

# Influence of Acoustic Anisotropy on Frequency Bandwidths of Bragg Diffraction in Two Orthogonally Polarized Diffraction Orders

A.V. ZAKHAROV\* AND V.B. VOLOSHINOV

Faculty of Physics, Lomonosov Moscow State University, GSP-2, Leninskiye Gory, 119992 Moscow, Russia

This paper theoretically and experimentally examines a specific regime of acousto-optic diffraction during which an arbitrarily polarized incident light in a paratellurite crystal is scattered simultaneously into two orthogonally polarized diffraction maxima. We examined acoustic energy walk-off in the crystal and its influence on phase matching as well as on distribution of light energy between the two diffraction orders. This influence was theoretically considered by means of wave vector diagrams illustrating momentum conservation law during the photon-phonon interaction. We obtained expressions for mismatch parameters in the ordinary +1 and extraordinary -1 diffraction orders both depending on the walk-off angle. The obtained parameters were used in the Raman-Nath system of coupled-wave equations to calculate the diffracted light intensities and frequency bandwidths of diffraction. We measured the bandwidths in experiments carried out in the (110) plane of paratellurite crystal at the walk-off angle equal to 54°.

DOI: [10.12693/APhysPolA.127.63](https://doi.org/10.12693/APhysPolA.127.63)

PACS: 42.25.Fx, 42.79.Jq, 43.35.Sx

## 1. Introduction

The majority of crystals used for design of acousto-optic (AO) devices is characterized by a strong dependence of their elastic and optic properties on direction of acoustic and optic wave propagation [1–3]. For example, the angle between directions of phase and group velocity vectors of ultrasonic wave (acoustic walk-off angle) in a well-known paratellurite crystal may reach the value  $\Psi = 74^\circ$ . The influence of acoustic anisotropy on parameters of AO interaction is being a subject of stable interest during the recent years [4–10].

In this report, we discuss a particular regime of anisotropic Bragg AO diffraction when arbitrarily polarized incident light is deflected simultaneously into two diffraction maxima with orthogonal polarizations. Numerical and experimental investigation of influence of acoustic anisotropy on this type of diffraction was performed in AO cell based on (110) plane of paratellurite crystal, where the walk-off angle of ultrasound equals to 54° for the slow shear acoustic wave with the wave vector directed at 10° to [110] axis of the crystal. It should be mentioned that the examined interaction geometry in paratellurite differs from that in the papers [7–10] because the considered diffraction applies some other frequencies of ultrasound and especially optical angles of incidence as compared to the cited references.

## 2. Theoretical background

We consider diffraction of an incident arbitrary polarized optical wave on a plane-parallel ultrasonic column with width  $l$  and a wave vector of sound  $\mathbf{K}$  in (110) plane of paratellurite crystal. In this plane, refraction indices are equal to  $n_o = 2.26$  and  $n_e = 2.41$  for light

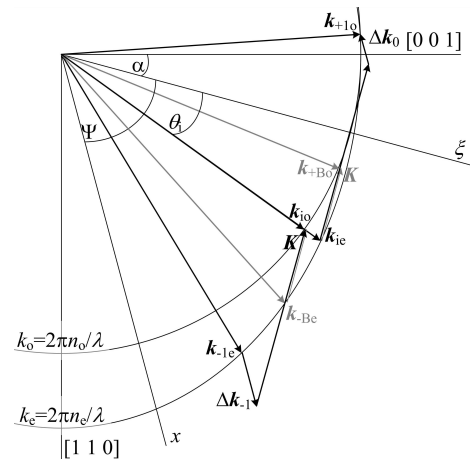


Fig. 1. Wave vector diagrams of Bragg diffraction of arbitrarily polarized incident light in two orthogonally polarized diffraction orders in (110) plane of paratellurite crystal.

at the optical wavelength  $\lambda = 633$  nm. As one may see from the wave vector diagrams of interaction in Fig. 1, the wave vectors of sound  $\mathbf{K}$  are directed at angle  $\alpha$  to the axis [110] of the crystal. The incident optical wave in the crystal is separated into two parts: the extraordinary wave with the wave vector  $\mathbf{k}_{ie}$  and the ordinary wave with the wave vector  $\mathbf{k}_{io}$ . These waves experience diffraction with change of polarization into +1 ordinary maximum with the wave vector  $\mathbf{k}_{+1o}$  and -1 extraordinary maximum with the wave vector  $\mathbf{k}_{-1e}$ . The wave vectors of the incident and the diffracted light and also of the ultrasound are combined by the following relations:

$$\begin{aligned} \mathbf{k}_{-1e} + \Delta\mathbf{k}_{-1} + \mathbf{K} &= \mathbf{k}_{io}, \\ \mathbf{k}_{ie} + \mathbf{K} + \Delta\mathbf{k}_0 &= \mathbf{k}_{+1o}. \end{aligned} \quad (1)$$

