

# Research on the Spin-Hamiltonian Parameters and Local Structure for the Tetragonal $\text{Mo}^{5+}$ Centers in $\text{CaWO}_4$ Crystal

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The spin-Hamiltonian parameters ( $g$  factors  $g_{\parallel}$ ,  $g_{\perp}$  and hyperfine structure constants  $A_{\parallel}(A)$ ,  $A_{\perp}(A)$ ,  $A_{\parallel}(B)$  and  $A_{\perp}(B)$ ,  $A(A)$  and  $A(B)$  belonging to isotopes  $^{95}\text{Mo}^{5+}$  and  $^{97}\text{Mo}^{5+}$ ) of  $\text{Mo}^{5+}$  ion at the tetragonally-compressed tetrahedral  $\text{W}^{6+}$  site in  $\text{CaWO}_4$  crystal are calculated from the high-order perturbation formulae based on the two-mechanism model, where besides the contributions to spin-Hamiltonian parameters due to the crystal-field mechanism concerning the crystal-field excited states in the extensively-applied crystal-field theory, those due to charge-transfer mechanism concerning charge-transfer excited states (which are omitted in crystal-field theory) are included. The calculated results are in reasonable agreement with the experimental values. The calculations show that for the high-valence state  $d^n$  ions (e. g.,  $\text{Mo}^{5+}$  considered) in crystals, the contributions due to charge-transfer mechanism should be taken into account in the studies of spin-Hamiltonian parameters. The local structure of  $\text{Mo}^{5+}$  center in  $\text{CaWO}_4$  crystal due to the impurity-induced local lattice relaxation is estimated from the calculations. The results are discussed.

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## 1. Introduction

Tungstates crystals  $\text{AWO}_4$  (where A indicates a divalent cation) with sheelite-type structure have attracted investigative efforts because they can be readily grown as good-sized, stable, colorlessly transparent and hard crystals. On being doped with rare earth and transition metal ions, they have the potential applications in solid state lasers, luminescence, phosphor and scintillator devices [1–6]. This fact caused a number of spectroscopic studies for  $\text{AWO}_4$  crystals doped with rare earth and transition metal ions [1–12]. Decades ago, Azarbayejani and Merlo [12] measured the electron paramagnetic resonance (EPR) spectra of  $\text{Mo}^{5+}$ -doped  $\text{CaWO}_4$  crystal and found that  $\text{Mo}^{5+}$  ion occupies the tetragonally-compressed  $\text{W}^{6+}$  site. The spin-Hamiltonian parameters ( $g$  factors  $g_{\parallel}$ ,  $g_{\perp}$  and hyperfine structure constants  $A_{\parallel}(A)$ ,  $A_{\perp}(A)$ ,  $A_{\parallel}(B)$  and  $A_{\perp}(B)$ ,  $A(A)$  and  $A(B)$  belong to isotopes  $^{95}\text{Mo}^{5+}$  and  $^{97}\text{Mo}^{5+}$ ) of this  $\text{Mo}^{5+}$  tetrahedral center were reported from the measurement [12]. Up to date there is still lack of the theoretical explanations for these spin-Hamiltonian parameters.

For  $d^n$  ions in crystals, the spin-Hamiltonian parameters are generally calculated theoretically by the extensively-applied crystal-field (CF) theory where only the contributions due to CF mechanism concerning the interactions of CF excited states with the ground state are considered [13–15]. However, strictly speaking,

