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Effectiveness Analysis of the Beam Modes Active Vibration Protection with Different Number of Actuators

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This paper deals with an active vibration protection (p-reduction) of the beam-actuators mechanical system, hence it concerns separate modes. The paper's aim is an effectiveness analysis of the p-reduction assuming different number of actuators. It is assumed a priori that actuators are bonded to the beam in the sub-domain with the largest curvatures and they are exactly the same. The beam clamped at one end is chosen as the research object. Next, as required by the p-reduction condition, the number and distribution of actuators are changed. It turns out that the best reduction effectiveness, measured via any effectiveness coefficient, is obtained for one actuator bonded in the sub-domain with the largest curvature. The validation of theoretical considerations is confirmed numerically.

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1. Introduction

The p-reduction is a particular case of active vibration reduction (a-reduction) [1–3]. Both a- and p-reduction are realized with actuators. Instead of the active vibration reduction, there is the passive one [4, 5]. The quantity of the reduction depends on many factors. One of them, and at the same time the more important, is appropriate distribution of actuators on the structure [5–7]. A question about an optimal distribution of actuators is justified. Up to now, a great number of papers has been published on this subject and they pointed out a lot of optimization techniques, e.g. [8]; a survey is given in [9]. However, these techniques do not provide explicit distribution of the actuators and consistently, do not assure maximum effectiveness of the reduction. However, this problem is solved in [1, 10–12]. It was proved there, that the most effective actuators distribution is on the structure sub-domains with the largest curvatures; such distribution is called quasi-optimal one (QO). In the quoted papers, the QO-distribution is deduced based on the heuristic reasons and it was confirmed theoretically in [13]. So hereafter such distribution may be regarded as optimal one (O-distribution) and it is proved that the maximum effectiveness of the p-reduction of the beam separate modes may be achieved for O-distribution. Sub-domains with such curvatures are called as O-subdomains.

The beam clamped at end and free at the other one is chosen as the research object. It is excited with evenly spread and harmonic force. The force acts with first three natural frequencies separately. The internal damping coefficient of the material is introduced. All actuators are identical from the geometrical and technical point of view, so they interact on the beam with the same forces. The force values depend on the number and distribution of actuators on the beam. Applying the p-reduction condition, force values are derived. Assuming that the number of actuators is the same as the number of O-subdomains, the values of forces are found theoretically in [13]. It turned out that the values of forces were minimal, so this means that the minimum energy is added to the system and consequently, the maximum effectiveness of the p-reduction is assured. An effectiveness measure of the reduction is an effectiveness coefficient defined in [1, 2, 4, 7, 9–15].

Useful, from the point of view application to the active reduction of general beam vibration, is an effectiveness analysis of p-reduction for different number of actuators placed in O-subdomains. It is proved in [1] that even one actuator bonded anywhere on the beam provides p-reduction of separate modes but the effectiveness may be poor.

The aim of the paper is an effectiveness analysis of p-reduction for different combinations of actuators distribution in O-subdomains. It is possible for the second and third mode, since they have more then one O-subdomain. The first mode is taken into account for comparison. To the author's knowledge, such problem has not been considered yet.

2. Forced vibration of the beam with damping

This theory is repeated after papers [13, 15, 16]. Let the beam be clamped at one side, Fig. 1; and geometrical data of the beam are: ℓ – length; $S = bh$ – surface of the rectangular cross-section; b – width; h – thickness; $q_E = q_E(x,t)$ – excited force. The beam vibration equation is given by

$$EJ\left(D_x^4 u + \mu D_x^4\left(D_t u\right)\right) + \rho S D_t^2 u = -q_E, \qquad (1)$$

where $u = u(x,t)$ is the beam deflection at the point x and the moment t, E – the Young moduli, J – sur-

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face moment of inertia of the beam cross-section, ρ – mass density, μ – internal damping factor, $D_x^4(\ldots) = \partial^4(\ldots)/\partial x^4$, $D_t(\ldots) = \partial(\ldots)/\partial t$.

