Determination of Stresses and Forces on the Orthodontic System by Using Numerical Simulation of the Finite Elements Method

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This study was addressed to use knowledge about the orthodontic system with numerical simulation of the finite elements method. For the first time we simulated the stresses on the orthodontic system and, in this manner, calculated the orthodontic force on the tooth. A 3D orthodontic model or orthodontic system was designed resembling moderate crowding in the dental arch with all supporting structures. CATIA V5 computer software was used to set up a model for the orthodontic system and ABAQUS was used for simulation of the stresses on the orthodontic system. Our attention was focused on the stresses on the tooth lateral incisor and its periodontal ligament. The results of the numerical simulation showed complex stresses on the tooth lateral incisor and its periodontal ligament. In this paper there is presented a calculation of the orthodontic force acting on the tooth lateral incisor due to the orthodontic wire. This orthodontic force was calculated from the stresses on the bracket. The calculated orthodontic force was in the area which is considered as the optimal orthodontic force for movement of the tooth.

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1. Introduction

Malocclusion is the misalignment of teeth, or when the relationship between the upper and lower dental arches is incorrect. Malocclusions occur in all three planes of space and can affect each tooth in all three planes. Total malocclusion frequency varies with a mean of 46% [1]. Of all malocclusions, crowding is the predominant intra-arch problem in patients seeking orthodontic treatment in the United States and Western Europe [2, 3].

In order to correct malocclusions orthodontic treatment is needed. The preferred treatment option in the correction of malocclusions is the utilization of fixed orthodontic appliances. Contemporary orthodontics relies on the use of fixed orthodontic appliances to solve the misalignment of teeth and bite problems by moving the teeth gradually into the normal position in the dental arch. The fixed orthodontic appliance consists of brackets that are bonded to the teeth, as well as orthodontic wires. When the wire is engaged in the slot of the brackets it generates the forces necessary for orthodontic tooth movement [4–6].

Each tooth is attached to the alveolar bone by a strong network of parallel collagen fibres: the periodontal ligament (PDL) is remodelled and renewed constantly during normal function. PDL has two major components: (1) cellular elements, and (2) the tissue fluid. Both components play an important role in normal function and allow the orthodontic movements of teeth [7]. The sequence of events carried out by applying forces within the limits of physiological tolerance begins with the decreased blood flow through the PDL, followed by the resorption and apposition of the bone. A periodontal ligament placed under pressure will result in bone resorption, whereas a periodontal ligament under tension results in bone formation. Within a few hours of applying a light force, a series of chemical changes in the PDL begins stimulating the cells to differentiate into osteoclasts (responsible for bone resorption) and osteoblasts (responsible for bone apposition). The bone that opposes the motion undergoes frontal resorption to allow for dental displacement, whereas on the opposite side, the stress of the periodontal fibres results in the deposition and production of a new bone. If orthodontic forces stay light, frontal resorption on one side and apposition on the other will occur at the same rate. When a force of great intensity is applied on the tooth, it causes a vascular occlusion and cuts the blood supply to the PDL. In this case, aseptic necrosis occurs, resulting in an undermining bone resorption that does not start from the dental side, but comes from the alveolar region, causing the tissue damage, hyalinization and pain. The process of underlying resorption is faster and more damaging compared to frontal resorption [8–12].

In order to prevent undermining resorption light forces should be used during orthodontic treatment. The optimum force used in orthodontic treatment should be enough to produce tooth movement without tissue damage and with maximum comfort for the patient. Excessive forces can lead to severe pain, damage of the periodontal ligament and root resorption [13]. Insufficient forces extend the duration of the treatment. Delivering optimal force levels for controlled tooth movement remains of the *utmost importance* during *orthodontic treatment*. Light continuous orthodontic forces are preferred. The force needed to move the tooth is different for each tooth and depends on the kind of movement that is required during orthodontic treatment [14]. For example; the force of 10–20 g/cm^2 is needed for intrusion, and 70–100 g/cm^2 is the desirable force for translation [6]. Optimum force level for tooth movement usually varies in the range of 0.09 to 0.98 N (9–100 g/cm^2).