the spin-Hamiltonian parameters originate from two contributions or mechanisms, one is the above CF mechanism and another is the charge-transfer (CT) mechanism concerning with the interactions of CT excited states with the ground state [16–18]. The neglect of CT mechanism in CF theory is due to the CT energy levels being often much higher than the CF energy levels [19], which results in the very weak influence of CT energy levels on the ground state. However, since the CT energy levels lower with the increase of the valence state of  $d^n$  ions [19], for the high-valence state  $d^n$  ions (e. g.,  $\text{Mo}^{5+}$  considered here) in crystals, the reasonable and exact calculations of spin-Hamiltonian parameters should also take the contributions due to CT mechanism into account and so the two (CF and CT)-mechanism model should be used. In this paper, we calculate the spin-Hamiltonian parameters of  $\text{Mo}^{5+}$ -doped  $\text{CaWO}_4$  crystals from the high-order perturbation formulae based on the two-mechanism model. In view of the fact that the spin-Hamiltonian parameters of a paramagnetic ion in crystals depend sensitively upon its immediate environment, the local (or defect) structure of  $\text{Mo}^{5+}$  centers caused by the impurity-induced local crystal relaxation in  $\text{CaWO}_4$  crystals can be estimated from the calculations. The results are discussed.

## 2. Calculation

The one-electron basis functions in the two-mechanism model for a tetrahedral  $d^n$  cluster can be expressed as the linear combinations of  $d$  orbitals  $|d_{\gamma}\rangle$  of  $d^n$  ions and the  $p$  orbitals  $|\pi_{\gamma}\rangle$  and  $|\sigma_{\gamma}\rangle$  of ligand [17, 18]:

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$$\psi_e^\chi = N_e^\chi \left( |d_e\rangle + \sqrt{3}\lambda_\pi^\chi |\pi_e\rangle \right),$$

$$\psi_t^\chi = N_t^\chi \left( |d_t\rangle + \lambda_\sigma^\chi |\sigma_t\rangle + \lambda_\pi^\chi |\pi_t\rangle \right), \quad (1)$$

where the subscript  $\gamma$  ( $= t$  or  $e$ ) indicates the irreducible representation of  $T_d$  group and the superscript  $\chi$  ( $= a$  or  $b$ ) stands for the anti-bonding orbitals related to CF mechanism and bonding orbitals concerning CT mechanism.  $N_\gamma$  (normalization coefficients) and  $\lambda_\beta$  (the orbital mixing coefficients,  $\beta = \sigma$  or  $\pi$ ) are the molecular orbital (MO) coefficients.

The perturbation formulae of spin-Hamiltonian parameters for  $d^1$  center in crystals depend upon its ground state and symmetry [13–15]. From the observed  $g_{\parallel} > g_{\perp}$ , one can conclude that the ground state of  $\text{Mo}^{5+}$  in  $\text{CaWO}_4$  is  $|d_z^2\rangle$ . Thus, from the above one-electron basis functions and by adding the spin-orbit interaction term  $H_{\text{SO}}^{\text{CT}}$ , the Zeeman (or magnetic) interaction term  $H_{\text{Ze}}^{\text{CT}}$  and the hyperfine interaction term  $H_{\text{hf}}^{\text{CT}}$  connected with CT mechanism to the traditional perturbation Hamiltonian in the CF mechanism, the high-order perturbation formulae based on the two-mechanism model for  $g$  factors of tetragonal  $d^1$  tetrahedral cluster with the ground state  $|d_z^2\rangle$  are derived in Ref. [18] and those for hyperfine structure constants  $A_i$  are derived here. They are