\*corresponding author; e-mail: [azakharov@physics.msu.ru](mailto:azakharov@physics.msu.ru)

Simultaneous scattering may be observed under certain directions of light incidence and certain values of ultrasonic frequency, when the magnitudes of phase mismatch vectors  $\Delta\mathbf{k}_{-1}$  and  $\Delta\mathbf{k}_0$  in Eq. (1) are considerably small. At light incidence under the Bragg angle  $\theta_B$  relatively to wave fronts of the ultrasonic wave and at certain magnitude of the acoustic wave vector  $\mathbf{K}$ , the exact satisfaction of the Bragg phase matching condition is possible for both diffraction maxima, when the phase mismatch vectors turn into zero (wave vectors corresponding this case are gray-coloured in Fig. 1). In a general case, the phase mismatch parameters are determined by the differences of projections of the wave vectors of light corresponding to the neighbouring diffraction maxima and of the ultrasound on the  $x$  axis. This axis is orthogonal to the direction of group velocity of sound and tilted at the acoustic walk-off angle  $\Psi$  to the wave fronts of the ultrasound (and to the axis  $\xi$  in Fig. 1).

Our analysis of the wave vector diagrams allow obtaining expressions for the phase mismatch parameters which depend on the incidence angle of light  $\theta_i$ , the acoustic walk-off angle  $\Psi$  and the angle of crystalline cut  $\alpha$ :

$$\begin{aligned} \Delta k_0 &= -[k_{oe} \cos(\theta_i + \Psi) - K \sin \Psi] \\ &+ \sqrt{k_o^2 - [k_{oe} \sin(\theta_i + \Psi) + K \cos \Psi]^2}, \\ \Delta k_{-1} &= \left\{ A_{-1} k_o^2 \sin(\Psi - \alpha) + B_{-1} k_e^2 \cos(\Psi - \alpha) \right. \\ &\left. - k_o k_e \sqrt{a - D_{-1}^2} \right\} / a, \end{aligned} \quad (2)$$

where  $k_o = \frac{2\pi n_o}{\lambda}$ ,  $k_e = \frac{2\pi n_e}{\lambda}$ ,  $k_{oe} = k_o k_e / \sqrt{k_o^2 \sin^2(\alpha + \theta_i) + k_e^2 \cos^2(\alpha + \theta_i)}$ ,  $A_{-1} = K \cos \alpha - k_o \sin(\alpha + \theta_i)$ ,  $B_{-1} = K \sin \alpha + k_o \cos(\alpha + \theta_i)$ ,  $D_{-1} = K \cos \Psi - k_o \sin(\Psi + \theta_i)$ ,  $a = k_o^2 \sin^2(\Psi - \alpha) + k_e^2 \cos^2(\Psi - \alpha)$ . The dependence of the phase mismatch parameters on the acoustic walk-off angle results in the influence of the acoustic anisotropy of a crystal on characteristics of the diffraction.

In order to find the amplitudes of the transmitted and diffracted optical waves, we applied the Raman-Nath system of coupled-wave equations describing the particular case of interaction

$$\begin{aligned} \frac{dC_{ie}}{dx} &= -\frac{q}{2l} C_{+1o} \exp(-j\Delta k_0 x), \\ \frac{dC_{io}}{dx} &= \frac{q}{2l} C_{-1e} \exp(j\Delta k_{-1} x), \\ \frac{dC_{+1o}}{dx} &= \frac{q}{2l} C_{ie} \exp(j\Delta k_0 x), \\ \frac{dC_{-1e}}{dx} &= -\frac{q}{2l} C_{io} \exp(-j\Delta k_{-1} x). \end{aligned} \quad (3)$$

The following border conditions for the complex amplitudes:  $C_{i\circ,e}(0) = 0.5$ ,  $C_{\pm 1}(0) = 0$  were considered in the analysis. The coupling coefficients  $q$  and the mismatch parameters  $\Delta k$  characterize efficiency of light energy exchange between the neighbor maxima. The intensity of light in the diffraction maxima may be found as the product of the conjugate amplitudes  $I_p = C_p C_p^*$  at  $x = l$  [1, 2].