A variety of wires are used to generate the necessary biomechanical forces associated with tooth movement, such as: stainless steel; nickel–titanium (NiTi); beta--titanium; and cobalt–chromium. Once the wire is activated or bent, it is the unloading or deactivating forces that produce the orthodontic tooth movement. With current orthodontic treatment nickel–titanium wires are often used due to their superior mechanical properties, biocompatibility, ductility, resistance to corrosion, lower elastic modulus, and special characteristics such as superelasticity and shape memory effect. The effect that the wire produces is a summary of properties of the wire itself and geometrical factors. Geometrical factors such as: the cross-section of the wire (round, rectangular) and the distance between the brackets, have a great impact on the force level [9]. All of these factors should be addressed when the magnitude of orthodontic force is measured. There is still insufficient knowledge of the direction, magnitude and distribution of the forces applied in orthodontic therapy, as well as their effect on the tooth and surrounding supportive structures. Until recently, much of the orthodontic biomechanics literature was restricted to 2-dimensional experimental studies of the biomechanical aspects of orthodontic force systems and, more recently, to 3-dimensional (3D) computer modelling.

There is little evidence regarding 3D experimental measurements and analysis of orthodontic force systems [15–17]. Solutions using numerical methods began after 1970, leading to the development of specific software packages. Research conducted by the finite element method (FEM) in dental practice has been related mainly to dental implants, stress in periodontal ligaments and displacements of teeth under the influence of external forces [7, 18, 19]. The FEM enables the investigation of the biomechanical issues involved in orthodontic treatment. In addition, it stimulates currently increasing scientific interest in tooth movement [20]. The development of a numerical model makes it possible to quantify and evaluate the effects of orthodontic loads applied in order to achieve tooth movement. One of the main features of the FEM lies in its potential to analyse complex structures. In the case of tooth movement, the numerical model should resemble the clinical setting, including the type of malocclusion and choice of brackets, as well as arch wires.

Simulation of orthodontic tooth movement with a fixed orthodontic appliance using FEM can help in the determination of the forces produced by the orthodontic wire.

The purpose of this article was to simulate the stresses on the orthodontic system in the case of moderately crowded frontal teeth in the upper dental arch and to quantify the forces applied to teeth when different NiTi wires were engaged in fixed orthodontic appliances.

2. Materials and methods model

The orthodontic 3D model for this study was built using CATIA V5 software. A 3D model of a crowded central incisor, lateral incisor and canine in the upper dental arch simulated real malocclusion (Fig. 1).

Fig. 1. Model of moderate crowding in the upper dental arch.

A fixed orthodontic appliance was used to simulate the orthodontic treatment in a case with moderate crowding. Metal brackets were bonded to the teeth and wire inserted into the slots. Data for tooth dimensions were obtained from the dental anatomy literature [21]. The teeth were modelled using the orthographic views (top, front and side view) of the tooth. The teeth crowns in the model had the following heights and mesio-distal widths respectively: 11.2 mm and 8.6 mm for the central incisor, 9.8 mm and 6.6 mm for the lateral incisor and 10.6 mm and 7.6 mm for the canine. The root lengths were: 13 mm for the central incisor, 13.4 mm for the lateral incisor and 16.5 mm for the canine. The position of the teeth in the model resembled moderate crowding in the upper dental arch. The lateral incisor was moved 4 mm lingually and 3 mm gingivally from its normal position in the dental arch in order to present moderate crowding.

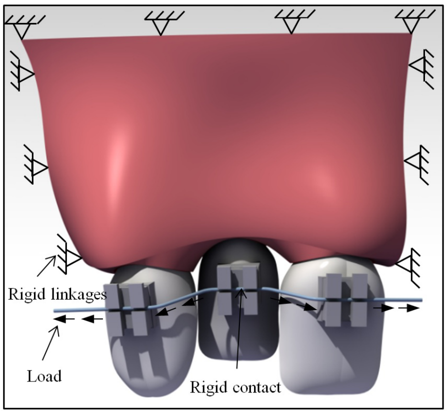

Fig. 2. CAD model of tooth position with boundary conditions.

A 3D model of the upper frontal teeth was obtained, with the periodontal ligament modelled for the length of the whole root (0.25 mm in width) (Fig. 2). Supportive bone was modelled in a 2 mm thick layer, with underlying cortical bone. The orthodontic NiTi wire was inserted into the bracket slot of each tooth in the model. For the model a wire of a diameter of 0.012″ (0.305 mm) was taken into consideration.

