Regular Trends of Acousto-Optic Interaction in Terahertz Region of Electromagnetic Radiation

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1. Introduction

Acousto-optic (AO) interaction is widely used to control such parameters of electromagnetic radiation as direction of propagation, phase, frequency, intensity and state of polarization [1–3]. Since nature of the phenomenon does not depend on frequency of the electromagnetic radiation, AO devices have already been used in the ultraviolet, visible, infrared and even in the terahertz (THz) spectral ranges. Usually, these devices are AO modulators, deflectors and tunable filters. However, so far there are no publications describing these devices and their operation in the THz range. Unfortunately, a number of papers related to the AO interaction in the THz range is limited to a few contributions [1, 2]. The main reason for this drawback is a number of disappointing features of the AO interaction in the THz region. These features namely are the low diffraction efficiency and the substantial divergence of light, i.e. the THz radiation, due to its relatively long wavelengths $\lambda$ [3–5]. The second characteristic feature is related to the strong absorption of light at the values of the absorption coefficient $\alpha = 4\pi n''/\lambda$ of the order from 1.0 to 10 cm$^{-1}$. Here $n''$ is the imaginary part of the refractive index $n = n' - in''$, which is usually much smaller than the real part of $n'' \ll n'$. The purpose of our study was generalization of the well-known theory of the AO interaction over media in part transparent in the THz region. Experimental examination of the AO interaction in solid and liquid media promising for the AO applications in the THz region was also one of the goals of the research.

2. Theory

We follow general approach of the plane-wave theory of the AO interaction [3] to formulate the total electric field $E$ in terms of complex amplitudes of the diffracted $C_1$ and transmitted $C_0$ light waves:

$$E(x, y) = \sum_{p=0,1} C_p(x) \exp (i (k_pr - \omega_pt)), \quad (1)$$

where $k_p = \alpha \cdot \sin \theta_p$ is the wave vector and $\alpha$ is the angular frequency of light and $x$ — the coordinate across the sound beam. In our analysis, changes in the wave number $k$ is the essential limitation in the direction of propagation $\theta_p \ll 1$, which is satisfied only at the quasi-orthogonal and the quasi-collinear geometry of AO interaction.

It is known that a plane acoustic wave changes permitivity $\varepsilon$ of a medium:

$$\varepsilon = n'^2 - 2in'n'' + 2n'\Delta n' cos(Kr - \Omega t), \quad (2)$$

where $K$ is the acoustic wave vector which is equal to $K(1, 0, 0)$ at the quasi-collinear geometry, and $K(0, 1, 0)$ at the quasi-orthogonal geometry.

In order to derive a system of coupled wave equations, one has to substitute Eq. (2) in the Maxwell equations and collect terms with equal time dependent exponential factors. This procedure is possible only if the angular frequencies satisfy the condition $\omega_n = \omega_0 + n\Omega$. The Bragg matching condition is evaluated by means of the mismatch parameter $\Delta k = k_1 - k_0 - K$ dependent only on $x$-component [3]. Therefore, we obtain the following equations describing the AO interaction in the Bragg regime:

$$\begin{align*}
\frac{\partial C_0}{\partial x} &= 0.5 \left[ -\alpha_0 C_0 + i\eta_0 C_1 \exp (i\Delta k x) \right], \\
\frac{\partial C_1}{\partial x} &= 0.5 \left[ -\alpha_1 C_0 + i\eta_1 C_1 \exp (-i\Delta k x) \right],
\end{align*} \quad (3)$$

where $\alpha_p = 2k_p n''/n' \cos \theta_p$ is the absorption coefficient and $\eta_p = k_p \Delta n'/n' \cos \theta_p$ is the well-known coupling coefficient.

Analytical solution of the system Eq. (3) with the boundary conditions $C_0(0) = 1$ and $C_1(0) = 0$ is as follows:

$$C_0(L) = \cos \left( \frac{\alpha_0 L}{2} \right) \sin \left( \frac{\Delta k L}{2} \right)$$

$$C_1(L) = \frac{2i\Delta k}{\eta_1} \sin \left( \frac{\alpha_0 L}{2} \right) \sin \left( \frac{\Delta k L}{2} \right)$$

(49)
where \( s = \sqrt{q_1 q + \Delta k^2 + (\alpha_1 - \alpha_0)(4 i \Delta k + \alpha_0 - \alpha_1)/4} \) and \( L \) is a width of the sound beam usually equal to a length of a piezoelectric transducer.

Fig. 1. Dependence of the optimal dimensionless length of AO interaction on the dimensionless absorption coefficient. The solid line corresponds to the orthogonal geometry, while the dashed line describes the collinear geometry.

3. Experimental results

We measured transparency of a few AO materials in the THz region of spectrum in a preliminary experiment carried out by means of the Novosibirsk free-electron laser (FEL) [4]. It was found that the majority of known AO-materials were opaque. Therefore, basic experiments were carried out in AO cell fabricated of a high-resistive single crystal germanium [5].

\[
\times \exp \left( \frac{(2i\Delta k - \alpha_0 - \alpha_1)L}{4} \right),
\]

\[
C_1(L) = i \frac{q_1}{s} \sin \left( \frac{SL}{2} \right) \times \exp \left( -\frac{(2i\Delta k + \alpha_0 + \alpha_1)L}{4} \right),
\]

\[
\text{if } \theta_1 \text{ and } \theta_2 \text{ are narrow and the acoustic angular frequency is much smaller than that of light, then } \alpha_p = \alpha = 2k_0n'/n' \text{ and } q_p = q = k_0\Delta n'/n'. \text{ Consequently, the derived expressions (4) and (5) become simpler:}
\]

\[
C_0(L) = \left[ \cos \left( \frac{SL}{2} \right) - \frac{\Delta k}{s} \sin \left( \frac{SL}{2} \right) \right] \times \exp \left( \frac{(i\Delta k - \alpha)L}{2} \right),
\]

\[
C_1(L) = \frac{q}{s} \sin \left( \frac{SL}{2} \right) \exp \left( \frac{(-i\Delta k - \alpha)L}{2} \right),
\]

where \( s \) is defined as \( s = \sqrt{q^2 + \Delta k^2} \).