### 3. Numerical and experimental implementation

Calculation and experimental verification of the frequency bandwidths of diffraction at the level 0.5 were carried out for the AO cell with the following parameters. In the  $(1\bar{1}0)(1\bar{1}0)$  plane of paratellurite crystal under crystalline cut angle  $\alpha = -10^\circ$  to  $[001]$  axis, the phase velocity of the slow shear acoustic wave equals  $V = 0.71 \times 10^5$  cm/s, and the acoustic walk-off angle equals  $\Psi = 54^\circ$ . The calculation showed that for the considered geometry of AO interaction, the Bragg incidence angle equals  $\theta_B = -12.16^\circ$  at the frequency of ultrasound  $f = 116.65$  MHz. It means that the optical beams propagated far away from the axis  $[001]$ . Consequently, the rotatory power of the crystal was neglected. The acoustic wave is generated in the crystal by a piezo-transducer having the length 1.1 cm, while the width of the ultrasonic column under influence of the acoustic walk-off equals to  $l = 0.65$  cm.

At these conditions, the calculated on base of Eqs. (2), (3), frequency bandwidths equaled 0.41 MHz and 0.34 MHz, respectively, for the +1 ordinary and -1 extraordinary maxima. To emphasize the influence of acoustic anisotropy, we also calculated the bandwidths of diffraction at the same conditions excluding the walk-off angle, which was then equal to zero. This calculation gave the bandwidths of diffraction for the +1 and -1 diffraction orders, respectively, 0.56 MHz and 0.44 MHz. Thus, due to the influence of the acoustic anisotropy, the bandwidths of diffraction are narrowing in 1.3–1.4 times in comparison with an isotropic medium.

The calculated values were also verified experimentally. It is important to mention that measuring of the frequency bandwidths of diffraction at the discussed geometry of interaction is rather complicated. The problem arises from the side of a certain angular fuzziness of a region of the simultaneous Bragg scattering (different from zero region of acceptance angles), while the calculated estimations were obtained at a particular Bragg angle corresponding to the optimal, i.e. the most efficient diffraction of light into both maxima. Therefore, in order to get an exact experimental determination of the optimal light incidence angle, we measured the frequency bandwidths of diffraction in a certain range of light incidence angles.

The measured dependences of the frequency bandwidths of diffraction on the incidence angle and ultrasonic frequency are presented in Fig. 2. In the figure, the set of gray curves corresponds to the scattering from the extraordinary zero into the ordinary +1 maximum, while the set of black curves describes the scattering of the ordinary zero beam into the extraordinary -1 maximum. The central frequencies of diffraction are presented by dash-dot curves and the borders of the frequency bandwidths are illustrated by solid curves. We can see in Fig. 2 that the experimentally found magnitude of the Bragg angle, in the limits of the acceptance angles area, and the central ultrasonic frequency corresponding to the optimal scattering into both diffraction maxima are

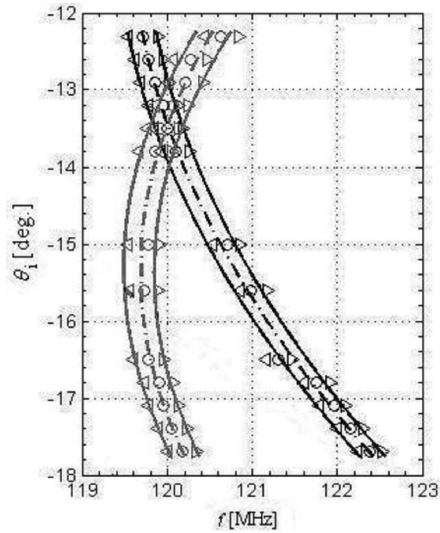


Fig. 2. Frequency-angular curves of frequency bandwidths in  $(1\bar{1}0)$  plane of paratellurite crystal: gray curves — diffraction from zero extraordinary polarized order into +1 ordinary polarized order; black curves — diffraction from zero ordinary polarized order into -1 extraordinary polarized order.

slightly different from the calculated ones:  $\theta_B \approx -13.4^\circ$  and  $f \approx 120$  MHz.