$$\begin{aligned} g_{\parallel} &= g_e + \Delta g_{\parallel}^{\text{CF}} + \Delta g_{\parallel}^{\text{CT}}, \\ \Delta g_{\parallel}^{\text{CF}} &= \frac{3\zeta_{\text{CF}}'^2 (k_{\text{CF}} - g_e)}{(E_1^{\text{CF}})^2}, \\ \Delta g_{\parallel}^{\text{CT}} &= \frac{3\zeta_{\text{CT}}^2 k'_{\text{CT}}}{E_1^{\text{CT}} E_2^{\text{CT}}}, \\ g_{\perp} &= g_e + \Delta g_{\perp}^{\text{CF}} + \Delta g_{\perp}^{\text{CT}}, \\ \Delta g_{\perp}^{\text{CF}} &= \frac{6\zeta_{\text{CF}} k'_{\text{CF}}}{E_1^{\text{CF}}} - \frac{3\zeta_{\text{CF}}' (\zeta_{\text{CF}} k'_{\text{CF}} + \frac{1}{2} \zeta_{\text{CF}}' g_e)}{(E_1^{\text{CF}})^2}, \\ \Delta g_{\perp}^{\text{CT}} &= \frac{6\zeta_{\text{CT}} k_{\text{CT}}}{E_1^{\text{CT}}}, \\ A_{\parallel} &= A_{\parallel}^{(1)} + A_{\parallel}^{(2)\text{CF}} + A_{\parallel}^{(2)\text{CT}}, \\ A_{\parallel}^{(1)} &= P_{\text{CF}} \left( -\kappa + \frac{4}{7} \right), \\ A_{\parallel}^{(2)\text{CF}} &= P'_{\text{CF}} \left[ (g_{\parallel}^{\text{CF}} - g_e) - \frac{1}{7} (g_{\perp}^{\text{CF}} - g_e) \right], \\ A_{\parallel}^{(2)\text{CT}} &= P'_{\text{CT}} \left( \frac{3\zeta_{\text{CT}}^2 k'_{\text{CT}}}{E_1^{\text{CT}} E_2^{\text{CT}}} \right), \\ A_{\perp} &= A_{\perp}^{(1)} + A_{\perp}^{(2)\text{CF}} + A_{\perp}^{(2)\text{CT}}, \\ A_{\perp}^{(1)} &= P_{\text{CF}} \left( -\kappa - \frac{2}{7} \right), \\ A_{\perp}^{(2)\text{CF}} &= P'_{\text{CF}} \left[ \frac{15}{14} (g_{\perp}^{\text{CF}} - g_e) \right], \\ A_{\perp}^{(2)\text{CT}} &= P'_{\text{CT}} \left( \frac{6\zeta_{\text{CT}} k_{\text{CT}}}{E_1^{\text{CT}}} \right) \end{aligned} \quad (2)$$

with the spin-orbit parameters  $\zeta$ ,  $\zeta'$ , the orbit reduction factors  $k$ ,  $k'$  and the dipolar hyperfine structure constants  $P$ ,  $P'$  in CF and CT mechanisms [17, 18]:

$$\begin{aligned} \zeta_{\text{CF}} &= (N_t^a)^2 \left[ \zeta_d^0 + \left( \sqrt{2}\lambda_\pi^a \lambda_\sigma^a - \frac{\lambda_\pi^{a2}}{2} \right) \zeta_p^0 \right], \\ \zeta'_{\text{CF}} &= N_t^a N_e^a \left[ \zeta_d^0 + \left( \frac{\lambda_\pi^a \lambda_\sigma^a}{\sqrt{2}} + \frac{\lambda_\pi^{a2}}{2} \right) \zeta_p^0 \right], \\ \zeta_{\text{CT}} &= N_t^a N_t^b \left[ \zeta_d^0 + \left( \frac{\lambda_\pi^a \lambda_\sigma^b + \lambda_\pi^b \lambda_\sigma^a}{\sqrt{2}} - \frac{\lambda_\pi^a \lambda_\pi^b}{2} \right) \zeta_p^0 \right], \\ \zeta'_{\text{CT}} &= N_e^b N_t^a \left[ \zeta_d^0 + \left( \frac{\lambda_\pi^a \lambda_\sigma^b}{\sqrt{2}} + \frac{\lambda_\pi^a \lambda_\pi^b}{2} \right) \zeta_p^0 \right], \\ k_{\text{CF}} &= (N_t^a)^2 \left[ 1 - \frac{\lambda_\pi^{a2}}{2} + \sqrt{2}\lambda_\pi^a \lambda_\sigma^a + 2\lambda_\sigma^a S_{dp}(\sigma) + 2\lambda_\pi^a S_{dp}(\pi) \right], \\ k'_{\text{CF}} &= N_t^a N_e^a \left[ 1 + \frac{\lambda_\pi^{a2}}{2} + \frac{\lambda_\pi^a \lambda_\sigma^a}{\sqrt{2}} \right. \\ &\quad \left. + 4\lambda_\pi^a S_{dp}(\pi) + 2\lambda_\sigma^a S_{dp}(\sigma) \right], \\ k_{\text{CT}} &= N_t^a N_t^b \left[ 1 + \left( \frac{\lambda_\pi^a \lambda_\sigma^b + \lambda_\pi^b \lambda_\sigma^a}{\sqrt{2}} - \frac{\lambda_\pi^a \lambda_\pi^b}{2} \right) \right. \\ &\quad \left. + (\lambda_\sigma^a + \lambda_\sigma^b) S_{dp}(\sigma) + (\lambda_\pi^a + \lambda_\pi^b) S_{dp}(\pi) \right], \\ k'_{\text{CT}} &= N_e^b N_t^a \left[ 1 + \left( \frac{\lambda_\pi^a \lambda_\sigma^b}{\sqrt{2}} - \frac{\lambda_\pi^a \lambda_\pi^b}{2} \right) \right. \\ &\quad \left. + \lambda_\sigma^b S_{dp}(\sigma) + (3\lambda_\pi^a + \lambda_\pi^b) S_{dp}(\pi) \right], \\ P_{\text{CF}} &= (N_t^a)^2 P_0, \quad P'_{\text{CF}} = N_t^a N_e^a P_0, \\ P_{\text{CT}} &= N_t^b N_t^a P_0, \quad P'_{\text{CT}} = N_e^b N_t^a P_0. \end{aligned} \quad (3)$$

