Violation of Bell–CHSH & CH Inequalities by Superposition of Two Coherent States ($\pi/2$ Out of Phase)

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We study the tests of the Bell–CHSH & CH inequalities by superposition of two coherent states ($\pi/2$ out of phase), a class of non-Gaussian state, using photon parity and on/off measurements. Large violations of the Bell-type inequalities have been observed theoretically confirming the interpretation and validity of quantum mechanics against the local-realistic theories.

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1. Introduction

Einstein et al. [1] realized that in many states, when measuring either linear momentum or position of one of the two particles, one can infer precisely either momentum or position of the other. They advocated that if the local realism is taken for granted, then quantum theory is an incomplete description of the physical world. Following this line, Bell demonstrated that a contradiction arises between the EPR assumptions of *realism and locality* and quantum physics and termed as Bell's theorem. Quantum nonlocality confirms the interpretation and validity of quantum mechanics against the local-realistic theories by violations of the constraints on the correlation between local measurement outcomes. Mathematical expression of such a constraint is known as Bell inequality [2], of which many variants exist [3–5]. For example, well known Clauser, Horne, Shimony, and Holt (CHSH) inequality [3] and Clauser and Horne (CH) [4] are used for the verification of nonlocal correlations in a two-dimensional Hilbert space. The Bell inequalities concern measurements made by observers on pairs of particles that have interacted and then separated. According to the quantum mechanics they are entangled, while local realism would limit the correlation of subsequent measurements of the particles. Nonlocal correlations play crucial role for the device-independent versions of quantum information protocols, such as cryptography, random number generation, state estimation, and entangled measurement certification.

Quantum continuous variables (CV) [6] of light have been successfully used to realize some of the standard informational tasks traditionally based on qubits. Bell-inequality tests were performed by using qubits [7] which satisfy the space-like separation between two local parties. But, the difficulty of the detection loophole [8, 9] demanded to look at other approaches for Bell-inequality tests. Continuous variable (CV) states are of recent interest in order to suggest proposals for loophole-free Bell-inequality tests [10]. CV states have advantage over discrete variable states due to the highly efficient and well experimentally developed method of detection for CV states. Bell's inequality tests in the phase space have been studied by Banaszek and Wódkiewicz (BW) [11] in terms of the Wigner (Q) function based upon the photon number parity (on/off) measurements and the displacement operation. The Wigner function approach supports the Bell inequality version of CHSH [3], while the Q function supports the version of CH [4].

The Schrödinger cat states are superposition of two coherent states in free-traveling optical fields, non-Gaussian CV states, called [12], has been generated and detected [13, 14], where the size of the states was reasonably large for fundamental tests of quantum theory and quantum-information processing [15]. Application of superposed coherent states in quantum computing is of interest because the encoding of qubits in coherent states (and the resulting need for their superposition states, i.e., the cat states) only requires relatively small coherent state amplitudes to be sufficiently distinguishable by homodyne detection. Nonclassical features [16] of the Schrödinger cat states are of great interest in the quantum information processing applications. Recently, Zeng et al. [17] discussed nonclassical features, such as sub-Poissonian photon statistics, quadrature squeezing, and the negativity of the Wigner function, of the superposition of two coherent states which are $\pi/2$ out of phase (we will abbreviate this state as SCSP for simplicity throughout the paper),

$$|\psi\rangle = (N/\sqrt{2})\left(|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\,|\mathrm{i}\alpha\rangle\right), \quad (1)$$

with the normalization constant $N^2 = [1 + \exp(-|\alpha|^2)\cos(\phi + |\alpha|^2)]^{-1}$. Here, $|\alpha\rangle$ is the coherent state defined by the eigenvalue equation, $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

In this paper, we study the quantum nonlocality test for SCSP [7] by using photon parity and on/off measurements [18]. We study violations of Bell–CHSH and Bell–CH inequalities for the SCSP state and observe strong violations establishing the quantum nonlocality.

