisions, the electron spin precesses by an amount $\delta\Phi \sim \alpha p_F\tau$. If $\delta\Phi$ is small, the spin undergoes a diffusive motion. $\tau_{\rm DP}$ is the time at which the total precession after $N = \tau_{\rm DP}/\tau$ scattering events becomes of order unity yielding $\tau_{\rm DP} \sim (\alpha^2 p_F^2\tau)^{-1}$ in agreement with (14). This justifies the use of the scattering time as the smallest time over which integrate the precession equation (2), if the condition $\alpha p_F\tau \ll 1$ is satisfied. Hence the heuristic derivation of the expression (12) for the diffusive current in terms of the covariant derivative associated to the the SOC induced $SU(2)$ field is in perfect agreement with the standard physical picture of the DPSR and, moreover, with more technically rigourous microscopic derivations [5–7].

A further consequence of adopting the language of the $SU(2)$ gauge potential is the appearance of the associated $SU(2)$ magnetic field $\boldsymbol{B} = \sum_a \mathcal{B}^a\sigma^a/2$ defined as

$$\mathcal{B}^a_i = \frac{1}{2}\varepsilon_{ijk}\left(\partial_j A^a_k - \partial_k A^a_j + \mathrm{i}\,[A_j, A_k]^a\right). \tag{15}$$

Hence, even though the SOC vector potential of Eq.(10) is independent of space and time, it yields a non-zero field with only non vanishing components $\mathcal{B}^z_z = -(2m\alpha)^2$. This field gives rise to a Lorentz-like force acting in opposite way for electrons with opposite z component of the spin. As a result charge and spin currents, flowing perpendicularly to each other, are coupled in much the same way as charge currents are in the ordinary Hall effect:

$$J^\uparrow_y = \frac{\mathcal{B}^z_z\tau}{2m}J^\uparrow_x, \quad J^\downarrow_y = -\frac{\mathcal{B}^z_z\tau}{2m}J^\downarrow_x,$$

where $J^{\uparrow(\downarrow)}_i$ are the number currents for electrons with up and down spin along the z axis. All this leads to the addition of a further term in the spin current (12)

$$J^a_i = -D(\nabla_i S)^a - \varepsilon_{ija}\frac{\gamma}{e}J_j, \tag{16}$$

with the spin Hall angle $\gamma = m\alpha^2\tau$ and $e > 0$ the unit charge. This last term is responsible for the spin Hall effect. The charge current can be written in a similar way as

$$J_i = -D\partial_i(-en) - \varepsilon_{ija}4e\gamma J^a_j, \tag{17}$$

n being the electron density. In the presence of an external electric field one must add the standard drift terms on the expressions for both the charge and spin currents. Since we are not interested here in such a case, we do not include them to keep our expressions as simple as possible. Together with the obvious continuity equation for charge, Eqs.(14), (16)–(17) are all we need to discuss the coupled dynamics of spin and charge, provided we add a term responsible for the initial non equilibrium spin polarization. To understand the origin of a non equilibrium spin polarization we assume a time-dependent magnetic field $B^a(t)$ coupling linearly with the spin density S^a. By using linear response theory we may write

$$\delta S^a(t) = \int_{-\infty}^{\infty}\mathrm{d}t'\chi(t-t')B^a(t'), \tag{18}$$

$\chi(t)$ being the generalized spin susceptibility. The ordinary static spin susceptibility is given by the zero frequency Fourier transform of $\chi(t)$

$$\chi_0 = \int_{-\infty}^{\infty}\mathrm{d}t'\chi(t-t'). \tag{19}$$

The non equilibrium spin polarization is obtained by subtracting from $\delta S^a(t)$ the instantaneous spin polarization $\delta S^a_{\rm equi}(t) = \chi_0 B^a(t)$. By using Eqs.(18) and (19) the rate of change of the non equilibrium spin polarization reads

$$\frac{\mathrm{d}}{\mathrm{d}t}(\delta S^a(t) - \delta S^a_{\rm equi}(t)) = \frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty}\mathrm{d}t'(B^a(t')$$
$$-B^a(t))\chi(t-t') \approx -\partial_t B^a(t)\chi_0. \tag{20}$$

Hence, by assuming a magnetic field varying in time at a constant rate $\partial_t B \equiv \dot{B}^y$ along the y axis, we may write the continuity equation for the non equilibrium spin density $S^a \equiv \delta S^a(t) - \delta S^a_{\rm equi}(t)$ in the presence of a steady

†Under a $SU(2)$ transformation $U = \exp(\mathrm{i}\,\Psi)$, invariance of the Hamiltonian requires that the vector potential transforms as $\boldsymbol{A} \to U\boldsymbol{A}U^\dagger + U(\boldsymbol{p}U^\dagger)$. For infinitesimal Ψ, one has $\boldsymbol{A} \to \boldsymbol{A} - \mathrm{i}\,[\boldsymbol{A}, \Psi] - \mathrm{i}\,(\boldsymbol{p}\Psi)$. After using the expansion in terms of Pauli matrices for $\boldsymbol{A}$ as given in (10), a similar one for Ψ, one obtains the definition (11).

spin injection

$$\partial_t S^y = -\chi_0 \dot{B}^y - \frac{1}{\tau_{\rm DP}} S^y. \qquad (21)$$

In the absence of any external field breaking translational invariance, we have neglected the space derivative term appearing on the left hand side of Eq.(14).