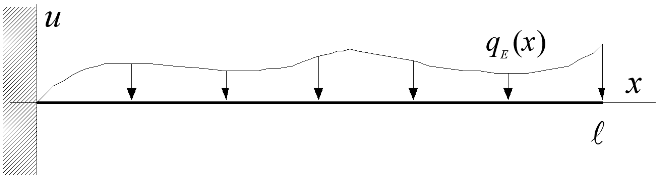

Fig. 1. The geometry of the problem.

The boundary conditions are described by the following equations:

$$u(x=0,t)=0, \quad D_x u(x=0,t)=0, \tag{2}$$

$$D_x^2 u(x=\ell,t)=0, \quad D_x^3 u(x=\ell,t)=0. \tag{3}$$

Besides, initial conditions are assumed to be equal to zero. The solution of the formulated problem is forced harmonic vibrations with damping. Let the lateral load force

$$q_E(x,t) = q_E(x)\exp(\mathrm{i}\omega_q t), \tag{4}$$

where $\mathrm{i}^2 = -1$, and ω_q is the angular frequency.

Applying the Fourier method, the solution of Eq. (1) is assumed as

$$u(x,t) = X(x)\exp(i\omega_q t). \tag{5}$$

After some calculation, the solution of the above problem is

$$X(x) = X_q(x) = \sum_\nu C_\nu X_\nu(x) = \sum_\nu X_{q;\nu}(x), \quad \nu = 1,2,\ldots,n, \tag{6}$$

where C_ν are certain constants, $X_q(x)$ – forced vibrations, $X_\nu(x)$ – ν-modes (eigenfunctions), and

$$X_\nu(x) = K_2(\lambda_\nu\ell)K_2(\lambda_\nu x) - K_1(\lambda_\nu\ell)_1 K_3(\lambda_\nu x), \tag{7}$$

where the Krylov functions have the form

$$\begin{aligned} K_1(z) &= (\mathrm{ch}(z)+\cos(z))/2,\\ K_2(z) &= (\mathrm{sh}(z)-\sin(z))/2,\\ K_3(z) &= (\mathrm{ch}(z)-\cos(z))/2,\\ K_4(z) &= (\mathrm{sh}(z)+\sin(z))/2, \end{aligned} \tag{8}$$

$\{\lambda_\nu\}$ – set of eigenvalues: $\{\lambda_\nu\ell\} = \{1.8751, 4.6941, \ldots, (2\nu-1)\pi/2\}$.

The constants C_ν are expressed by

$$C_\nu = \frac{1}{(1+i\mu\omega_f)\omega_\nu^2 - \omega_q^2} D_\nu = \frac{1}{\alpha_\nu^2} D_\nu = \frac{1}{\rho S}\frac{1}{\alpha_\nu^2}\frac{1}{\beta_\nu^2} I_{\nu;E} = C_\nu^* I_{\nu;E}, \tag{9}$$

$$C_\nu^* = \frac{1}{\rho S}\frac{1}{\alpha_\nu^2}\frac{1}{\beta_\nu^2}, \quad I_{\nu;E} = -\int_0^\ell q_E X_\nu(x)\,\mathrm{d}x,$$

$$\omega_\nu^2 = \frac{EJ}{\rho S}\lambda_\nu^4, \quad \beta_\nu^2 = \int_0^\ell X_\nu^2(x)\,\mathrm{d}x. \tag{10}$$

Thus, the problem of the beam vibration with damping, excited with the force $q_E(x,t)$, is solved. The first three modes, Eq. (7), are depicted in Fig. 2. Henceforth, the spread load force with constant amplitude q_E is considered, i.e. $q_E(x) = q_E$.