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2. Violations of Bell–CHSH inequality with photon parity measurement scheme

The Wigner function [19] offers the appealing possibility of being able to describe quantum phenomena using the classical-like concept of a phase space distribution function. The Wigner function of a quantum state described by the density operator, $\hat{\rho}$, is

$$W(\alpha) = \mathrm{Tr}\left(\hat{\rho}\int \exp(\alpha\xi^* - \alpha^*\xi)D(\xi)\pi^{-1}\,\mathrm{d}^2\xi\right). \quad (2)$$

Here the displacement operator [20], $\hat{D}(\xi)$, is defined as $\hat{D}(\xi) = \exp(\xi\hat{a}^\dagger - \xi^*\hat{a})$. The Wigner function $W(\alpha)$ is real-valued, uniformly continuous, and square-integrable function of α for all density operators $\hat{\rho}$. In a coherent state representation, the Wigner function can be written as $W(\alpha) = (2/\pi)\langle\psi|\hat{\Pi}(\alpha)|\psi\rangle$, where $\hat{\Pi}(\alpha) = \hat{D}(\alpha)\hat{\Pi}\hat{D}^\dagger(\alpha)$ is the parity operator $\hat{\Pi} = (-1)^{\hat{a}^\dagger\hat{a}}$, shifted in the phase space by α, with the help of displacement operator, $\hat{D}(\alpha)$. The displacement operator can be experimentally realizable by a beam splitter with the transmission coefficient close to one and a strong coherent state being injected into the other input port [11]. Correlated parity measurement [11] can be described by the following positive operator-valued measure operators [21]:

$$\hat{\Pi}^+(\beta) = \hat{D}(\beta)\sum_{k=0}^{\infty}|2k\rangle\langle 2k|\,\hat{D}^\dagger(\beta), \quad (3)$$

$$\hat{\Pi}^-(\beta) = \hat{D}(\beta)\sum_{k=0}^{\infty}|2k+1\rangle\langle 2k+1|\,\hat{D}^\dagger(\beta). \quad (4)$$

Corresponding operator for the correlated measurement of the parity on modes "a" and "b" of two parties, say Alice and Bob, may be defined as

$$\hat{\Pi}(\beta,\gamma) = \left[\hat{\Pi}_{\mathrm{a}}^{(+)}(\beta) - \hat{\Pi}_{\mathrm{a}}^{(-)}(\beta)\right] \otimes \left[\hat{\Pi}_{\mathrm{b}}^{(+)}(\gamma) - \hat{\Pi}_{\mathrm{b}}^{(-)}(\gamma)\right]. \quad (5)$$

The outcome of the measurements is either $+1$ or -1 (i.e., dichotomic). Then the Bell–CHSH inequality is

$$B \equiv |B_{\mathrm{CHSH}}| = \left|\left\langle \hat{\Pi}(\beta,\gamma) + \hat{\Pi}(\beta,\gamma') + \hat{\Pi}(\beta',\gamma) - \hat{\Pi}(\beta',\gamma')\right\rangle\right| \le 2, \quad (6)$$

where we call $B \equiv |B_{\mathrm{CHSH}}|$, the Bell–CHSH function. The two-mode Wigner function at a given phase point described by β and γ is $W(\beta,\gamma) = \frac{4}{\pi^2}\mathrm{Tr}[\hat{\rho}\hat{\Pi}(\beta,\gamma)]$, where $\hat{\rho}$ is the density operator of the field. Then the Wigner representation of the Bell–CHSH inequality is

$$B = \frac{\pi^2}{4}\left|\langle W(0,0) + W(\beta,0) + W(0,\gamma) - W(\beta,\gamma)\rangle\right| \le 2. \quad (7)$$

The Cirel'son bound is $B = |B_{\mathrm{CHSH}}| \le 2\sqrt{2}$ in the generalized BW formalism. The Wigner function of the state SCSP, $|\psi\rangle$, is [17]:

$$W(\beta) = (N^2/2)\left[W_{|\alpha\rangle}(\beta) + W_{|\mathrm{i}\alpha\rangle}(\beta) + W_{\mathrm{int}}(\beta)\right], \quad (8)$$

where

$$W_{|\alpha\rangle}(\beta) = \frac{2}{\pi}\exp\left(-2(|\alpha|^2 + |\beta|^2 - \alpha^*\beta - \alpha\beta^*)\right), \quad (9)$$

$$W_{|\mathrm{i}\alpha\rangle}(\beta) = \frac{2}{\pi}\exp\left(-2(|\alpha|^2 + |\beta|^2 + \mathrm{i}\alpha^*\beta - \mathrm{i}\alpha\beta^*)\right), \quad (10)$$

$$W_{\mathrm{int}}(\beta) = \frac{4}{\pi}\cos(\phi - |\alpha|^2 + \Delta) \times \exp\left(-|\alpha|^2 - 2|\beta|^2 + \Delta\right), \quad (11)$$

and

$$\Delta = \alpha^*\beta + \alpha\beta^* - \mathrm{i}\alpha^*\beta + \mathrm{i}\alpha\beta^*. \quad (12)$$

