

# Boson Fields in Ordered Magnets

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Spin wave theory of magnetism reveals two severe shortcomings. First, it considers non quantized classical spins and, second, the predicted temperature power functions for the thermal decrease of the magnetic order parameter hold asymptotically at  $T = 0$  only. As experiments unambiguously show the dynamics is different for magnets with integer and half-integer spin and the “critical” power functions at  $T = 0$  of type  $\Delta M \approx T^\epsilon$  or at  $T = T_c$  of type  $\approx (T_c - T)^\beta$  hold over a finite temperature range, independent of spin structure. The finite critical range unequivocally indicates that the dynamics of the spins is controlled by a field of freely propagating bosons instead by exchange interactions. Consequently, field theories are necessary for description of the thermodynamics of ordered magnets. The experimental indications will be discussed that the field quanta are essentially magnetic dipole radiation emitted by the precessing magnetic moments. Since integer and half-integer spins precess differently the generated field quanta and the dynamics of the field are correspondingly different.

DOI: [10.12693/APhysPolA.127.350](https://doi.org/10.12693/APhysPolA.127.350)

PACS: 75.10.-b, 75.30.Ds

## 1. Introduction

Typical for atomistic or local theories of magnetism is that they make power series expansion for the magnetic order parameter either at the critical temperature,  $T_c$  or at  $T = 0$ . Famous examples are Bloch  $T^{3/2}$  function for the thermal decrease of the spontaneous magnetization of the isotropic ferromagnet and Onsager’s critical power function of the 2D Ising model of type  $\approx (T_N - T)^{1/8}$ . The two power functions are the first term of a power series expansion at  $T = 0$  and at  $T = T_N$ , respectively. As a consequence, the width of the critical range at the two points is zero. Experimentally it is observed that the critical power functions hold up to a considerable distance from critical temperature. For  $T = 0$  the critical power functions are power functions of absolute temperature. The critical power functions at  $T = 0$  and at  $T = T_c$  overlap and give complete description of the magnetic order parameter for all temperatures [1].

Spin structure independence of the dynamics indicates that the dynamics is determined by a field of freely propagating bosons. The bosons are the excitations of the continuous magnetic medium. Note that there are no spins in the magnetic continuum. We have called all types of bosons of the magnetic continuum GSW bosons, giving tribute to J. Goldstone, A. Salam and S. Weinberg [2]. In contrast to the local exchange interactions the field bosons are delocalized excitations. The spin in the GSW boson field has much similarity with an electric dipole put into the electromagnetic radiation field. The electric dipole gets excited according to the spectral distribution of the electromagnetic field. In a similar way the non relevant spin system is a passive system and follows the dynamics of the boson guiding field. In the ordered state

all thermal energy is in the field. The spins are indicators of the field dynamics.

As was proven by renormalization group (RG) theory, spins and interactions between spins (magnons) are unimportant for the critical dynamics (of the spins) [3]. A severe consequence of this finding is that the magnetic ordering transition is executed not by exchange interactions but by the GSW boson field. In RG theory the boson driven phase transition is called a stable fixed point (SFP). Quite generally, the SFP is at a lower temperature than the ordering temperature estimated by atomistic theories on the basis of the exchange interactions between neighboring spins. It is evident that the observed critical exponents at SFP are specific for the dynamics of the boson field, and should not be compared with atomistic model predictions.

A general obstacle for the development of realistic field theories of magnetism is that the GSW field quanta have not been identified unambiguously as yet. Available field theories of the critical dynamics rely on hypothetical assumptions on the field [4]. The as calculated critical exponents cannot be reliable. As experiments on genuine Ising magnets suggest, the GSW bosons seem to be essentially magnetic dipole radiation emitted upon precession of the magnetic moments. Since Ising spins cannot precess they are unable to create field quanta. The field gets not populated by bosons. As a consequence, the dynamics is atomistic. In fact the few known genuine Ising magnets are the only ordered magnets showing atomistic dynamics. Onsager exact solution of the 2D Ising model seems to be the only atomistic model that is realized in nature [5].