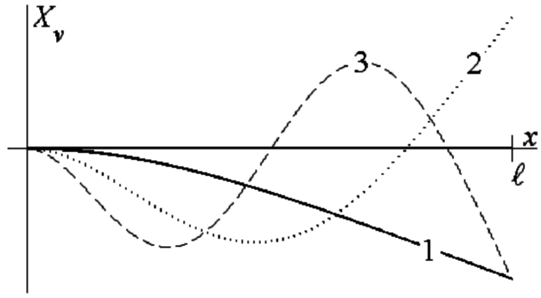

Fig. 2. Eigenfunctions (modes): 1 – X_1, 2 – X_2, 3 – X_3.

3. Beam vibration reduction by actuators

It is well known from [13, 15, 17] and references cited therein, that actuators-beam interact with moments of the couples of forces approximately. Since the beam vibration equation is the equation of forces, then to consider the action of actuators on the beam, two moments are replaced with two couples of forces, Fig. 3. Next, the separate forces are taken into account in the Eq. (1). Hence, the total load is the sum of the load forces expressed by Eq. (4) and the forces interacting between actuators and the beam, and it is given by

$$f(x) = -q_E + \big(f_a\delta(x-x_{1a}) - 2f_a\delta(x-x_a) + f_a\delta(x+x_{2a})\big), \tag{11}$$

where $x_{1a} = x_a - \ell_a/2$; $x_{2a} = x_a + \ell_a/2$; x_a is the location of the actuator centre; expression in the bracket is the sum of interacting forces actuators-beam; and $\delta(.)$ is the Dirac delta function. In this case, instead of $I_{\nu;E}$ in Eq. (10), for $f(x)$ given by Eq. (11) one has

$$I_\nu = -\int_0^\ell f(x)X_\nu(x)\,\mathrm{d}x = -q_E\int_0^\ell X_\nu(x)\,\mathrm{d}x + f_a\left[X_\nu(x_{1a}) - 2X_\nu(x_a) + X_\nu(x_{2a})\right] = -I_{q;\nu} + I_{a;\nu}. \tag{12}$$

The expression in square bracket constitutes the second-order central finite difference. Since the distance between nodes ℓ_a is constant, then the difference can be transformed into

$$\frac{1}{\ell_a^2}\left[X_\nu(x_{1a}) - 2X_\nu(x_a) + X_\nu(x_{2a})\right] = D^2X_\nu(x_a) = \kappa_\nu(x_a), \tag{13}$$

where $\kappa_\nu(x_a)$ is the curvature of the mode $X_\nu(x)$ at the point $x = x_a$ [1, 2, 4]. Substituting Eq. (13) into Eq. (12) one obtains

$$I_\nu = -q_E\int_0^\ell X_\nu(x)\,\mathrm{d}x + f_a\ell_a^2\kappa_\nu(x_a) = -I_{q;\nu} + I_{a;\nu}. \tag{14}$$

For several actuators, instead of Eq. (14),

$$I_\nu = -I_{q;\nu} + \sum_a f_a \ell_a^2 \kappa_\nu(x_a) = -I_{q;\nu} + \sum_a I_{a;\nu} = -I_{q;\nu} + I_{\Sigma;\nu}, \tag{15}$$

where $a = 1, 2, \ldots, n_a$ – number of actuators.

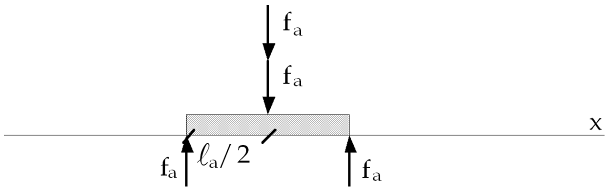

Fig. 3. The idea of interaction of the PZT-beam via couples of forces.

