On the Buckling of the Layered Cylindrical Shell with FGM Face Sheets Subjected to the Axial Load*

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In this study, the buckling analysis of layered cylindrical shells with functionally graded material face sheets subjected to an axial compressive load is investigated. The dimensionless axial buckling load of layered cylindrical shells with functionally graded material face sheets is obtained. Effects of volume fractions of functionally graded material face sheets and cylindrical shell characteristics on the dimensionless axial buckling load have been studied.

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1. Introduction

Layered composite shells are widely used as structural elements in modern construction engineering, aircraft construction, ship building, rocket construction, and the nuclear, aerospace and aeronautical industries. As the layered structure offers advantages over other types of structures, it is important to develop new types of materials in order to obtain the absolute minimum weight for given conditions. These new three-layered structures should be compared with other layered construction and with alternative structures in order to select the best configuration. The considerable advantages offered by functionally graded materials (FGMs) over conventional materials have prompted an increased use of layered structures, and incorporation in their construction of FGMs as face sheets. The concept of FGMs was first introduced in 1984 [1]. The buckling behavior of FGM cylindrical shells has been addressed by many investigators [2, 3]. Some studies have been reported on the buckling of layered shells containing an FGM layer [4] and sandwich plates with FGM face sheets [5]. In this study, the buckling behavior of cylindrical shells with FGM face sheets under the axial load is investigated.

2. Formulation of the problem

This study examines three-layered shell consisting of two thin FGM face sheets and a relatively thick middle layer composed of a metal. The simply-supported three-layered cylindrical shell under axial compressive load and in plane geometry of the layered structure are shown in Fig. 1a,b. The coordinate system has its origin at the end of the cylindrical shell on the reference surface, where x and y are the longitudinal and circumferential direction, z is perpendicular to the cylindrical shell. The length, radius and total thickness of the three-layered cylindrical shell are L, R, and h, respectively. The thickness of each FGM face sheet is h_{F}, while the thickness of the middle metal layer is h_{m}, as shown in Fig. 1c.

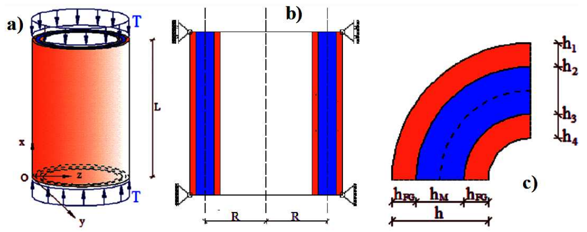

Fig. 1. Geometry of the cylindrical shell with FGM face sheets.

The FGM face sheets are made from a mixture of ceramics and metals, the mixing ratio of which is varied continuously and smoothly in z direction. This is achieved by using a simple rule of mixture of composite materials. The effective Young modulus, E_{Fg}, and Poisson's ratio, ν_{Fg}, can be expressed as [2, 5]:

$$E_{\mathrm{Fg}} = E_{\mathrm{s}} + (E_{\mathrm{m}} - E_{\mathrm{s}})V_{\mathrm{m}},$$
$$\nu_{\mathrm{Fg}} = \nu_{\mathrm{s}} + (\nu_{\mathrm{m}} - \nu_{\mathrm{s}})V_{\mathrm{m}}, \tag{1}$$

where $E_{\mathrm{m}}, \nu_{\mathrm{m}}$ and $E_{\mathrm{c}}, \nu_{\mathrm{c}}$ are the Young modulus and Poisson ratio of the metal and ceramic surfaces of the FGM face sheets, respectively. We assume that the volume fraction, V_{m}, for the top and bottom face sheets follows as [5]:

$$V_{\mathrm{m}}^{\mathrm{top}} = \left(\frac{t - t_1}{t_3 - t_2}\right)^d \quad \text{and} \quad V_{\mathrm{m}}^{\mathrm{bot}} = \left(\frac{t_3 - t}{t_4 - t_3}\right)^d, \tag{2}$$

where $t = z/h$, $t_1 = h_1/h$, $t_2 = h_2/h$, $t_3 = h_3/h$, $t_4 = h_4/h$ and the volume fraction index d $(0 \le d < +\infty)$ dictates the material variation profile through the FGM