## 2. Analysis of experimental data

Genuine Ising magnets are extremely rare. A well investigated example is the 2D Ising antiferromagnet  $K_2CoF_4$  [5]. The paramagnetic susceptibility of  $K_2CoF_4$  is extremely anisotropic [6]. Using elastic neutron

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scattering it could be verified that the order parameter of  $\text{K}_2\text{CoF}_4$  follows Onsager's exact solution of the atomistic 2D Ising model perfectly (see Fig. 1). Typical for the atomistic nature of this model is that power series expansions can be made at  $T = 0$  and at  $T = T_N$ . For all other magnets with three-dimensional spin the critical power functions of type  $\approx (T_c - T)^\beta$  hold over a finite temperature range. From the observation that only in Ising magnets the dynamics is atomistic it can be concluded that the field quanta are essentially magnetic dipole radiation.

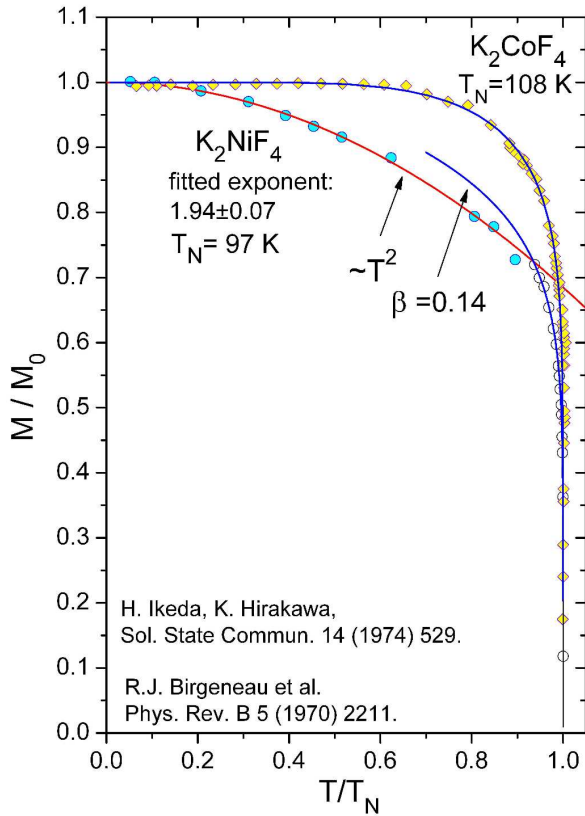


Fig. 1. Order parameter of the 2D Ising antiferromagnet  $\text{K}_2\text{CoF}_4$  (upper curve) as a function of reduced temperature [5]. These data follow Onsager's exact solution of the atomistic 2D Ising model perfectly (solid line). For the 2D antiferromagnet  $\text{K}_2\text{NiF}_4$  [7] with three dimensional spin ( $S = 1$ ) the universal power function  $\approx T^2$  at  $T = 0$  and  $\approx (T_N - T)^{0.14}$  overlap and give complete description of the order parameter for all temperatures.

As a typical example demonstrating the finite width of the two universal power functions at  $T = T_c$  and at  $T = 0$  Fig. 2 shows data of the reduced order parameter squared of the cubic ferromagnet  $\text{ErAl}_2$  as a function of temperature. These data have been obtained using zero field elastic neutron scattering. Note that in magnetization measurements the sample is in the saturated state and, in principle, one-dimensional. Linear temperature dependence of  $[(M(T)/M(T = 0))^2]$  in the critical