We can visualize the nonclassical nature [19] of the state $|\psi\rangle$ in terms of the negative regions of its Wigner function (Fig. 1). In order to make the Bell–CHSH inequality test,

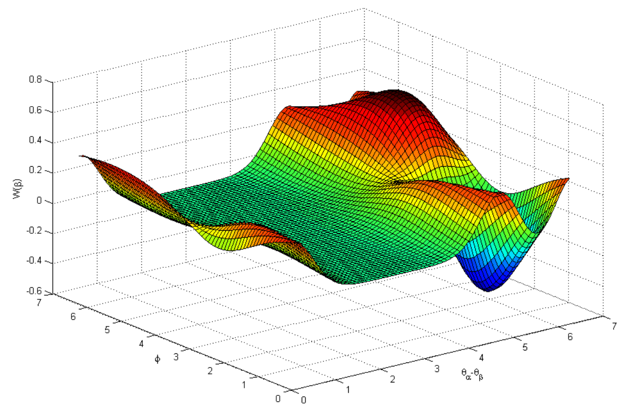

Fig. 1. Variation of Wigner function, $W(\beta)$, of SCSP with ϕ and $(\theta_\alpha - \theta_\beta)$ for $J = 1.0$.

the single-mode SCSP state $|\psi\rangle$ with the Wigner function given by Eq. (8) is divided by a beam splitter to generate a two-mode state shared by distant parties, say Alice and Bob. The beam splitter operator acting on modes $\hat{\mathrm{a}}$ and $\hat{\mathrm{b}}$ is represented as

$$\hat{B}(\theta) = \exp\left(\theta(\hat{\mathrm{a}}^\dagger\hat{\mathrm{b}} - \hat{\mathrm{a}}\hat{\mathrm{b}}^\dagger)/2\right), \quad (13)$$

where the beam splitter reflectivity and transmittivity are defined as $R = \sin^2(\theta/2)$ and $T = 1 - R$, respectively. When the state $|\psi\rangle$ passes through a 50:50 beam splitter, the Wigner function of the resulting state is

$$W_{\mathrm{out}}(\beta,\gamma) = W_{|\psi\rangle}\left(\frac{\beta-\gamma}{\sqrt{2}}\right)W_{|0\rangle}\left(\frac{\beta+\gamma}{\sqrt{2}}\right), \quad (14)$$

where $W_{|0\rangle}(\gamma)$ is the Wigner function of the vacuum

$$W_{|0\rangle}(\gamma) = \frac{2}{\pi}\exp\left(-2|\gamma|^2\right). \quad (15)$$

The two-mode state $W_{\mathrm{out}}(\beta,\gamma)$, Eq. (14), can be used to calculate the Bell–CHSH function given by Eq. (7). We can find several situations when the Bell–CHSH inequality, Eq. (7), is violated by the state $|\psi\rangle$. For example, let us consider, for simplicity, $\alpha \equiv |\alpha|\mathrm{e}^{\mathrm{i}\theta_\alpha}$, $\beta \equiv |\beta|\mathrm{e}^{\mathrm{i}\theta_\beta}$, $\gamma \equiv |\gamma|\mathrm{e}^{\mathrm{i}\theta_\gamma}$, and $|\alpha| = |\beta| = |\gamma| \equiv \sqrt{J}$. Then, e.g., for $\theta_\alpha = \pi/8$, $\theta_\beta = \pi/2$, $J = 0.2$, we plot variation of the Bell–CHSH function (B) with ϕ and θ_γ (see Fig. 2) and we observe strong violations of the Bell–CHSH inequal-

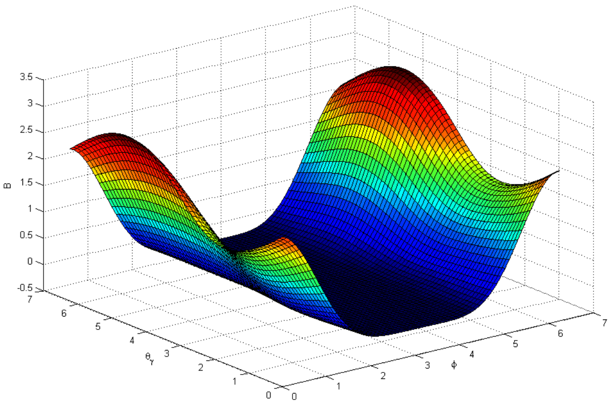

Fig. 2. Variation of Bell–CHSH function (B) with ϕ and θ_γ for $\theta_\alpha = \pi/8$, $\theta_\beta = \pi/2$, $J = 0.2$.

ity, Eq. (7), for different values of ϕ and θ_γ. The Cirel'son bound, $B = |B_{\rm CHSH}| \le 2\sqrt{2}$, is also followed.