In above formulae, the superscripts and subscripts CF and CT stand for the parameters in the CF and CT mechanisms.  $g_e$  ( $\approx 2.0023$ ) is the free-electron  $g$  value.  $\kappa$  is the core polarization constant.  $E_1^{\text{CF}}$  and  $E_j^{\text{CT}}$  are the CF and CT energy levels.  $\zeta_d^0$  and  $\zeta_p^0$  are the spin-orbit parameters of free  $d^n$  ion and free ligand.  $P_0$  is the corresponding parameter of free  $d^n$  ion. For  $(\text{MoO}_4)^{3-}$  clusters under study, we have  $\zeta_d^0(\text{Mo}^{5+}) \approx 1030 \text{ cm}^{-1}$  [14],  $\zeta_p^0(\text{O}^{2-}) \approx 150 \text{ cm}^{-1}$  [20],  $P_0(^{95}\text{Mo}^{5+}) \approx -66.7 \times 10^{-4} \text{ cm}^{-1}$  and  $P_0(^{97}\text{Mo}^{5+}) \approx -68.2 \times 10^{-4} \text{ cm}^{-1}$  [21].  $S_{dp}(\beta)$  are the group overlap integrals which can be calculated from the Slater-type self-consistent field (SCF) functions [22, 23] with the metal-ligand distance  $R$ . Since the ionic radius  $r_i$  of impurity is unlike the radius  $r_h$  of the host ion it replaces, the metal-ligand distance  $R$  in the impurity center should differ from the corresponding distance  $R_h$  in the host crystal. As an approximation, we estimate the distance  $R$  by using the empirical formula  $R \approx R_h + \frac{1}{2}(r_i - r_h)$  [24]. For  $\text{Mo}^{5+}$  at the  $\text{W}^{6+}$  site of  $\text{CaWO}_4$  crystal, from  $r_i(\text{Mo}^{5+}) \approx 0.60 \text{ \AA}$ ,  $r_h(\text{W}^{6+}) \approx 0.56 \text{ \AA}$  [25] and  $R_h \approx 1.782 \text{ \AA}$  [26], we obtain  $R \approx 1.802 \text{ \AA}$ . Thus, we have  $S_{dp}(\pi) \approx 0.03320$  and  $S_{dp}(\sigma) \approx -0.10744$ .