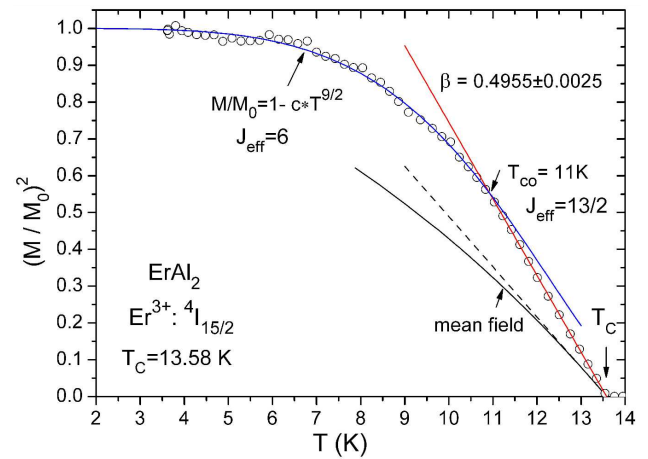


Fig. 2. Normalized spontaneous magnetization squared as a function of temperature of the cubic ferromagnet  $\text{ErAl}_2$ . At  $T_{CO} = 11$  K crossover from critical power function of type  $\approx (T_c - T)^{1/2}$  to critical power function of type  $\Delta M \approx T^{9/2}$  occurs. The two universal power functions overlap and give complete description of the order parameter for all temperatures.

range  $T_{CO} = 11$  K  $< T < T_C = 13.6$  K proves critical exponent of  $\beta = 1/2$ . In this temperature range deviations from the asymptotic critical power function of the classical mean field model (dashed line) are quite noticeable. It is a bit confusing that the atomistic model exponents can occur at a SFP. However, the finite width of the critical range indicates that the agreement with the classical mean field model is fortuitous. Mean field exponent of  $\beta = 1/2$  is observed in other cubic magnets with half-integer spin such as  $\text{GdZn}$  and  $\text{GdMg}$  [1]. In  $\text{ErAl}_2$   $T_C$  is rather low and crystal field interaction is relevant. A relevant crystal field has the effect of reducing the magnetic moment with respect to the value of the free  $\text{Er}^{3+}$  ion. As we have argued earlier, the effective moment within critical range near  $T_C$  is  $J_{\text{eff}} = 13/2$  but  $J_{\text{eff}} = 6$  in the critical range near  $T \approx 0$  extending over the range  $0 < T < T_{CO}$  [8]. In other words, at  $T_{CO} = 11$  K not only crossover to critical power function at  $T = 0$  of type  $\Delta M \approx T^{9/2}$  occurs but additionally a change in effective spin quantum number. The finite validity range of the power function  $\Delta M \approx T^{9/2}$  shows that the dynamics near  $T \approx 0$  also is due to a boson field. A finite width of the  $T^c$  power function results when the relevant bosons are freely propagating and therefore have dispersion that is a simple power function of wave vector over a large range of energy.

Figure 3 shows order parameter data of  $\text{EuO}$ ,  $\text{Y}_3\text{Fe}_5\text{O}_{12}$  and  $\text{CrI}_3$  as a function of reduced temperature squared revealing universal exponent of  $\varepsilon = 2$ . For the three compounds the spin is half-integer.  $T^2$  function has been identified as universality class of isotropic magnets with half-integer spin.  $\text{EuO}$  and  $\text{Y}_3\text{Fe}_5\text{O}_{12}$  (yttrium iron garnet) are cubic but  $\text{CrI}_3$  is hexagonal. As a conclusion, in  $\text{CrI}_3$  anisotropy of the GSW boson field seems to be below threshold to be relevant.