2.1. Material properties

The 3D model was based on the thesis that all materials are isotropic materials, which means that there are two independent material constants. In order to simplify the model, materials were considered to be homogeneous, meaning that linear and elastic material behaviour included two constants: Young's modulus and Poisson's ratio. The value of Young's modulus and Poisson's ratio for the alveolar bone, periodontal ligament, tooth and bracket were taken from literature (Table I) [22].

TABLE I

Young's modulus and Poisson's ratio on separate segments of the orthodontic system.

Linear-elastic material parameters used for:	Young's modulus of elasticity, E [MPa]	Poisson's ratio
alveolar bone	13800 [18]	0.30 [18]
teeth	20000 [18]	0.30 [18]
periodontal ligament	1 [22]	0.45 [22]
bracket (stainless steel)	180000	0.3

In the numerical simulation that was performed, three different NiTi orthodontic wires with various modulus of elasticity (Table II) were used. The purpose of the numerical simulation was to determine the initial stresses

TABLE II

Material parameter of wires for ABAQUS model.

Wire	Young's modulus of austenite, E [MPa]
1.	50000
2.	54000
3.	58000

and, consequently, the force when the wire was inserted into the slot of the brackets and ligated. Although the behaviour of NiTi wires in terms of superelasticity is complex, we simplified the material properties and the Young modulus of austenite for numerical analysis was taken [23]. However, this theory says that when, under certain stresses austenitic structures change in the martensitic structure, we can suppose, as in our case, that the stresses on the wire are in the elastic region of austenite. Data for Young's modulus of austenite for NiTi for various wires are presented in Table II [24]. In all three wires Poisson's ratio was 0.3 [24].

2.2. Finite element model generation

The constructed model was transferred into the ABAQUS/CAE 6.10-1 software for the numerical simulation by the FEM. In our model we performed a static analysis. The boundary conditions in our model are shown in Fig. 2. Our model is a fixed one mounted in the alveolar bone. To simplify the numerical calculation we fixed the orthodontic wire rigidly in the bracket of the lateral incisor. Loads were placed on both ends of the wire with the tension load as shown in Fig. 2. With this kind of load we are closer to the real case. The numerical model consists of 86315 finite elements.

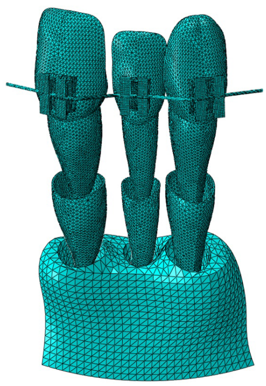

Fig. 3. The finite-element mesh of the model.

With the automatic mesh generation of parts by tetrahedral element, the following parts were meshed: alveolar bone, PDLs and teeth. The brackets and wire were meshed by hexahedral finite elements (Fig. 3). In real orthodontic treatment, especially in the levelling stage, it is desirable for the wire to slide through the slot of the bracket as freely as possible having the least friction [25]. In order to choose orthodontic wire wisely it is important to know the friction properties between the wire and the bracket. Information about these properties in the literature has not been consistent. It is very difficult to determine accurately the friction characteristics between the wire and the bracket because the contact conditions between them vary widely. For the friction coefficient between wire (NiTi) and bracket (stainless steel) a value of 0.3 [26] was taken from the literature. Other contacts between segments: alveolar bone-PDL, PDL-tooth, tooth-bracket were rigid.

2.3. Calculation of the force

The tooth was moved through the use of force applied by the wire inserted into the slot of the bracket. By using the FEM the stresses were converted into force.

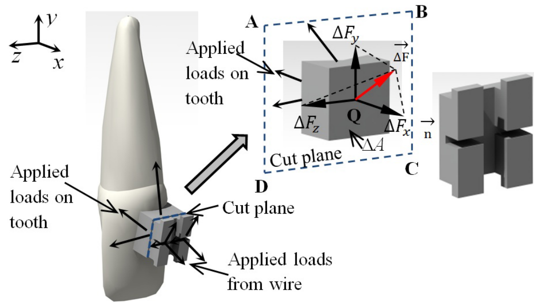

Fig. 4. Cutting a bracket of the lateral incisor, projecting the internal resultant force onto normal and tangential components.