The MO coefficients  $N_\gamma^\chi$  and  $\lambda_\beta^\chi$  needed for the calculations of the parameters in Eq. (3) can be related by the normalization correlations

$$N_e^\chi = [1 + 3(\lambda_\sigma^\chi)^2 + 6\lambda_\pi^\chi S_{dp}(\pi)]^{-\frac{1}{2}},$$

$N_t^\chi = [1 + (\lambda_\sigma^\chi)^2 + (\lambda_\pi^\chi)^2 + 2\lambda_\sigma^\chi S_{dp}(\sigma) + 2\lambda_\pi^\chi S_{dp}(\pi)]^{-\frac{1}{2}}$  (4) and the orthonormal relations [11]:

$$\lambda_{\pi}^b = -\frac{1 + 3\lambda_{\pi}^a S_{dp}(\pi)}{3[\lambda_{\pi}^a + S_{dp}(\pi)]},$$

$$\lambda_{\sigma}^b = -\frac{1 + \lambda_{\pi}^a \lambda_{\pi}^b + (\lambda_{\pi}^a + \lambda_{\pi}^b) S_{dp}(\pi) + \lambda_{\sigma}^a S_{dp}(\sigma)}{\lambda_{\sigma}^a + S_{dp}(\sigma)} \quad (5)$$

and the approximate relationships

$$f_e = (N_e^a)^4 [1 + 6\lambda_{\pi}^a S_{dp}(\pi) + 9(\lambda_{\pi}^a)^2 S_{dp}^2(\pi)]$$

$$f_t = (N_t^a)^4 [1 + 2\lambda_{\sigma}^a S_{dp}(\sigma) + 2\lambda_{\pi}^a S_{dp}(\pi)$$

$$+ 2\lambda_{\sigma}^a S_{dp}(\sigma) \lambda_{\pi}^a S_{dp}(\pi) + (\lambda_{\sigma}^a)^2 S_{dp}^2(\sigma) + (\lambda_{\pi}^a)^2 S_{dp}^2(\pi)], \quad (6)$$

in which we assume the covalence factor  $f_t \approx f_e \approx f_{\gamma}$  for decreasing the number of adjustable parameter and take  $f_{\gamma}$  as an adjustable parameter.

The CT energy levels of  $(\text{MoO}_4)^{3-}$  clusters have not been reported. Since the CT energy levels of the  $(\text{CuCl}_4)^{2-}$  tetrahedral clusters are near to those of the  $(\text{CuCl}_6)^{4-}$  octahedral clusters [27, 28], for  $(\text{MoO}_4)^{3-}$  tetrahedral clusters, we approximately take  $E_1^{\text{CT}} \approx 32400 \text{ cm}^{-1}$  and  $E_2^{\text{CT}} \approx 36400 \text{ cm}^{-1}$ , the values of  $(\text{MoO}_6)^{7-}$  octahedral clusters [29].

In the CF theory, the CF energy level

$$E_1^{\text{CF}} = E(|d_{xy,yz}\rangle) - E(|d_z\rangle) \approx 10Dq - Ds - 10Dt, \quad (7)$$

where the tetragonal field parameters  $Ds$  and  $Dt$  can be calculated from the superposition model [30]. In the model, they can be written as

$$Ds = -\frac{1}{7}B_{20} = -\frac{4}{7}\bar{A}_2(R)(3\cos^2\theta - 1)$$

$$Dt = -\frac{1}{21} \left( B_{40} - \frac{\sqrt{70}}{5} B_{44} \right) = -\frac{4}{21}\bar{A}_4(R)(35\cos^4\theta - 30\cos^2\theta + 3 + 7\sin^4\theta), \quad (8)$$

where  $B_{kl}$  are the CF parameters in Wybourne notation [31, 32].  $\bar{A}_k(R)$  ( $k = 2, 4$ ) are the intrinsic

parameters. For  $4d^n$  ions in crystals, the ratio  $\bar{A}_2(R)/\bar{A}_4(R) \approx 6 \pm 2$  was found [29, 33–35], we take the average value  $\bar{A}_2(R)/\bar{A}_4(R) \approx 6$  here. The parameter  $\bar{A}_4(R)$  for  $d^n$  tetrahedral cluster can take the form  $\bar{A}_4(R) \approx -\frac{27}{16}Dq$  [36], where  $Dq$  is the cubic field parameter. The value of  $Dq$  for  $(\text{MoO}_4)^{3-}$  cluster in crystals has not been reported and is estimated approximately as follows.