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layers thickness. The variation of Young's modulus and Poisson's ratio of three-layer system with FGM face sheets are given as [4]:

$$E(t),\nu(t)=\begin{cases} E_{\mathrm{Fg}},\nu_{\mathrm{Fg}}, & -t_1\le t\le -t_2 \\ & \text{and}\quad t_3\le t\le t_4, \\ E_{\mathrm{m}},\nu_{\mathrm{m}}, & -t_2\le t\le t_3. \end{cases} \tag{3}$$

The stability and compatibility equations of the cylindrical shell with FGM face sheets can be obtained as

$$L_{11}\Phi+L_{12}w=0,$$
$$L_{21}\Phi+L_{22}w=0, \tag{4}$$

where Φ is the Airy stress function, w is displacement of the reference surface in the normal direction and the following definitions apply:

$$L_{11}=A_2\left(\frac{\partial^4}{\partial x^4}+\frac{\partial^4}{\partial y^4}\right)+2\left(A_1-A_5\right)\frac{\partial^4}{\partial x^2\partial y^2}+\frac{1}{R}\frac{\partial^2}{\partial x^2},$$

$$L_{12}=-A_3\left(\frac{\partial^4}{\partial x^4}+\frac{\partial^4}{\partial y^4}\right)-2\left(A_4+A_6\right)\frac{\partial^4}{\partial x^2\partial y^2}-T\frac{\partial^2}{\partial x^2},$$

$$L_{21}=B_1\left(\frac{\partial^4}{\partial x^4}+\frac{\partial^4}{\partial y^4}\right)+2\left(B_2+B_5\right)\frac{\partial^4}{\partial x^2\partial y^2},$$

$$L_{22}=\frac{1}{R}\frac{\partial^4}{\partial x^2}-B_4\left(\frac{\partial^4}{\partial x^4}+\frac{\partial^4}{\partial y^4}\right)-2\left(B_3+B_6\right)\frac{\partial^4}{\partial x^2\partial y^2}, \tag{5}$$

in which A_j, B_j $(j=1,2\ldots,6)$ are FGM properties and shell characteristics [3].

3. Solution of basic equations

Consider a cylindrical shell is simply-supported boundary condition. The solution of equations set (4) is sought as follows [3]:

$$w=A\sin\frac{m_1x}{R}\sin\frac{ny}{R},\qquad \Phi=B\sin\frac{m_1x}{R}\sin\frac{ny}{R}, \tag{6}$$

where A and B are the amplitudes, $m_1=m\pi R/L$, m is the half wavelength in the direction of x axis and n is the wave number in the direction of y axis. Substituting expressions (6) in the equations set (4), then applying the Galerkin method, for the dimensionless axial buckling load, the following equation is obtained:

$$T_{1buc}^{ax}=\frac{A_3\left(m_1^4+n^4\right)+2\left(A_4+A_6\right)m_1^2n^2}{m_1^2R^2E_{\mathrm{m}}h} +\left\{\left[m_1^2R-A_2\left(m_1^4+n^4\right)-2\left(A_1-A_5\right)m_1^2n^2\right]\times\left[m_1^2R+B_4\left(m_1^4+n^4\right)+2\left(B_3+B_6\right)m_1^2n^2\right]\right\} \Big/\left\{\left[B_1\left(m_1^4+n^4\right)+2\left(B_2+B_5\right)m_1^2n^2\right]m_1^2R^2E_{\mathrm{m}}h\right\}. \tag{7}$$

The minimum values of the dimensionless axial buckling load are obtained by minimizing Eq. (7) with respect to (m,n).

TABLE

The variation of $T_{1buc}^{ax}\times 10^3$ (n_{cr}) for the fully metal, fully FGM, FG/S/FG and FG/T/FG shells versus $h_{\mathrm{m}}/h_{\mathrm{Fg}}$, R/h $(L/R=2)$.