Substituting Eq. (10) into Eq. (6) via Eq. (15), the reduction vibration is obtained:

$$X_f(x) = \sum_\nu C_\nu^* I_\nu X_\nu(x) = \sum_\nu C_\nu^* (-I_{q;\nu} + I_{\Sigma;\nu}) X_\nu(x) = \sum_\nu A_{f;\nu} X_\nu(x), \tag{16}$$

where, in explicit form,

$$A_{f;\nu} = C_\nu^* (-I_{q;\nu} + I_{\Sigma;\nu}) = C_\nu^* \left(-I_{q;\nu} + \sum_a f_a \ell_a^2 \kappa_\nu(x_a) \right). \tag{17}$$

As it was pointed out in [1, 13, 15], reduction of $A_{f;\nu}$ leads to reduction of the curvature $\kappa_f(x)$, the shear force $Q(x)$, and the bending moment $M(x)$; hereafter all these quantities are described jointly as

$$\Psi(x) = \sum_\nu \Psi_\nu(x) = C \sum_\nu A_{f;\nu} \Phi_\nu(x), \tag{18}$$

where

$$\Psi(x) = \{u(x), Q(x), M(x)\},$$

$$\Phi_\nu(x) = \{X_\nu(x), \kappa_\nu(x), D\kappa_\nu(x),\}$$

$$C = \{1, \pm EJ, \pm EJ\}. \tag{19}$$

4. Reduction effectiveness coefficients

Let the difference between any quantities of the beam vibration [15]

$$\Delta \Psi(x) = \Psi_q(x) - \Psi_R(x) = \Psi_\Sigma(x), \tag{20}$$

where $\Psi_q(x)$, $\Psi_R(x)$ are quantities calculated without and with actuators, respectively. Hence

$$\Psi_R(x) = \sum_\nu \Psi_\nu(x) = C \sum_\nu A_{R;\nu} \Phi_\nu(x) = \Psi_q(x) - \Psi_\Sigma(x), \tag{21}$$

where

$$A_{R;\nu} = -A_{q;\nu} + A_{\Sigma;\nu} = C_\nu^* I_\nu = C_\nu^* (-I_{q;\nu} + I_{\Sigma;\nu}). \tag{22}$$

It is stressed that the amplitude $A_{q;\nu}$ arises from q_E acting alone, whereas $A_{\Sigma;\nu}$ is a result of acting of actuators only; this is exactly the amplitude called the reduction amplitude.

The difference $\Delta \Psi(x)$ is interpreted as the quantity of the vibration reduction and it is the first measure of this reduction called the quantity reduction coefficient. The second measure of the vibration reduction is defined as

$$R_\psi(x) = \frac{\Delta \Psi(x)}{\Psi_q(x)} = \frac{\Psi_q(x) - \Psi_R(x)}{\Psi_q(x)}. \tag{23}$$

It is called the reduction coefficient. The effectiveness of the vibration reduction is defined as a quotient of some vibration reduction measure by an amount of the energy W provided to the system in order to excite actuators. Hence, first measure of the vibration reduction may be defined by the so-called effectiveness coefficient

$$E_\Psi(x) = R_\Psi(x)/W. \tag{24}$$

As mentioned above, the energy W provided to the system is transformed into couples of forces. Therefore, the energy W may be replaced by sum of forces $f_\Sigma = 4\sum_a f_a$, hence

$$E_\Psi(x) = R_\Psi(x)/f_\Sigma. \tag{25}$$

Eqs. (20)–(25) define the appropriate factors of the vibration reduction at the point x. In many cases it is convenient to calculate mean values of these coefficients at the whole beam domain or at the beam sub-domains. For this purpose, the mean reduction coefficients are defined; more details are given in references cited above. Other measures are presented in [14, 18, 19].

5. The p-reduction condition

The condition of the p-reduction may be expressed in different form, namely: $A_{\Sigma;\nu} = A_{q;\nu}$, $\Psi_R(x) = 0$, $\Delta \Psi(x) = \Psi_q(x)$, or $R_\Psi(x) = 1$. It leads to the maximum of effectiveness coefficient, i.e. $E_\Psi(x) = 1/f_\Sigma = \max$. The last condition is met, if the sum of forces f_Σ attains its minimum. Under circumstances given above, it comes down to the determination of n_a and x_a. The problem of the distribution of actuators, i.e. $\{x_a\}$, is solved, even analytically, in [13]. It is pointed out there, that the set $\{x_a\}$ constitutes both points $\{x'_a\}$ in which the curvature $\kappa_\nu(x)$ achieves its extreme and the points $\{x_{\max}, x_{\min}\}$ in which $\kappa_\nu(x)$ attains the biggest and the lowest values. But the influence of the number of actuators distributed in O-subdomains on the effectiveness of the p-reduction is not solved so far. This problem is solved below in numerical way.