The optical spectral data for various  $d^n$  ions in crystals suggest that the value of  $Dq$  for  $4d^n$  clusters is about 1.5(1) times that of the isoelectronic  $3d^n$  clusters [19]. So, from  $Dq \approx 1350(50) \text{ cm}^{-1}$  of the  $3d^1(\text{CrO}_4)^{3-}$  tetrahedral clusters [37], we obtain for the corresponding  $4d^1(\text{MoO}_4)^{3-}$  tetrahedral cluster,  $Dq \approx 1900 \text{ cm}^{-1}$ .  $\theta$  refers to the angle between the metal–ligand distance  $R$  and  $C_4$  axis. Analogous to the bonding length  $R$ , the bonding angle  $\theta$  in the impurity center may be different from the corresponding angle  $\theta_h$  in the host crystal. We assume  $\theta \approx \theta_h + \Delta\theta$ , where  $\theta_h \approx 56.89^\circ$  [26] in the host  $\text{CaWO}_4$  crystal and  $\Delta\theta$  represents the impurity-induced angular distortion.  $\Delta\theta$  is also treated as an adjustable parameter. Thus, in the above formulae, we have three parameters  $f_{\gamma}$ ,  $\Delta\theta$  and  $\kappa$  left as the adjustable parameters. From the calculated (with the above high-order perturbation formulae)-to-experimental fitting of spin-Hamiltonian parameters for  $\text{CaWO}_4: \text{Mo}^{5+}$ , we obtain

$$f_{\gamma} \approx 0.525, \quad \Delta\theta \approx 0.4^\circ, \quad \kappa \approx 0.41. \quad (9)$$

The MO coefficients based on the value of  $f_{\gamma}$  are given in Table I. The parameters in Eq. (3) calculated from these MO coefficients are listed in Table II. The calculated spin-Hamiltonian parameters are compared with the experimental values in Table III.

TABLE I

The molecular orbital (MO) coefficients for  $(\text{MoO}_4)^{3-}$  tetrahedral clusters in  $\text{CaWO}_4: \text{Mo}^{5+}$  crystal.

| $N_t^a$ | $N_e^a$ | $N_t^b$ | $N_e^b$ | $\lambda_{\sigma}^a$ | $\lambda_{\pi}^a$ | $\lambda_{\sigma}^b$ | $\lambda_{\pi}^b$ |
|---------|---------|---------|---------|----------------------|-------------------|----------------------|-------------------|
| 0.8767  | 0.8745  | 0.3622  | 0.2350  | 0.3709               | -0.5274           | -2.3792              | 0.6391            |

TABLE II

The spin-orbit parameters  $\zeta$ ,  $\zeta'$  ( $\text{cm}^{-1}$ ), the orbital reduction factors  $k$ ,  $k'$  and the dipolar hyperfine structure constants  $P$ ,  $P'$  (in  $10^{-4} \text{ cm}^{-1}$ ) in CF and CT mechanisms for  $(\text{MoO}_4)^{3-}$  tetrahedral clusters in  $\text{CaWO}_4: \text{Mo}^{5+}$  crystal.