3. Violations of Bell–CH inequality with the on/off measurement scheme

According to Banaszek and Wódkiewicz, the Husimi–Kano Q-function [22] can be used for test of the Bell–CH inequality violation using photon presence measurements [11]. It is important to be noted that this method is more practical for an experimental Bell inequality test using the currently available photodetectors. The Q function for a two-mode state $\hat{\rho}_{\rm ab}$ is defined as [22]:

$$Q_{\rm ab}(\beta,\gamma) = {}_{\rm b}\langle\gamma|_{\rm a}\langle\beta|\,\hat{\rho}_{\rm ab}\,|\beta\rangle_{\rm a}|\gamma\rangle_{\rm b}/\pi^2, \tag{16}$$

where $|\beta\rangle$ and $|\gamma\rangle$ are coherent states of amplitudes β and γ, respectively. Then, the Bell–CH function in terms of Q-function is

$$\begin{aligned} B_{\rm CH} = \left\langle \hat{B}_{\rm CH} \right\rangle &= \pi^2 \big[Q_{\rm ab}(\beta,\gamma) + Q_{\rm ab}(\beta',\gamma) \\ &+ Q_{\rm ab}(\beta,\gamma') - Q_{\rm ab}(\beta',\gamma')\big] - \pi\left[Q_{\rm a}(\beta) + Q_{\rm b}(\gamma)\right], \end{aligned} \tag{17}$$

where $Q_{\rm a}(\beta)$ and $Q_{\rm b}(\gamma)$ are marginal Q functions in the corresponding modes. For local theories, the Bell–CH inequality [4], $-1 \le B_{\rm CH} \le 0$, must be satisfied and, therefore, any violation establishes the nonlocal theory. The Q-function for the SCSP state $|\psi\rangle$ can be written as

$$\begin{aligned} Q_{|\psi\rangle}(\beta) &= \frac{N^2}{2\pi}\exp\left(-|\alpha|^2 - |\beta|^2\right) \\ &\times \Big[\exp(\alpha^*\beta + \alpha\beta^*) + \exp({\rm i}\alpha^*\beta - {\rm i}\alpha\beta^*) \\ &+ \exp(\alpha\beta^* - {\rm i}(\varphi + \alpha^*\beta)) \\ &+ \exp\left(\alpha^*\beta + {\rm i}(\varphi + \alpha\beta^*)\right)\Big]. \end{aligned} \tag{18}$$

Then, by using the method similar to that for the case of Eq. (14), with

$$Q_{|0\rangle}(\gamma) = \frac{1}{\pi}\exp(-|\gamma|^2)$$

and

$$Q_{\rm ab}(\beta,\gamma) = Q_{|\psi\rangle}\left(\frac{\beta-\gamma}{\sqrt{2}}\right) Q_{|0\rangle}\left(\frac{\beta+\gamma}{\sqrt{2}}\right),$$

we can get the Q-function of the two-mode state shared by two parties, say Alice (mode "a") and Bob (mode "b") as

$$\begin{aligned} Q_{\rm ab}(\beta,\gamma) &= \frac{N_{\rm ab}^2}{2\pi^2}\exp(-3J) \\ &\times \Big[\exp\left(\sqrt{2}\,J\left(\cos(\theta_\alpha-\theta_\beta) - \cos(\theta_\alpha-\theta_\gamma)\right)\right) \\ &+ \exp\left(-\sqrt{2}\,J\left(\sin(\theta_\alpha-\theta_\beta) - \sin(\theta_\alpha-\theta_\gamma)\right)\right) \\ &+ 2\cos(\varphi + \sqrt{2}JX)\exp(\sqrt{2}JX)\Big], \end{aligned} \tag{19}$$

where

$$\begin{aligned} N_{\rm ab}^2 &= \Big[1 + \exp\left(-J\left(1-\cos(\theta_\beta-\theta_\gamma)\right)\right) \\ &\times \cos\left(\varphi + J\left(1-\cos(\theta_\beta-\theta_\gamma)\right)\right)\Big]^{-1}, \end{aligned} \tag{20}$$