It is obvious that the absorption of light results in a decrease of the optimal length of the AO interaction \( L_{\text{opt}} \) required reaching maximum of the diffraction efficiency. Here we consider the AO interaction between a plane electromagnetic wave incident on ultrasound at the Bragg incidence \( \Delta k = 0 \) and a sound beam having constant acoustic power. In case of the orthogonal AO interaction geometry we have \( q = 2A/\sqrt{L} \), where \( A = \text{const} \). In case of the collinear geometry \( q \) remains constant \( q = 2B \), where \( B = A/\sqrt{L} \) and \( l \) is a size of the acoustic beam in the direction orthogonal to the plane of the AO interaction.

We found that the intensity of the diffracted light \( I_1 = |C_1|^2 \) in the quasi-orthogonal and the quasi-collinear interaction is given by the following equation in terms of the dimensionless parameters \( X_1 = \alpha/A \), \( Y_1 = A^2L_1 \), \( X_2 = \alpha/B \) and \( Y_2 = BL_2 \):

\[
I_{\text{opt}}^\text{ort} = e^{-X_1Y_1}\sin^2\sqrt{Y_1}, \quad I_{\text{opt}}^\text{col} = e^{-X_2Y_2}\sin^2Y_2.
\]

We used these expressions to evaluate the efficiency of the AO interaction in the terahertz region of spectrum.

Fig. 2. Scheme of the experimental setup: (1) THz free-electron laser, (2) polarizer, (3) diaphragm, (4) AO deflector, (5) Golay detector, (6) pulse generator, (7) high-frequency generator, (8) lock-in amplifier, and (9) personal computer.
applied to extract and amplify a diffracted signal coming from the Golay detector. The experimental data was collected and processed using the personal computer (9).

The germanium crystal was chosen for the analysis due to its high refractive index \( n = 4 \) and relatively low absorption coefficient \( \alpha = 0.75 \, \text{1/cm} \). Unfortunately, in our experiment, the length of the piezoelectric transducer \( L = 2.0 \, \text{cm} \) was longer than the optimal length \( L_{\text{opt}} = 1.3 \, \text{cm} \). We measured two different dependences in the carried out experiments [5]. The goal of the first experiment was related to measurement of the frequency dependence of the Bragg angle \( \theta(F) \) in the cell. We registered the dependence in the acoustic frequency band 25–38 MHz. The second experiment gave evaluation of the acoustic frequency bandwidth of diffraction \( \Delta F \) registered at a fixed angle \( \theta \) of the radiation incidence. It was proved that the examined device could be used as a reliable deflector providing scanning of the monochromatic THz radiation at the wavelength \( \lambda = 140 \, \mu\text{m} \). The diffraction efficiency \( \xi = I_1(L)/I_0(L) \) was equal to \((5.0 \pm 0.3) \times 10^{-4}\) at 1.0 W of the driving electrical power. Spatial resolution of the deflection was evaluated as \( N = \Delta F \tau = 8 \) pixels, where the quick-action of the device was about \( \tau = 2 \, \mu\text{s} \) and 3 dB acoustic frequency bandwidth was \( \Delta F = 4 \, \text{MHz} \) at the central frequency of about 30 MHz. It should be noted that the registered value of the diffraction efficiency correlated with the general theoretical estimation. As for a more detailed information on the experimental investigation as well as on a graphical representation of the obtained results, they may be found in Ref. [5].

In the most recent experiments, we examined the AO interaction in a number of non-polar liquids such as cyclohexane \( \text{C}_6\text{H}_{12} \), white spirit WS and tetra-chloro-methane \( \text{CCl}_4 \). The study proved the general trend that the efficiency of the AO diffraction in germanium was one order higher than that in the liquids: \( \xi = (2.7 \pm 0.2) \times 10^{-5} \) for \( \text{C}_6\text{H}_{12} \) and \( \xi = (5.0 \pm 1.2) \times 10^{-5} \) for WS. Moreover, the acousto-optic deflection of light in \( \text{CCl}_4 \) was not detected at all. We note that the large values of experimental errors originated from intensity fluctuations of the THz radiation at the operating frequencies of about 10–15 Hz. We had to use these extremely low operating frequencies because they corresponded to maximal sensitivity of the used Golay detector GC-1T [6].

4. Conclusion

To summarize the foregoing, we state that the AO interaction in the THz range of electromagnetic spectrum is rather weak and difficult to observe due to the strong absorption and divergence of radiation. More sophisticated theoretical and experimental investigations should be performed to understand principal aspects of the diffraction phenomenon in the THz region. It is expected that further investigations will result in development of new and efficient methods to control parameters of the THz radiation. Another direction of future investigations is related to search of new acousto-optic materials having high diffraction efficiency in the THz range. As for the crystalline germanium, for today it seems that it is the best available material to realize the acousto-optic interaction in the terahertz spectral range.

Acknowledgments

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References