| $\zeta_{\text{CF}}$         | $\zeta'_{\text{CF}}$         | $\zeta_{\text{CT}}$         | $\zeta'_{\text{CT}}$         | $k_{\text{CF}}$             | $k'_{\text{CF}}$             | $k_{\text{CT}}$             | $k'_{\text{CT}}$             |
|-----------------------------|------------------------------|-----------------------------|------------------------------|-----------------------------|------------------------------|-----------------------------|------------------------------|
| 743.8                       | 789.8                        | 250.0                       | 360.3                        | 0.3610                      | 0.4698                       | 0.7757                      | 0.7222                       |
| $P_{\text{CF}}(\text{A})^a$ | $P'_{\text{CF}}(\text{A})^a$ | $P_{\text{CT}}(\text{A})^a$ | $P'_{\text{CT}}(\text{A})^a$ | $P_{\text{CF}}(\text{B})^a$ | $P'_{\text{CF}}(\text{B})^a$ | $P_{\text{CT}}(\text{B})^a$ | $P'_{\text{CT}}(\text{B})^a$ |
| -51.3                       | -51.1                        | -21.2                       | -21.1                        | -52.4                       | -52.3                        | -21.7                       | -21.6                        |

<sup>a</sup>  $P(\text{A})$  and  $P(\text{B})$  belong to isotopes  $^{95}\text{Mo}^{5+}$  and  $^{97}\text{Mo}^{5+}$ .

TABLE III

The spin-Hamiltonian parameters ( $g$  factor  $g_{\parallel}$ ,  $g_{\perp}$  and hyperfine structure constants  $A_{\parallel}(A)$ ,  $A_{\perp}(A)$ ,  $A_{\parallel}(B)$  and  $A_{\perp}(B)$ ,  $A(A)$  and  $A(B)$  belong to isotopes  $^{95}\text{Mo}^{5+}$  and  $^{97}\text{Mo}^{5+}$ , constants  $A$  are in units of  $10^{-4} \text{ cm}^{-1}$ ) for the tetragonal  $(\text{MoO}_4)^{3-}$  clusters in  $\text{CaWO}_4$  crystals.

|                                    |                                    |                                   |                                    |                                       |
|------------------------------------|------------------------------------|-----------------------------------|------------------------------------|---------------------------------------|
| $\Delta g_{\parallel}^{\text{CF}}$ | $\Delta g_{\parallel}^{\text{CT}}$ | $g_{\parallel}(\text{calc.})$     | $g_{\parallel}(\text{expt. [12]})$ |                                       |
| -0.0175                            | 0.0001                             | 1.9849                            | 1.987                              |                                       |
| $\Delta g_{\perp}^{\text{CF}}$     | $\Delta g_{\perp}^{\text{CT}}$     | $g_{\perp}(\text{calc.})$         | $g_{\perp}(\text{expt. [12]})$     |                                       |
| -0.1508                            | 0.0359                             | 1.8874                            | 1.887                              |                                       |
| $A_{\parallel}^{(1)}(A)$           | $A_{\parallel}^{(2)\text{CF}}(A)$  | $A_{\parallel}^{(2)\text{CT}}(A)$ | $A_{\parallel}(A)(\text{calc.})$   | $A_{\parallel}(A)(\text{expt. [12]})$ |
| -8.27                              | -0.21                              | -0.002                            | -8.48                              | 8.39 <sup>a</sup>                     |
| $A_{\perp}^{(1)}(A)$               | $A_{\perp}^{(2)\text{CF}}(A)$      | $A_{\perp}^{(2)\text{CT}}(A)$     | $A_{\perp}(A)(\text{calc.})$       | $A_{\perp}(A)(\text{expt. [12]})$     |
| 35.66                              | 8.26                               | -0.76                             | 43.16                              | 41.18 <sup>a</sup>                    |
| $A_{\parallel}^{(1)}(B)$           | $A_{\parallel}^{(2)\text{CF}}(B)$  | $A_{\parallel}^{(2)\text{CT}}(B)$ | $A_{\parallel}(B)(\text{calc.})$   | $A_{\parallel}(B)(\text{expt. [12]})$ |
| -8.45                              | -0.21                              | -0.002                            | -8.66                              | 8.64 <sup>a</sup>                     |
| $A_{\perp}^{(1)}(B)$               | $A_{\perp}^{(2)\text{CF}}(B)$      | $A_{\perp}^{(2)\text{CT}}(B)$     | $A_{\perp}(B)(\text{calc.})$       | $A_{\perp}(B)(\text{expt. [12]})$     |
| 36.46                              | 8.44                               | -0.78                             | 44.12                              | 42.52 <sup>a</sup>                    |

<sup>a</sup>The values are actually the absolute values.

