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Exact Ground States of Frustrated Spin-1 Ising-Heisenberg and Heisenberg Ladders in a Magnetic Field

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Ground states of the frustrated spin-1 Ising-Heisenberg two-leg ladder with Heisenberg intra-rung coupling and only Ising interaction along legs and diagonals are rigorously found by taking advantage of local conservation of the total spin on each rung. The constructed ground-state phase diagram of the frustrated spin-1 Ising-Heisenberg ladder is then compared with the analogous phase diagram of the fully quantum spin-1 Heisenberg two-leg ladder obtained by density matrix renormalization group (DMRG) calculations. Both investigated spin models exhibit quite similar magnetization scenarios, which involve intermediate plateaux at one-quarter, one-half and three-quarters of the saturation magnetization.

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1. Introduction

Over the last few decades, quantum spin ladders have been actively studied mainly in connection with spin-liquid behaviour, quantum critical points and superconductivity under hole doping of some cuprates (see Ref. [1] for a review). In particular, the frustrated spin-1/2 Heisenberg two-leg ladder exhibits a striking dimerized ground state [2] and a low-temperature magnetization process with an intermediate plateau and magnetization jumps [3].

Another challenging topic of current research interest consists of the theoretical investigation of related models such as the quantum spin-1 Heisenberg two-leg ladder [4, 5]. The main goal of the present work is to find the exact ground states of a simpler spin-1 Ising-Heisenberg ladder and to contrast them with the respective ground states of the pure quantum spin-1 Heisenberg ladder. Note that the former model is analytically tractable using the procedure developed in Refs. [6, 7] and it brings insight into the relevant behaviour of the latter not fully integrable model.

2. Frustrated Ising-Heisenberg ladder

Consider first the frustrated spin-1 Ising-Heisenberg ladder with the Heisenberg intra-rung interaction and the unique Ising interaction along the legs and diagonals. The total Hamiltonian of the investigated model is given by

$$\hat{H} = \sum_{i=1}^{N} [J \hat{\boldsymbol{S}}_{1,i} \cdot \hat{\boldsymbol{S}}_{2,i} + J_1 (\hat{S}^z_{1,i} - h(\hat{S}^z_{1,i} + \hat{S}^z_{2,i}) + \hat{S}^z_{2,i}) \cdot (\hat{S}^z_{1,i+1} + \hat{S}^z_{2,i+1})], \quad (1)$$

where $\hat{\boldsymbol{S}}_{\alpha,i} \equiv (\hat{S}^x_{\alpha,i}, \hat{S}^y_{\alpha,i}, \hat{S}^z_{\alpha,i})$ denotes spatial components of the spin-1 operator, the former suffix $\alpha = 1$ or 2 enumerates the leg and the latter suffix specifies a lattice position within a given leg. The coupling constant J denotes the isotropic Heisenberg intra-rung interaction, the parameter J_1 determines the Ising interaction along the legs and diagonals, h is an external magnetic field.

For further convenience, let us introduce the spin operator $\hat{\boldsymbol{T}}_i = \hat{\boldsymbol{S}}_{1,i} + \hat{\boldsymbol{S}}_{2,i}$, which corresponds to the total spin angular momentum of the ith rung. It can be easily proved that the operators $\hat{\boldsymbol{T}}_i^2$ and $\hat{T}_i^z$ commute with the Hamiltonian (1), i.e. $[\hat{\boldsymbol{T}}_i^2, \hat{H}] = [\hat{T}_i^z, \hat{H}] = 0$, which means that the total spin of a rung and its z component represent conserved quantities with well defined quantum numbers. The complete energy spectrum of the frustrated spin-1 Ising-Heisenberg ladder then readily follows from the relation

$$E = -2NJ + \frac{J}{2} \sum_{i=1}^{N} T_i (T_i + 1) + J_1 \sum_{i=1}^{N} T_i^z T_{i+1}^z - h \sum_{i=1}^{N} T_i^z, \quad (2)$$

which depends just on the quantum numbers $T_i = 0, 1, 2$ and $T_i^z = -T_i, -T_i+1, ..., T_i$ determining the eigenvalues of the total spin of the ith rung and its z spatial projection, respectively. Using this procedure, the spin-1 Ising-Heisenberg two-leg ladder has been rigorously mapped to some classical chain of composite spins and accordingly, we can readily find all available ground states by looking for the lowest-energy state of Eq. (2).