In the presence of a steady-state spin injection along the y axis, by using Eqs.(16-17) and (21), we get the charge current induced along the x axis

$$J_x = (-4e\gamma)(2\alpha m D)(\tau_{\rm DP}\chi_0)(-\dot{B}^y) = -e\alpha N_0 \tau \dot{B}^y, \qquad (22)$$

having included the Bohr magneton and the gyromagnetic ratio in the definition of the magnetic field so that $\chi_0 = -N_0/2$. Eq.(22) is the the mathematical expression of IEE, whereby a charge current is induced by a non equilibrium spin polarization. We notice that the DPSR time $\tau_{\rm DP}$ has dropped out from the final expression of the Edelstein conductivity $\sigma_{\rm IEE} = J_x/\dot{B}^y = -e\alpha N_0 \tau$. This is because, as the derivation of Eq.(4) has shown, the precession which generates the out-of-plane spin density occurs during the time between two successive collisions. The longer DPSR time $\tau_{\rm DP}$ controlling the value of the steady-state spin injection, $S^y = -\tau_{DP}\chi_0 \dot{B}^y$, is compensated by the fact that the coupling between spin density and spin current develops over longer time and length scales as shown in Eq.(8), which can also be written as $J^z_y = (L_{\rm SO}/\tau_{\rm DP}) S^y$, $L_{\rm SO} = (2m\alpha)^{-1}$ being a typical length scale associated to the SOC. Finally, the spin Hall angle controlling the conversion of the generated spin current into the charge current can be written as $\gamma \sim (\alpha\tau)/L_{\rm SO}$, thus explaining how in the final result (22) only the length scale $\alpha\tau$ remains. This suggests that Eq.(22) is actually valid beyond the diffusive regime where $\alpha p_F \tau \ll 1$. This can be checked by computing the Kubo expression for $\sigma_{\rm IEE}$, whose evaluation is briefly sketched. The conductivity for the IEE reads

$$\sigma_{\rm IEE} = \lim_{\omega \to 0} \frac{\langle\langle J_x; S^y \rangle\rangle}{{\rm i}\omega} = -\frac{e}{2\pi} \int \frac{{\rm d}^2 p}{(2\pi)^2} {\rm Tr}\left[\hat{J}_x \hat{G}^R \frac{\sigma^y}{2} \hat{G}^A\right], \qquad (23)$$

where the retarded Green function is $\hat{G}^R = G^R_0 \sigma^0 + G^R_1 \sigma^x + G^R_2 \sigma^y$, $G^R_0 = (G^R_+ + G^R_-)/2$, $G^R_{1,2} = (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{z}})(G^R_+ - G^R_-)/2$, and $G^R_\pm = (\omega - \varepsilon_\pm + {\rm i}/2\tau)^{-1}$, with $\varepsilon_\pm = p^2/2m \pm \alpha p$ the eigenvalues of the Hamiltonian (1). Similar relations exist for the advanced Green function G^A. The scattering time τ enters via the self-consistent Born approximation of the impurity technique. The charge current vertex $\hat{J}_x = p_x/m$ does not contain anomalous contributions due to vertex corrections cancellations [8]. Following the methods of Ref. [8] one obtains

$$\sigma_{\rm IEE} = \frac{e}{8\pi m} \int \frac{{\rm d}^2 p}{(2\pi)^2} p \left(G^R_+ G^A_+ - G^R_- G^A_-\right) = -eN_0 \alpha\tau, \qquad (24)$$

thus confirming the validity of the result (22) even beyond the diffusive regime. The expression (22) can be further extended to include the effect of SOC from the impurities [4] as well as from bulk asymmetry of the Dresselhaus type. We do not consider these extensions here. To make contact with the experiment [1], we notice that the steady state spin injection, $\dot{S}^y = -\chi_0 \dot{B}^y$, must be replaced by the injected spin current density J^y_s (polarized along the y direction). Hence, Eq.(22) can be written as $J_x/(-2eJ^y_s) = \alpha\tau = \lambda_{\rm IEE}$, where the measured length is $\lambda_{\rm IEE} = 3$ Å. From the measured Fermi wave vector in a silver-bismuth interface [9], $k_F \sim 0.2$ Å^{-1}, one estimates $\alpha k_F \tau \sim 1$ at the border of the diffusive regime, but within the domain of validity of the present theory.

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References

[1] J.C. Rojas Sánchez, L. Vila, G. Desfonds, S. Gambarelli, J.P. Attané, J.M. De Teresa, C.Magén, A. Fert, *Nature Commun.* **4**, 2944 (2013).

[2] The Onsager reciprocal of this effect has been proposed by V. M. Edelstein, *Solid State Commun.* **73**, 233 (1990); A.G. Aronov, Y.B. Lyanda-Geller, *JETP Lett.* **50**, 431 (1989).

[3] S.D. Ganichev, E.L. Ivchenko, V.V. Bel'kov, S.A. Tarasenko, M. Sollinger, D. Weiss, W. Wegscheider, W. Prettl, *Nature* **417**, 153 (2002).

[4] Ka Shen, G. Vignale, R. Raimondi *Phys. Rev. Lett.* **112**, 096601 (2014).

[5] C. Gorini, P. Schwab, R. Raimondi, A.L. Shelankov, *Phys. Rev. B* **82**, 195316 (2010).

[6] C. Gorini, R. Raimondi, P. Schwab, *Phys. Rev. Lett.* **109**, 246604 (2012).

[7] R. Raimondi, P. Schwab, C. Gorini, G. Vignale, *Ann. Phys.* **524**, 153 (2012).

[8] R. Raimondi, P. Schwab, *Phys. Rev. B* **71**, 033311 (2005).

[9] C.R. Ast, J. Henk, A. Ernst, L. Moreschini, M.C. Falub, D. Pacilé, P. Bruno, K. Kern, M. Grioni, *Phys. Rev. Lett.* **98**, 186807 (2007).