In Fig. 3, one can see the dependences of the diffracted light intensity in both diffraction orders: in the ordinary +1 (Fig. 3a) and in the extraordinary -1 (Fig. 3b). These data were obtained at the incidence of light optimal for the simultaneous Bragg scattering. Experimentally measured bandwidths of diffraction turned out to be equal to 0.38 MHz and 0.33 MHz for, respectively, the +1 ordinary and the -1 extraordinary maxima. Thus, the experimental data demonstrate quite good agreement with the theoretically calculated ones.

### 5. Conclusion

We theoretically and experimentally investigated the influence of acoustic anisotropy on parameters of AO interaction. This influence was analyzed in the particular case of the simultaneous Bragg diffraction of arbitrarily polarized incident light into  $\pm 1$  diffraction orders having orthogonal polarizations. We evaluated phase matching condition during the photon-phonon interaction in the particular case of diffraction. The obtained formulae for the phase mismatch parameters depending on the acoustic walk-off angle together with the Raman-Nath coupled wave differential equations provided calculation of light intensities in the diffraction maxima. We used this theoretical background to determine frequency bandwidths of light diffraction in the  $(1\bar{1}0)$  plane of paratellurite crystal and sound propagating at the angle  $-10^\circ$  with respect to the  $[110]$  axis. Due to the elastic anisotropy of the crystal, in the particular case of interaction, the calculated and measured bandwidths proved to be 1.3–1.4 times narrower than those in a hypothetical isotropic case. The obtained results may not directly be generalized over other

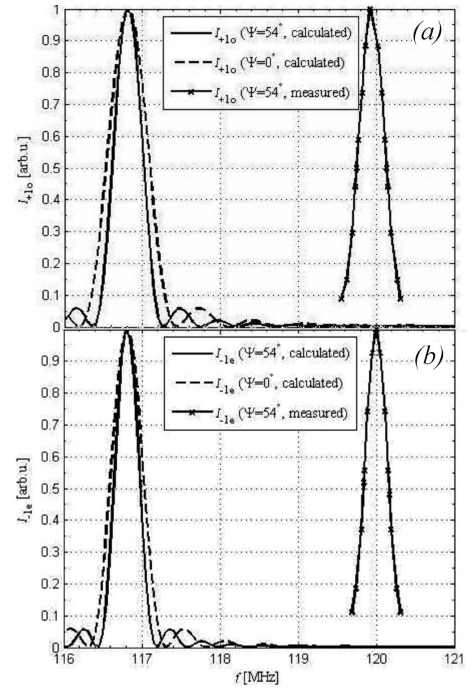


Fig. 3. Frequency bandwidths of diffraction: (a) +1 ordinary order; (b) -1 extraordinary order; solid curves — calculated at walk-off angle  $\Psi = 54^\circ$ ; dashed curves — calculated at walk-off angle  $\Psi = 0^\circ$ ; solid curves marked as “x” describe measured data.

geometries of light and sound interaction in paratellurite. However it is evident that the strong elastic anisotropy of the crystal should be taken into consideration while designing new acousto-optic devices.

### Acknowledgments

The financial support of RSF grant 14-12-00380 is greatly appreciated.

### References

- [1] V.I. Balakshy, V.N. Parygin, L.E. Chirkov, *Physical Fundamentals of Acousto-Optics*, Radio i Svyaz', Moscow 1985 [in Russian].
- [2] A. Korpel, *Acousto-Optics*, Marcel Dekker, New York 1988.
- [3] A.P. Goutzoulis, D.R. Pape, S.V. Kulakov, *Design and Fabrication of Acousto-Optic Devices*, Marcel Dekker, New York 1994.
- [4] A.V. Zakharov, V.B. Voloshinov, E. Blomme, *Ultrasonics* **51**, 745 (2011).
- [5] V.I. Balakshy, A.S. Voloshin, *Opt. Spectrosc.* **110**, 788 (2011).
- [6] J.-C. Kastelik, S. Dupont, K.B. Yushkov, J. Gazalet, *Ultrasonics* **53**, 219 (2013).
- [7] V.I. Balakshy, D.E. Kostyuk, *Appl. Opt.* **48**, C24 (2009).
- [8] V. Pozhar, A. Machihin, *Appl. Opt.* **51**, 4513 (2012).
- [9] V.B. Voloshinov, V.Ya. Molchanov, T.M. Babkina, *Techn. Phys.* **45**, 1186 (2000).
- [10] J.-C. Kastelik, K.B. Yushkov, S. Dupont, V.B. Voloshinov, *Opt. Express* **17**, 12767 (2009).