To calculate the forces on the lateral incisor we used the stresses which act on the bracket of the lateral incisors. For the stresses which act on the cut plane $ABCD$ (Fig. 4) bracket of the lateral incisor we used Eqs. (1.1)–(1.3) to calculate the forces on the tooth

$$\sigma_{xx} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}, \tag{1.1}$$

$$\tau_{xy} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}, \tag{1.2}$$

$$\tau_{xz} \overset{\text{def}}{=} \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}. \tag{1.3}$$

The cut plane $ABCD$ is oriented by its unit normal direction vector $\boldsymbol{n}$ or normal. Normal $\boldsymbol{n}$ has been chosen to be parallel to the x-axis. At the point Q we refer to a rectangular Cartesian coordinate system of axes $\{x, y, z\}$. To the cut portion of the bracket we applied a system of internal forces that restores static equilibrium. $\Delta \boldsymbol{F}$ are the resultant of internal forces acting on the ΔA. The component ΔF_x is aligned with the cut-plane normal and is called the normal internal force component or normal force. Components ΔF_y and ΔF_z which lie on the cut plane are called the tangential internal forces components or tangential forces. We defined the stress components at point Q by taking the limits of internal force over elemental-area rations, as that area shrinks to zero (see Eqs. (1.1)–(1.3)). σ_{xx} is called a normal stress, whereas τ_{xy} and τ_{xz} are tangential stresses [27].

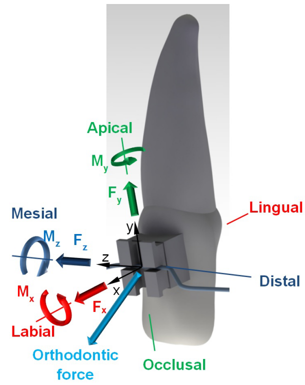

Fig. 5. Reference coordinate system in the tooth with showing the forces and moments.

The reference coordinate system was defined and presented in Fig. 5.

3. Results

By using ABAQUS software for numerical simulation a model with three different orthodontic wires was presented. The values of the stress on the lateral incisor, PDL and the bracket itself produced by different types of wires are shown in Figs. 6 and 7. It can be seen that by increasing the elastic modulus of the wire, the stresses on the bracket, PDL and tooth became more intense.

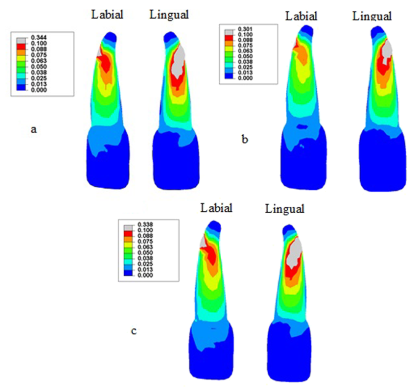

Fig. 6. Von Mises stresses on the tooth lateral incisor: (a) E = 50000 MPa; (b) E = 54000 MPa; (c) E = 58000 MPa.

Due to the action of several components of stresses on the tooth lateral incisor and PDL stresses are shown with *equivalent* or the *Von Mises* stresses. The units in the figures are shown in MPa.

Figure 6 shows the Von Mises stresses on the upper lateral incisor. From the figure it could be seen that the

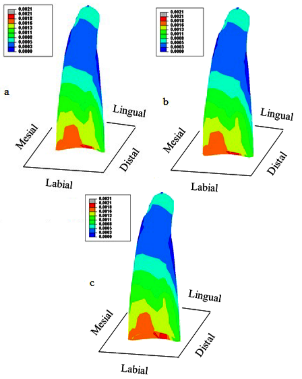

Fig. 7. Von Mises stresses on the periodontal ligament: (a) E = 50000 MPa; (b) E = 55000 MPa; (c) E = 58000 MPa.

stress state generated on the tooth complex was caused by a single component of force. The maximum Von Mises stresses occurred on the bottom of the root of the tooth. The maximal values of the Von Mises stress on the lateral incisor tooth for the separate wires are: wire 1: 0.344 MPa, wire 2: 0.301 MPa, wire 3: 0.338 MPa.

Maximum Von Mises stress on the lateral incisor tooth did not depend on the increasing modulus of elasticity of the wire. This means that the components of normal and tangential stresses on the tooth are different. But the surfaces with the highest stresses increased with the increase in Young's modulus of the wire. The highest stresses were caused on the lower side of the lingual side.