$h_{\mathrm{m}}/h_{\mathrm{Fg}}$ $(R/h=100)$	SUS304	FGSFG $d=0.5$	FGSFG $d=1$	FGSFG $d=2$	FGSFG $d=15$
fully FG	3.926(11, 4)	4.531(11, 4)	4.797(11, 4)	5.071(11, 4)	5.701(11, 4)
2		4.455(11, 3)	4.697(11, 3)	4.920(11, 3)	5.229(11, 3)
6		4.220(11, 3)	4.360(11, 3)	4.494(11, 3)	4.695(11, 3)
8		4.167(11, 3)	4.282(11, 3)	4.393(11, 3)	4.563(11, 3)
$h_{\mathrm{m}}/h_{\mathrm{Fg}}$ $(R/h=100)$	Ti-6Al-4V	FGTFG $d=0.5$	FGTFG $d=1$	FGTFG $d=2$	FGTFG $d=15$
fully FG	1.984(11, 4)	3.217(11, 4)	3.746(11, 5)	4.291(11, 5)	5.498(11, 5)
2		3.021(11, 1)	3.488(10, 4)	3.921(10, 4)	4.527(10, 4)
6		2.563(11, 2)	2.836(11, 2)	3.091(10, 4)	3.477(10, 4)
8		2.458(11, 2)	2.683(11, 2)	2.897(10, 5)	3.221(10, 4)
R/h $(h_{\mathrm{m}}/h_{\mathrm{Fg}}=6)$	SUS304	FGSFG $d=0.5$	FGSFG $d=1$	FGSFG $d=2$	FGSFG $d=15$
100	3.926(11, 4)	4.220(11, 3)	4.360(11, 3)	4.494(11, 3)	4.695(11, 3)
300	1.309(19, 7)	1.407(19, 6)	1.453(19, 6)	1.498(19, 5)	1.565(19, 5)
500	0.785(19, 18)	0.844(19, 17)	0.872(19, 17)	0.899(19, 17)	0.940(19, 16)
R/h $(h_{\mathrm{m}}/h_{\mathrm{Fg}}=6)$	Ti-6Al-4V	FGTFG $d=0.5$	FGTFG $d=1$	FGTFG $d=2$	FGTFG $d=15$
100	1.984(11, 4)	2.563(11, 2)	2.836(11, 2)	3.091(10, 4)	3.477(10, 4)
300	0.661(19, 7)	0.854(19, 4)	0.944(19, 2)	1.030(18, 5)	1.159(18, 5)
500	0.397(19, 17)	0.512(19, 16)	0.566(19, 16)	0.618(18, 17)	0.695(18, 16)

4. Numerical computations and results

Two sets of a material mixture of FGM face sheets are considered. One is silicon nitride and stainless steel, referred to as Si_3N_4/SUS304 and second is silicon nitride and titanium alloy referred to as Si_3N_4/Ti-6Al-4V. The effective Young modulus and Poisson ratio for Si_3N_4 are $E_{\mathrm{Fg}} = 3.22271 \times 10^{11}$ Pa, $\nu_{\mathrm{Fg}} = 0.24$, for SUS304 are $E_{\mathrm{Fg}} = 2.07788 \times 10^{11}$ Pa, $\nu_{\mathrm{Fg}} = 0.3178$, and for Ti-6Al-4V are $E_{\mathrm{Fg}} = 1.056982 \times 10^{11}$ Pa, $\nu_{\mathrm{Fg}} = 0.298$ at room temperature (300 K) [2]. The variation of T^{ax}_{1buc} for the fully metal, fully FGM, FGM/SUS304/FGM (or FGSFG) and FGM/Ti-6Al-4V/FGM (or FGTFG) three--layered cylindrical shells with different volume fraction index ($d = 0.5,\ 1,\ 2,\ 15$) of FGM face sheets, versus the ratios $h_{\mathrm{m}}/h_{\mathrm{Fg}}$ and R/h, are presented in Table. As $h_{\mathrm{m}} = 0$, the three-layered cylindrical shell with FGM face sheets is transformed to fully FGM cylindrical shell. As R/h and $h_{\mathrm{m}}/h_{\mathrm{Fg}}$ increase, the values of T^{ax}_{1buc} decrease, whereas, as d increases, T^{ax}_{1buc} increase for FGSFG and FGTFG shells. The chance of T^{ax}_{1buc} for the FGTFG shell is stronger than for the FGSFG shell. As R/h increases, the effect of profiles on T^{ax}_{1buc} remains constant, whereas this effect decreases as the ratio $h_{\mathrm{m}}/h_{\mathrm{Fg}}$ increases. Comparing the values of T^{ax}_{1buc} for FGSFG and FGTFG shells with the fully FGM shell, the higher effect of FGM on T^{ax}_{1buc} is for FGTFG shell.

5. Conclusions

In this study, the buckling response of layered cylindrical shells with FGM face sheets subjected to the uniform axial compressive load is investigated. The dimensionless axial buckling load of layered cylindrical shell with FGM face sheets is found. Effects of volume fractions of FGM face sheets and shell characteristics on the values of dimensionless axial buckling load have been studied.

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