### 3. Discussion

The signs of hyperfine structure constants  $A_i$  are hard to be determined solely by EPR experiment [14, 21, 38]. So, even though the values of  $A_i$  are frequently written as positive in EPR experiment, they are actually the absolute values. For  $\text{Mo}^{5+}$  ions in  $\text{CaWO}_4$  crystal, our calculations suggest that  $A_{\parallel}$  is negative and  $A_{\perp}$  is positive (see Table III).

The angular distortion  $\Delta\theta \neq 0$  confirms the expectation that the bonding angle  $\theta$  in the  $\text{Mo}^{5+}$  impurity center in  $\text{CaWO}_4$  is different from the corresponding angle  $\theta_h$  in the host crystal because of the impurity-induced local lattice relaxation. So, the local structure of a paramagnetic impurity center in crystals can be acquired by studying its EPR data.

Table III shows that by using three adjustable parameters, the calculated spin-Hamiltonian parameters  $g_{\parallel}$ ,  $g_{\perp}$ ,  $A_{\parallel}(A)$ ,  $A_{\perp}(A)$ ,  $A_{\parallel}(B)$  and  $A_{\perp}(B)$  are in reasonable agreement with the experimental values. This suggests that the high-order perturbation formulae based on the two-mechanism model are effective here. To characterize the relative importance of CT mechanism, we introduce the ratio  $|Q^{\text{CT}}/Q^{\text{CF}}|$ . From Table III, we obtain  $|Q^{\text{CT}}/Q^{\text{CF}}| \approx 0.6\%$ ,  $24\%$ ,  $1\%$  and  $9\%$  for  $Q = \Delta g_{\parallel}$ ,  $\Delta g_{\perp}$ ,  $A_{\parallel}^{(2)}$  and  $A_{\perp}^{(2)}$ . It can be seen that the values  $|Q^{\text{CT}}/Q^{\text{CF}}|$  of relative importance of CT mechanism for  $\Delta g_{\parallel}$  and  $A_{\parallel}^{(2)}$  are much smaller than those for  $\Delta g_{\perp}$  and  $A_{\perp}^{(2)}$ . The main reason of the above great difference in  $|Q^{\text{CT}}/Q^{\text{CF}}|$  may be due to the contributions to  $\Delta g_{\parallel}$  and  $A_{\parallel}^{(2)}$  in both the CF and CT mechanisms depending upon the third-order perturbation terms [note: the second-order terms are absent according to the derivation, see Eq. (2)], whereas those to  $\Delta g_{\perp}$  and  $A_{\perp}^{(2)}$  mainly from the second-order

perturbation terms. This leads the ratios  $|Q^{\text{CT}}/Q^{\text{CF}}|$  for  $\Delta g_{\parallel}$  and  $A_{\parallel}^{(2)}$  to be connected roughly with  $(E^{\text{CT}}/E^{\text{CF}})^2$ , but those for  $\Delta g_{\perp}$  and  $A_{\perp}^{(2)}$  roughly with  $E^{\text{CT}}/E^{\text{CF}}$  [see Eq. (2)]. Since  $E^{\text{CT}}/E^{\text{CF}} < 0.5$  (note: the calculated  $E_1^{\text{CF}} \approx 13240 \text{ cm}^{-1}$  here), the above large difference in the relative importance can be understood. In consideration of the great relative importance of CT mechanism to  $\Delta g_{\perp}$  and  $A_{\perp}^{(2)}$ , in the rational and precise calculations of spin-Hamiltonian parameters and the estimations of the local structure of impurity centers (by analyzing the spin-Hamiltonian parameters) for the high-valence state  $d^n$  ions in crystals, one should apply the method based on the two-mechanism model.

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