6. Numerical calculations

The aim of numerical tests is an analysis of the p-reduction via the effectiveness coefficient $E_\Psi(x)$. The different number of actuators is assumed. Since the actuators must be bonded only in O-subdomains, then the goal concerns only the second mode and the third one.

In numerical calculations the following data are assumed: $\ell = 0.5$ m, $b = 0.04$ m, $h = 0.005$ m, $J =$

$(bh^3)/12$ m^4, $E = 69 \cdot 10^9$ Pa, $\mu = 3.35 \cdot 10^{-4}$ s, $\rho_b = 2.7 \cdot 10^3$ kg/m^3, $q_E = 0.02$ N/m. The size of all actuators is the same, i.e. $\ell_0 = \ell_1 = \ell_2 = \ell_3 = 0.08\ell$. Furthermore, all actuators are excited by the same signal, hence $\{f_a\} = \{f_0\}$. The first actuators have to be moved away from clamped side within necessary distance $\ell_0/2$.

First of all, the curvatures of separate modes are calculated. The first mode is considered for comparison; the curvature $\kappa_1(x)$ is depicted in Fig. 4. This mode has only one O-subdomain outlined by $\{x_a\} = x_1 = \ell_0/2$. The value of the force, calculated based on Eq.(22), is $f_0 = 0.7365$. The effectiveness coefficient, Eq. (25), is $E_\Psi = 1/f_\Sigma = 1/(4 \cdot f_0) = 0.339$.

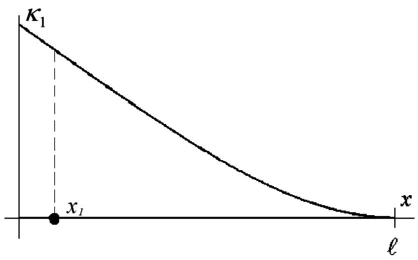

Fig. 4. Curvature $\kappa_1(x)$.

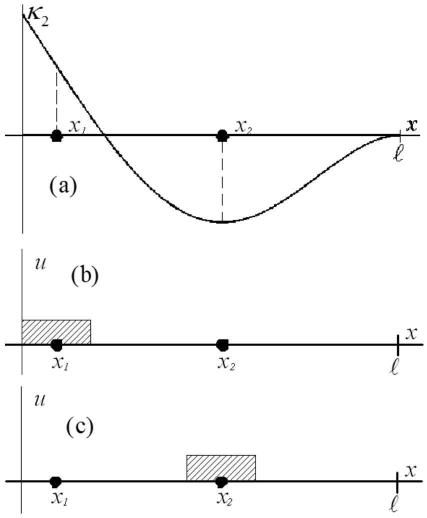

Fig. 5. (a) – curvature $\kappa_2(x)$, (b), (c) – distribution of separate actuators.

At the second mode case, there are two O-subdomains $\{x_a\} = \{x_1, x_2\} = \{\ell_0/2, 0.235\}$, see the curvature $\kappa_2(x)$ in Fig. 5(a). First, at these O-subdomains two actuators are placed. So they act on the beam by the same forces $f_0 = 0.041$ and the effectiveness coefficient is $E_\Psi = 1/f_\Sigma = 1/(2 \cdot 4 \cdot f_0) = 3.048$. But it is proved in [1] that p-reduction may be achieved by only one actuator. Having this conclusion in mind, first one actuator is placed in O-subdomain $\{x_a\} = x_1 = \ell_0/2$,