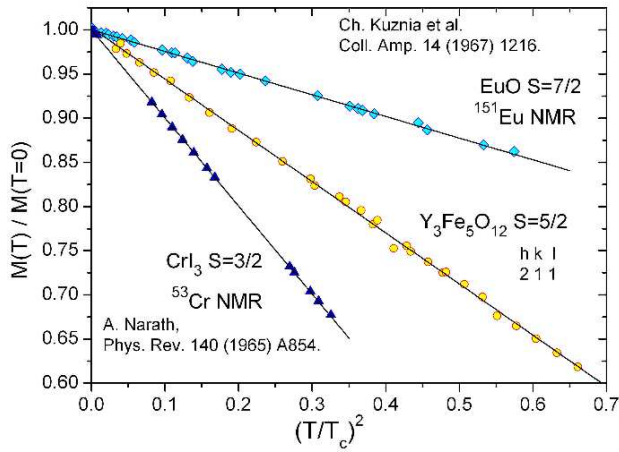


Fig. 3. Normalized order parameters of  $\text{EuO}$ ,  $\text{Y}_3\text{Fe}_5\text{O}_{12}$  and  $\text{CrI}_3$  as a function of reduced temperature squared proving  $\varepsilon = 2$  for isotropic magnets with half-integer spin. Data of  $\text{Y}_3\text{Fe}_5\text{O}_{12}$  (yttrium iron garnet) are from elastic neutron scattering. Data of  $\text{EuO}$  and  $\text{CrI}_3$  are obtained by zero field NMR and are not to scale.

As a result of many experimental investigations a scheme of six “critical” exponents  $\varepsilon$  could be established [1]. The six universality classes result from the three dimensionalities of the field and from whether the spin is integer or half-integer. For isotropic magnets with integer spin  $\varepsilon = 9/2$  (see Fig. 2) but for isotropic magnets with half-integer spin  $\varepsilon = 2$ . Since each power function represents a different universality class, it becomes evident that the power series expansions of the atomistic theories à la Bloch-Dyson are not justified in field theory.

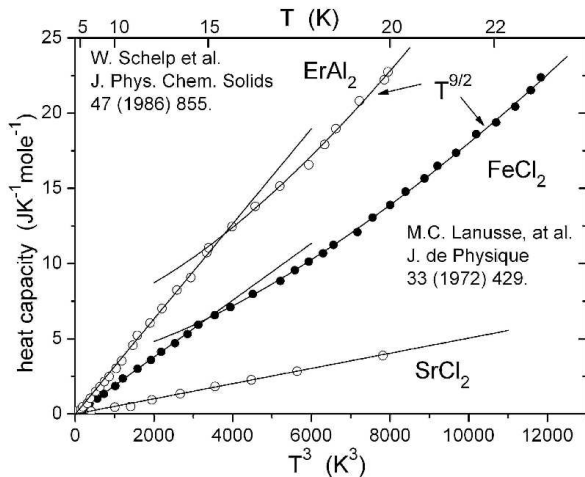


Fig. 4. Heat capacity of  $\text{ErAl}_2$  [9] and  $\text{FeCl}_2$  [10] as function of  $T^3$ . Crossover from  $T^3$  dependence due to relevant Debye bosons to  $T^{9/2}$  function due to relevant GSW bosons is clearly resolved. The two boson fields exclude each other and define the universal power function alternatively. The non asymptotic  $T^{9/2}$  function includes an absolute constant. Data of  $\text{ErAl}_2$  are not to scale.

Universality can be identified also in the heat capacity. At low temperatures Debye bosons and GSW bosons compete. It proves that the two boson fields exclude each other and that the field with the larger heat capacity defines the universal power function. For  $\text{ErAl}_2$  and  $\text{FeCl}_2$  with  $T^{9/2}$  function of the magnetic order parameter the heat capacity of the Debye boson field ( $T^3$  function) dominates at the lowest temperatures (see Fig. 4). For larger temperatures the heat capacity of the GSW boson field dominates and crossover to  $T^{9/2}$  can be identified. Consequently, in this temperature range thermal increase of heat capacity is by the same power function as thermal decrease of the magnetic order parameter.

### 3. Summary

The temperature dependence of the order parameter of the 2D Ising antiferromagnet  $\text{K}_2\text{CoF}_4$  is in perfect agreement with Onsager’s exact solution of the atomistic 2D Ising model [5]. The dynamics is atomistic. Since Ising spins cannot precess they do not generate field quanta (magnetic dipole radiation). For all magnets with three-dimensional spin the dynamics is governed by a boson guiding field. Since all thermal energy is in the field field theories need to consider the energy degrees of freedom of the field exclusively. Due to the weak coupling between field quanta and spins (given essentially by the emission probability for magnetic dipole radiation) the passive spin system follows the dynamics of the field. As a conclusion, only for genuine Ising magnets atomistic models are correct.

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