3. Frustrated Heisenberg ladder

Next, we will also consider the frustrated spin-1 Heisenberg two-leg ladder defined by the Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} [J \hat{\boldsymbol{S}}_{1,i} \cdot \hat{\boldsymbol{S}}_{2,i} - h(\hat{S}^z_{1,i} + \hat{S}^z_{2,i}) + J_1 (\hat{\boldsymbol{S}}_{1,i} + \hat{\boldsymbol{S}}_{2,i}) \cdot (\hat{\boldsymbol{S}}_{1,i+1} + \hat{\boldsymbol{S}}_{2,i+1})], \quad (3)$$

which represents the pure quantum analogue of the frustrated spin-1 Ising-Heisenberg ladder discussed previously. Taking advantage of the definition for the total

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spin of each rung, the Hamiltonian (3) of frustrated spin-1 Heisenberg two-leg ladder can be rewritten into the form

$$\hat{H} = -2NJ + \frac{J}{2}\sum_{i=1}^{N}\hat{\boldsymbol{T}}_i^2 + J_1\sum_{i=1}^{N}\hat{\boldsymbol{T}}_i \cdot \hat{\boldsymbol{T}}_{i+1} - h\sum_{i=1}^{N}\hat{T}_i^z. \qquad (4)$$

According to Eq. (4), the frustrated spin-1 Heisenberg ladder can be rigorously decomposed into the direct sum of quantum spin chains with spin 0, 1 or 2 at each site. The ground state of such a system can be shown to be either a homogeneous chain, with the same spin at all sites, or a chain with alternating spins on every other site. Comparing the energy of the different chains, obtained either analytically or using DMRG simulations, the exact ground-state phase diagram of the frustrated spin-1 Heisenberg ladder can be constructed.

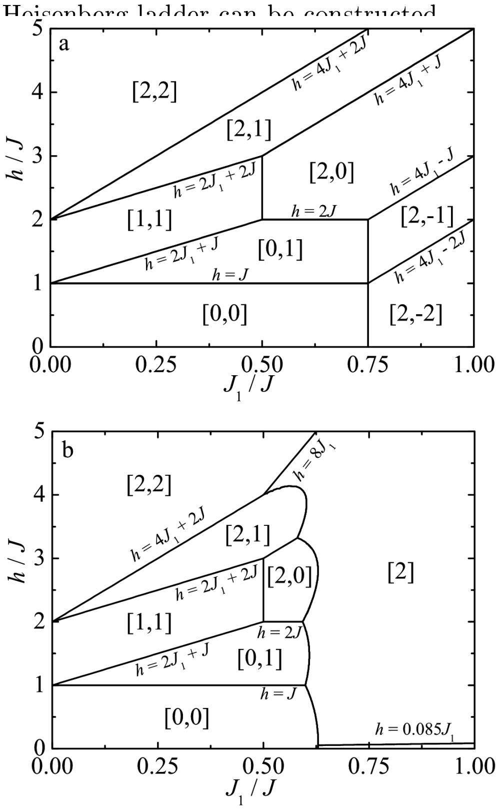

Fig. 1. Ground-state phase diagrams of the frustrated spin-1 ladder described within: (a) the Ising-Heisenberg model; (b) the pure Heisenberg model. For details see the text.

4. Results and discussion

The constructed ground-state phase diagrams of the frustrated spin-1 Ising-Heisenberg and Heisenberg ladders are depicted in Fig. 1a and Fig. 1b, respectively. The ground states of the spin-1 Ising-Heisenberg ladder can be discerned according to the z projection of the total spin on two consecutive rungs $[T_i^z, T_{i+1}^z]$, because $T_i = |T_i^z|$ holds for all available ground states. The quantum ground states [0,0], [0,1], [1,1], [2,±1] and [2,0] represent six different phases, whereas $T_i = T_i^z = 0$ implies a formation of two singlets on the ith rung, $T_i = |T_i^z| = 1$ entails only one singlet and $T_i = |T_i^z| = 2$ denotes fully polarized rungs without singlets. Besides, the two ground states [2,2] and [2,–2] are pertinent to the classical ferromagnetic and antiferromagnetic ordering of fully polarized rungs. The magnetization normalized with respect to its saturation equals zero for [0,0] and [2,–2], one-quarter for [0,1] and [2,-1], one-half for [1,1] and [2,0], three-quarters for [2,1] and unity for [2,2]. Altogether, it can be concluded that the frustrated spin-1 Ising-Heisenberg ladder always exhibits a stepwise magnetization curve, which involves intermediate plateaux at one-quarter, one-half and three-quarters of the saturation magnetization that are however of different origin.

It is quite clear from Fig. 1b that the ground-state phase diagram of the pure quantum Heisenberg ladder exactly coincides with that of the Ising-Heisenberg ladder just for sufficiently weak inter-rung interactions $J_1/J \leq 0.5$. A relatively good agreement between both phase diagrams is still observed in the parameter space $0.5 \leq J_1/J \leq 0.63$, where the gapless phase [2] with a continuously varying magnetization is present between the intermediate plateaux instead of direct magnetization jumps. The gapless phase [2] corresponds to the Luttinger-liquid phase of the effective spin-2 quantum Heisenberg chain. Finally, the gapped Haldane phase of the effective spin-2 quantum Heisenberg chain emerges for $J_1/J \geq 0.63$ at sufficiently low fields.

In conclusion, we have rigorously found the ground states of the frustrated spin-1 Ising-Heisenberg and Heisenberg ladders in a magnetic field. It has been verified that the Ising-Heisenberg ladder always exhibits a stepwise magnetization curve with three different intermediate plateaus, while the same quantum ground states can be identified in the pure quantum Heisenberg ladder provided the intra-rung coupling is sufficiently large.

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