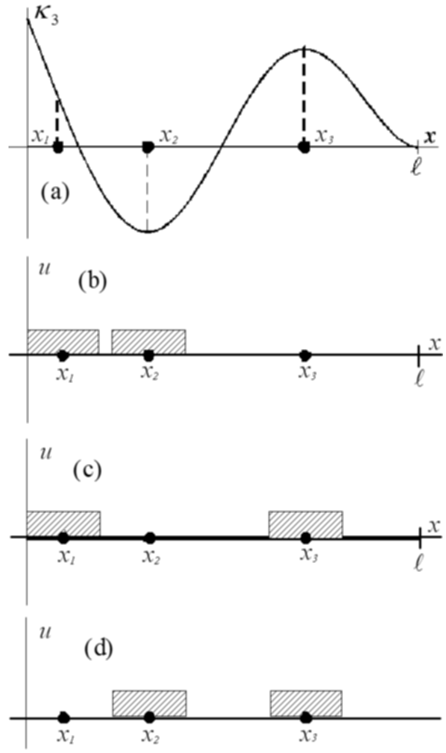

Fig. 6. (a) – curvature $\kappa_3(x)$, (b), (c), (d) – combinations of actuators pairs.

Fig. 5(b), and $f_0 = 0.0761$ and $E_\Psi = 3.285$ are obtained. Next, the actuator is placed in the second O-subdomain $\{x_a\} = x_2 = 0.235$, Fig. 5(c), and in this case $f_0 = 0.0887$ and $E_\Psi = 2.8185$.

At the third mode case, the procedure is quite the same as that given above; the curvature $\kappa_3(x)$ is given in Fig. 6(a). First of all, there are three O-subdomains $\{x_a\} = \{x_1, x_2, x_3\} = \{\ell_0/2, 0.1456, 0.3462\}$. In this case, all (three) actuators, each combination of two actuators, and separate one are taken into account. These combinations are described by $\{x_1, x_2\}$,$\{x_1, x_3\}$, $\{x_2, x_3\}$ and they are presented in Fig. 6(b)–(d). The separate actuators are distributed the same way as for the second mode.

All quantitative results are collected in Table.

TABLE

Effectiveness of p-reduction for different number of actuators.

ν	$\{x_a\}$	f_0	$\kappa_\nu(x_a)$	E_Ψ
1	$\ell_0/2$	0.7365	109.99	0.339
2	$\{\ell_0/2, 0.235\}$	0.041	$\{7.648, -6.560\} \times 10^3$	3.048
	$\ell_0/2$	0.0761	7.648×10^3	3.285
	0.235	0.0887	-6.560×10^3	2.835
3	$\{\ell_0/2, 0.1456, 0.3462\}$	0.0062	$\{4.3706, -4.1545, 4.7901\} \times 10^5$	13.4409
	$\{\ell_0/2, 0.1456\}$	0.0096	$\{4.3706, -4.1545\} \times 10^5$	13.0208
	$\{\ell_0/2, 0.3462\}$	0.0090	$\{4.3706, 4.7901\} \times 10^5$	13.8889
	$\{0.1456, 0.3462\}$	0.0092	$\{-4.1545, 4.7901\} \times 10^5$	13.5870
	$\ell_0/2$	0.0188	4.3706×10^5	13.2979
	0.1456	0.0198	-4.1545×10^5	12.6263
	0.3462	0.0171	4.7901×10^5	14.6199

7. Conclusions

The conclusions enumerated below are derived assuming that all actuators are the same and they are bonded at the O-subdomains. The conclusions are concern of separate modes of the beam vibration. Based on theoretical and numerical considerations, the following conclusions may be formulated:

1. If the domain has more than one O-subdomain, one actuator placed in O-subdomain with the maximal curvature assures maximum effectiveness of the p-reduction.

2. For many O-subdomains, the best effectiveness of the p-reduction is obtained if consecutive actuators are placed in O-subdomains with curvature decreasing one after the other.

Despite of conclusions are drawn for separate modes only and the p-reduction, it seems that they are correct for general vibrations of the beam and the a-reduction. The studies on these problems are carried on.

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