Alpha Head on Collision with a Fixed Gold Nucleus, Taking into Account the Relativistic Rest Mass Variation as Implied by Mass-Energy Equivalence

T. YARMAN, M. ARIK, A. Kholmetskii, A.A. ALTINTAS and F. OZAYDIN

Abstract: We reformulate the Rutherford scattering of alpha particle for a head on collision, taking into account the rest mass variation of the particle, as implied by the energy conservation law. Our relativistic reformulation (which includes the energy conservation) constitutes a new example for the breakdown of the Lorentz invariance. Briefly speaking, even at rest or during the whole scattering process, the distance between the alpha particle and the gold nucleus is not invariant but depends on the frame of the observer attached to either object. According to our relativistic reformulation, we also provide a new set of Lorentz transformations.

DOI: 10.12693/APhysPolA.125.618
PACS: 21.60.-n, 24.10.Jv

1. Introduction

Suppose we have an alpha particle of charge $Ze$ (where $z$ is 2, and $e$ is the fundamental charge), and of rest mass $m_{0\infty}$, at infinity. It moves with an instantaneous velocity $v_0$ at a distance $r_0$ from the gold nucleus of charge $Ze$, at the given time, as measured by an observer attached to the alpha particle. As a first approach, we suppose that the alpha particle heads right on $Ze$, just like one could have observed, as in the Rutherford’s nucleus discovery experiment [1]. Below, we will reformulate the head on collision, taking into account the rest mass variation of the alpha, throughout, as implied by the mass and energy equivalence of the special theory of relativity (STR). Thus, we first express the rest energy of alpha at the given distance from the gold nucleus, and the total relativistic energy of it, given that it is in motion, except at the moment it is stopped by the gold nucleus. Then we deal with the quest of how the internal dynamics of alpha and its size is affected through the process.

2. Rest and total relativistic energies of alpha

According to our approach the rest energy of the alpha particle is

$$m_0(r_0)c^2 = m_{0\infty}c^2 \left( 1 + \frac{Ze^2}{r_0 m_{0\infty}c^2} \right).$$  \hspace{1cm} (1)

In the above formula the observer is attached to alpha particle and measures the distance between alpha and gold nucleus as $r_0$. In our approach for an observer which is located on the gold nucleus the distance between gold nucleus and alpha particle is measured as $r$ which is different from $r_0$.

Now the total relativistic energy of alpha particle is written by applying Lorentz mass increase to Eq. (1)

$$m_1c^2 = m_{0\infty}c^2 \left( 1 + \frac{Ze^2}{r_0 m_{0\infty}c^2} \right) = m_{0\infty}c^2 \sqrt{1 - \frac{v_0^2}{c^2}}$$

where $v_0$ is the initial velocity of the alpha particle, far away from the gold nucleus.

Since the system is closed, the total energy is constant; this means that at the distance $R_0$, too (the alpha particle is stopped), Eq. (2) remains constant

$$m_1c^2 = m_{0\infty}c^2 \left( 1 + \frac{Ze^2}{R_0 m_{0\infty}c^2} \right) = \text{const.}$$  \hspace{1cm} (3)

Let us for simplicity pose

$$\chi(r_0) = \left( 1 + \frac{Ze^2}{r_0 m_{0\infty}c^2} \right), \quad \text{and} \quad \gamma(v_0) = \sqrt{1 - \frac{v_0^2}{c^2}}.$$  \hspace{1cm} (4)

In the head on collision the alpha particle is stopped by gold nucleus at $R_0$ and then fired back, after which Eq. (3) still remains valid. Via Eq. (3) one can write the equation of motion.

2.1. Equation of motion

Thus, by differentiating Eq. (3) one can get

$$\frac{Ze^2}{r_0^2} \sqrt{1 - \frac{v_0^2}{c^2}} = m_{0\infty} \left( 1 + \frac{Ze^2}{r_0 m_{0\infty}c^2} \right) \frac{dv_0}{dx} \sqrt{1 - \frac{v_0^2}{c^2}}.$$  \hspace{1cm} (5)

Using Eq. (2), we have

$$\frac{Ze^2}{r_0^2} \sqrt{1 - \frac{v_0^2}{c^2}} = m_1 \frac{dv_0}{dx}.$$  \hspace{1cm} (6)

In our approach, unlike the classical approach, the measured distances depend on observers [2, 3] attached to the gold nucleus, or the alpha particle. In such a framework, for the outside observer the measured distance is...
written as

\[ r = \frac{r_0}{1 + \frac{Zze^2}{r_0m_0c^2}}. \tag{7} \]

The velocities in both frames are the same, thus periods of time change as much as the distances are altered [4]. With the help of Eq. (7), Eq. (6) becomes

\[ \frac{Zze^2}{r^2 \left(1 + \frac{Zze^2}{r_0m_0c^2}\right)} \sqrt{1 - \frac{v_0^2}{c^2}} = m_0 \frac{dv}{dt} \left(1 + \frac{Zze^2}{r_0m_0c^2}\right). \tag{8} \]

Using Eq. (2), the equation of motion in the outside observer’s reference frame will be

\[ \frac{Zze^2}{r^2 \left(1 + \frac{Zze^2}{r_0m_0c^2}\right)} = m_0 \frac{dv}{dt}. \tag{9} \]

It is easy to see that equation of motions depends on the frame of observer. Equation (6) is the equation of motion written in the frame of alpha particle and Eq. (9) is equation of motion written in the frame of outside observer.

2.2. Differences in Lagrangians

Since the equation of motion varies according to observer’s frame, a natural quest arises on, what happens to Lagrangian? The definition of Lagrangian is server’s frame, a natural quest arises on, what happens to server. Is equation of motion written in the frame of outside observer. Equation (6) is the equation of motion written in the frame of alpha particle and Eq. (9) is equation of motion written in the frame of outside observer.

\[ U = \frac{1}{2} m_0 v^2 - \frac{Ze^2}{r} - \frac{Ze^2}{r_0m_0c^2}. \]

The original velocity \( v_0 \) of alpha in the Rutherford experiment is about \( 2 \times 10^7 \) m/s.

The framework we draw, anyway constitutes an example for the breakdown of the “Lorentz invariance”. Finally we have provided a modified set of Lorentz transformations, involving not only the motion but also the interaction between particles of concern.

Acknowledgments

This work was supported by the Scientific Research Projects Coordination Unit of Istanbul University, project number: BAP-5623.

References