Figure 7 shows the Von Mises stresses on the PDL of the lateral incisor. The maximal Von Mises stress on the PDL of the lateral incisor is caused on the circumference of the PDL by the contact with the tooth crown (Fig. 7).

The maximal Von Mises stresses for separate wires are very similar: wire 1: 0.00211434 MPa, wire 2: 0.00213878 MPa, wire 3: 0.00214612 MPa.

On the PDL the highest stresses occur on the lingual side of the tooth. This is due to the fact that, on this side, the tension stresses on the PDL. The difference comes between the mesial and the distal sides. As we can see in Fig. 7 the highest stresses are on the distal side. This is due to the smaller distance between the teeth or brackets.

Different stresses are caused in different directions (normal and tangential) on the tooth and PDL due to the various components of forces. Each component of force has a different influence on the stresses in the root of the tooth and PDL. The force F_x caused compressive and tension stresses on the labial and lingual side on the tooth and PDL. This force is mainly for moving the tooth into the correct position; it moves the tooth in a lingual or labial direction. The force F_y caused compression and tension stresses on the tooth and PDL in both the apical and occlusal direction. This force wants to move the tooth in an apical or occlusal direction. The force F_z caused compression and tension stresses in both the distal and mesial directions on the tooth and PDL. This

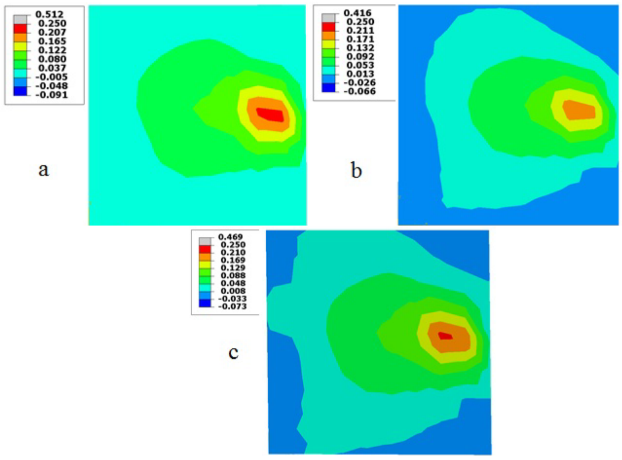

Fig. 8. Normal stresses σ_{xx} on the cut-plane of the bracket: (a) E = 50000 MPa; (b) E = 54000 MPa; (c) E = 58000 MPa.

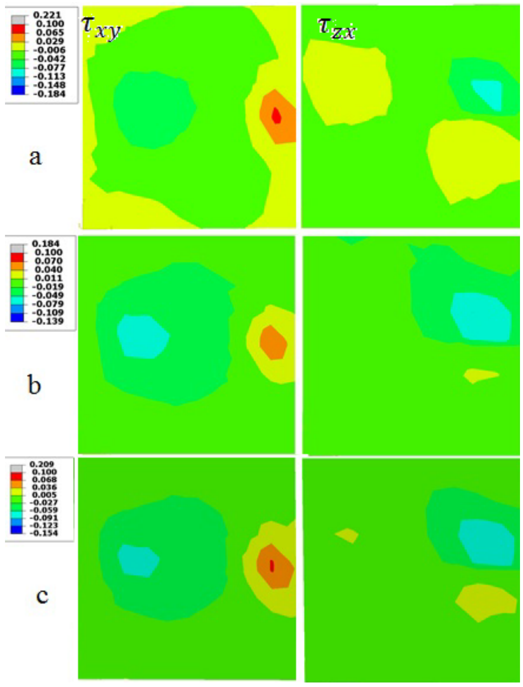

Fig. 9. Tangential stresses τ_{xy} and τ_{zx} on the cut--plane of the bracket: (a) E = 50000 MPa; (b) E = 54000 MPa; (c) E = 58000 MPa.

force is usually due to friction between the bracket and the wire. Due to the simultaneous activity of individual components of force a complex stress state was caused in both the tooth and PDL. All components of the forces acting on the tooth were dependent on the stiffness of the wire, for which we will also show the calculation of the forces in the following way.

The force caused on the bracket by the wire with given modulus was calculated using the stresses at the cut--plane (Fig. 4) of the bracket. Figure 8 shows the normal stresses on the cut-plane of the bracket for all three different wires. Maximum normal stress on the cut-plane of each wire was as follows: wire 1: 0.219192 MPa, wire 2: 0.219465 MPa, wire 3: 0.219735 MPa.