$$\begin{aligned} X &= \left[\sin\left(\theta_\alpha - \frac{1}{2}(\theta_\beta+\theta_\gamma)\right) + \cos\left(\theta_\alpha - \frac{1}{2}(\theta_\beta+\theta_\gamma)\right)\right] \\ &\times \sin\left(\frac{1}{2}(\theta_\beta-\theta_\gamma)\right), \end{aligned} \tag{21}$$

$$\begin{aligned} Q_{|\psi\rangle}(\beta) &= \frac{N^2}{2\pi}\exp(-2J)\Big[\exp\left(2J\,\cos(\theta_\alpha-\theta_\beta)\right) \\ &+ \exp\left(-2J\sin(\theta_\alpha-\theta_\beta)\right) \\ &+ 2\exp\left(J\left(\cos(\theta_\alpha-\theta_\beta) - \sin(\theta_\alpha-\theta_\beta)\right)\right) \\ &\times \cos\left(\varphi + J\left(\cos(\theta_\alpha-\theta_\beta) - \sin(\theta_\alpha-\theta_\beta)\right)\right)\Big], \end{aligned} \tag{22}$$

and for making simplicity of calculations, we denoted $\alpha = |\alpha|\exp({\rm i}\theta_\alpha)$, $\beta' = |\beta'|\exp({\rm i}\theta_{\beta'})$, $\gamma' = |\gamma'|\exp({\rm i}\theta_{\gamma'})$, $\beta = |\beta|\exp({\rm i}\theta_\beta)$, $\gamma = |\gamma|\exp({\rm i}\theta_\gamma)$, $|\alpha| = |\beta| = |\gamma| = |\beta'| = |\gamma'| = \sqrt{J}$. Using Eqs. (19)–(22) in Eq. (17), we can find several situations for violation of the Bell–CH inequality $-1 \le B_{\rm CH} \le 0$. For example, let us take $\theta_\beta - \theta_{\gamma'} = \theta_{\beta'} - \theta_\gamma = \theta_{\beta'} - \theta_{\gamma'} = \theta_{\beta'} - \theta_{\gamma'} = \pi$, $J = 1.5$ and we can see the variation of the Bell–CH function (see Fig. 3) with $(\theta_\alpha - \theta_\beta)$ and ϕ. We can see the strong violations of the CH inequality for different values of $(\theta_\alpha - \theta_\beta)$ and ϕ.

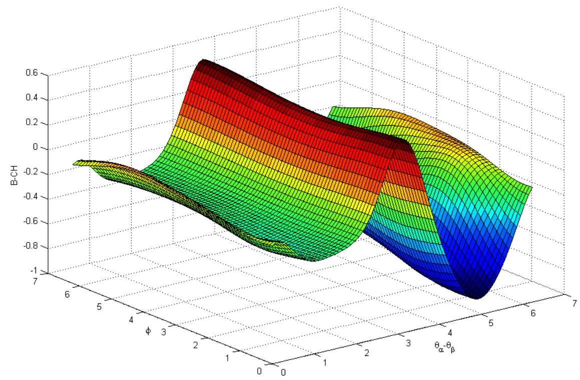

Fig. 3. Variation of Bell–CH function with $(\theta_\alpha - \theta_\beta)$ and ϕ for $\theta_\beta - \theta_{\gamma'} = \theta_{\beta'} - \theta_\gamma = \pi$, $J = 1.5$.

4. Concluding remarks

We discussed about the violation of the Bell–CHSH and Bell–CH inequalities using photon parity and on/off measurement schemes by the superposition of two coherent states which are $\pi/2$ out of phase (SCSP). Photon on/off measurements for a Bell-inequality test is more practical than that of photon number parity measurements but with the too large average photon number of the state under consideration, because of getting a negligible "off' result, Bell violations cannot be observed using photon on/off measurements.

This study may play an important role in usefulness of superposed coherent states for the quantum information processing applications. A key requirement of quantum information processing with cat states is the generation of cat states in free propagating optical fields. The state $|\psi\rangle$ can be generated by using the dispersive atom-field interactions [23]. A recent experimental progress [24] could be directly improved by the cat-amplification scheme to generate a cat state of a larger amplitude ($\alpha \approx 2$) and higher fidelity within reach of current technology.

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