Figure 9 shows the tangential stress at the cut-plane. Maximum tangential stress τ_{xy} on the cut-plane of each wire was as follows: wire 1: 0.07475 MPa, wire 2: 0.07493 MPa, wire 3: 0.08011 MPa.

Maximum tangential stress τ_{zx} on the cut-plane of each wire was as follows: wire 1: 0.0311 MPa, wire 2: 0.0320 MPa, wire 3: 0.0329 MPa.

To calculate the forces we used Eqs. (1.1)–(1.3). We first calculated the separate force in all axes, and then we calculated the resultant force acting on the tooth. We calculated separate forces from the average value of the average normal stress and two average tangential stresses from the cut-plane.

TABLE III

The average stresses in separate axes and calculated components of internal force with resultant force.

Wire	Average stresses [MPa]			Components of internal forces [N]			Orthodontic force [N]
	σ_{xx}	τ_{xy}	τ_{xz}	F_x	F_y	F_z	F
1	0.0341	−0.012	−0.0015	0.35	−0.12	−0.016	0.37
2	0.0346	−0.0121	−0.00152	0.354	−0.124	−0.0156	0.375
3	0.035	−0.0124	−0.00161	0.358	−0.127	−0.017	0.38

In Table III we can see the results of the components of internal forces and resultant force or orthodontic force. The components of internal forces F_y and F_z have negative values, which means that they act in a reverse direction as supposed in Fig. 5. The direction act of orthodontic force is shown in Fig. 5. From wire 1 we got the minimum orthodontic force 0.37 N and from wire 3 the maximum orthodontic force 0.38 N. In our model of the orthodontic system we can see that a higher stiffness of wire gives a higher level of orthodontic force on the tooth.

4. Discussions

FEM is a powerful tool for the analysis of complex structures, but the outcome is dependent on the formulation of the problem [28]. The acting forces in the beginning stage of orthodontic treatment in the case with moderate crowding were presented using FEM. This was an attempt to quantify and evaluate the effects of orthodontic loads applied to the bracket and teeth in order to achieve initial tooth movement. With the intention of simplifying the procedure the moment of force was not taken into account. The emphasis was put on the level of force produced by the NiTi wire. It was determined that, by the smallest elastic modulus of 50000 MPa, the calculated force on the lateral incisor was approximately 0.37 N. The bigger force was produced by wires with a higher value of elastic modulus. The force level was in area that is supposed to be optimal for orthodontic tooth movement (0.09–0.98 N) [6].

Different structures and materials used in orthodontics have had their properties identified, such as bones, teeth and stainless steel. When a numerical model is used choosing the right material properties, such as elastic modulus and Poisson's ratio, as well as presenting the characteristics of the alveolar bone, tooth and PDLs are the most important factors in obtaining precise results. Also, the reliability of FEM cannot be checked directly, due to the fact that the model of crowded teeth was made only as a replica of a real life problem.

The wide range of boundary conditions, contact definitions, material property definitions etc., used in current FEM analyses creates a genuine need for consistency [29]. Since the introduction of FEM into dental biomechanical research in 1973, the stress and strain fields in the alveolar support structures during orthodontic tooth movement have been analysed extensively [30–37]. Not much research has been done in simulating the orthodontic movement focusing on the properties of the orthodontic wire and its effect on supporting structures. In order to do that, the properties of wires made by different manufacturers should be available to enable simulation of the real life problem. The further simulations must be more accurate. Consequently, research work in the future should focus on the determination of the elastic modulus of different commercially available orthodontic wires, and friction coefficients between wires and brackets, as well as other properties that define orthodontic movement of the teeth. Contact points between single elements, especially between wire and bracket, should be constructed precisely. Contact points between wire and bracket are places of higher stress concentration and should be taken into consideration in determining the friction. Additionally, the force level produced by the fixed orthodontic appliance in different orthodontic cases of dental malposition should be measured and compared. With results obtained in such a manner, simulation would be more precise and efficient.

5. Conclusions

A 3D model can be used successfully for numerical simulation of modern orthodontic mechano-therapy. The initial orthodontic force produced by three different orthodontic wires at the beginning of the orthodontic treatment were quantified and qualified. Introduction of more variables into the future simulation of orthodontic treatment using FEM is needed for obtaining more accurate results.

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